A Delayed Choice Gedanken Experiment

M. W. Roberts

Abstract

A delayed choice experiment is proposed. Signal and idler photon pairs are sent to optical circulators. The fate experienced by an idler photon is described by two different cases. In case I, the idler photon has zero probability to reflect from the entrance beam splitter and therefore always enters its optical circulator. In case II, the idler photon has a non-zero probability to reflect from the entrance beam splitter without entering its optical circulator. Which case the idler photon actually experiences is selected by the method that is used to detect the signal photon of the pair. This is true, even if the detection of the signal photon occurs long after the detection of the idler photon.

## 1. Introduction

The original delayed choice experiment was discussed by Wheeler [1]. The gedanken experiment proposed herein employs non-local, "two-photon" interference like that described by Franson [2].

A representation of the experimental setup is shown in Figure 1. The setup is composed of a Source (Src), an idler photon path (IP), and a signal photon path (SP).

The Source (Src) contains a single-mode, continuous wave (cw) pump laser (LSR), a periodically-poled lithium niobate crystal (PPLN), a dichroic mirror (DM), a polarizing beam splitter (PBS1), and a beam stop (Stp).

The idler path (IP) contains optical circulator 2 (OC2) and detector D2. The signal path (SP) contains optical circulator 1 (OC1), a Pockels cell (PC), a polarizing beam splitter (PBS2), an unbalanced Mach-Zehnder interferometer (MZ), and four detectors (D1, D3, D4, and D5). All detectors are capable of photon counting.

1.1 Source (Src)
The PPLN is temperature-controlled, and is set to allow collinear, degenerate, type II spontaneous parametric down-conversion (SPDC) that creates a signal and idler pair of photons. The signal photon is horizontally (H) polarized, and the idler photon is vertically (V) polarized.

Short wavelength pump photons from laser LSR that are not down-converted in the PPLN are reflected at long-pass dichroic mirror DM and are incident on beam stop Stp.

Long wavelength signal and idler photons exit from the PPLN and are transmitted through DM to PBS1. The H polarized signal photons are transmitted through PBS1 and travel to OC1. The V polarized idler photons are reflected by PBS1 and travel to OC2.

## 1.2 Idler Path (IP)

OC2 consists of entrance amplitude beam splitter ABS2 and four mirrors. ABS2 may be a partially-silvered plate beam splitter. One of the four mirrors is moveable. This mirror allows the optical path length through the circulator to be controlled. The moveable mirror also allows for the adjustment of the phase of the idler photons at ABS2 after they travel through OC2.

The time required for an idler photon to make one cycle through OC2 is equal to X. The optical path length for one cycle through OC2 is equal to L.

The fixed time X should be much longer than the coherence time of an idler (or signal) photon [3]. Time X should also be much shorter than the coherence time of a pump photon from laser LSR. The coherence length of a pump photon should be greater than 100L (lasers with coherence length >50 meters are available).

Starting from PBS1, the time required for an idler photon to travel to ABS2, reflect from ABS2 without entering OC2, and then reach D2 is equal to τ2. Starting from PBS1, the time required for an idler photon to travel to ABS2, be transmitted through ABS2, travel N cycles through OC2, and then be transmitted through ABS2 to D2 is equal to (τ2 + NX).

## 1.3 Signal Path (SP)

OC1 consists of entrance amplitude beam splitter ABS1a, exit amplitude beam splitter ABS1b, and three mirrors. ABS1b is adjustable. Also, one of the mirrors is moveable. This mirror allows the optical path length through OC1 to be controlled. Fine adjustment of the moveable mirror along with ABS1b allows for the control of the phase of the signal photons at ABS1a and ABS1b after they travel through OC1.

The optical path length for one cycle through OC1 is set equal to L. This is the same optical path length for one cycle through OC2.

The MZ consists of amplitude beam splitters ABS3 and ABS4 and three mirrors. One of the mirrors is moveable. The unbalanced MZ provides a short path and a long path between ABS3 and ABS4 for the signal photons. The moveable mirror allows the optical path length difference between the long and short paths through the MZ to be controlled. This mirror also allows for the adjustment of

the phase difference of a signal photon between the two paths through the MZ.

The optical path length difference between the long path and the short path through the MZ is set equal to L. This is the same optical path length for one cycle through OC1. Consequently, as shown in Appendix 2, the signal photon may experience "one-photon" interference at the output from the MZ at ABS4.

Starting from PBS1, the time required for a signal photon to pass through both ABS1a and ABS1b without reflection, pass through PBS2, and then reach D3 is equal to τ3. Starting from PBS1, the time required for a signal photon to pass through both ABS1a and ABS1b without reflection, reflect from PBS2, travel through the short path of the MZ, and then reach either D4 or D5 is also equal to τ3.

Distances are adjusted so that τ3 > τ2 with (τ3 − τ2) = τ.


2. Addenda and Notation

The experiment considers those events in which the signal photon of a down-converted pair exits from OC1 via ABS1b and is detected in one of the detectors D3, D4, or D5. Events in which the signal photon is detected in detector D1 are ignored.

To simplify the description of this experiment, the effects of optical filters, detector quantum efficiency and dark counts, and other potential losses are not included in the following discussion.

In the following discussion and in the Appendices, both probability amplitude and probability will be calculated. As an example:

$$P[D2,D3;\Delta] = \left| pa[D2,D3;\Delta] \right|^2$$

In the above, pa[D2,D3;Δ] is the probability amplitude for the detection of the idler photon in D2 before the detection of the signal photon in D3 with time difference between detections equal to Δ. P[D2,D3;Δ] is the probability for the same detection events.

Both intensity and amplitude variables are used in the following. As an example, for amplitude beam splitter ABS2:

$$R_2 = \left| r_2 \right|^2 , \quad T_2 = \left| t_2 \right|^2 \quad \text{and} \quad R_2 + T_2 = 1$$

In the above, $R_2$ is the intensity reflectance, $T_2$ is the intensity transmittance, $r_2$ is the amplitude reflection coefficient, and $t_2$ is the amplitude transmission coefficient of ABS2.

For simplicity, the intensity and amplitude variables of amplitude beam splitters ABS1a and ABS1b are set equal:

$R_1 = |r_1|^2$ , $T_1 = |t_1|^2$  and  $R_1 + T_1 = 1$

### 3. Case I: Signal photon detection in D3

The signal photon of a down-converted pair reaches OC1 and ultimately exits from ABS1b. In case I, the PC is turned off, therefore the H polarized signal photon passes through the PC unchanged. It is then transmitted through PBS2 and is detected in D3. The V polarized idler photon of the pair reaches OC2 and is ultimately detected in D2.

As is shown in the Appendix 1, due to two-photon interference, the probability that the idler photon will be detected in D2 and then the signal photon will be detected in D3 with time difference between detections equal to τ is zero. Additionally, the probability that the idler and signal photons will be detected with a time difference greater than τ is also equal to zero.

In case I, the time difference between detections is always less than τ. The maximum time difference between the detection of the idler photon and the detection of the signal photon of a pair is equal to (τ − X).

The only way that the results described above can be realized is if the idler photon always enters and makes at least one cycle through OC2. In case I, the idler photon that travels from PBS1 to OC2 never initially reflects from ABS2 (for $R_2 < 1$). The idler photon is always initially transmitted through ABS2 into OC2.

### 4. Case II: Signal photon detection in D4 or D5

The signal photon of a down-converted pair reaches OC1 and ultimately exits from ABS1b. In case II, the PC is turned on, therefore the polarization direction of the signal photon is rotated to V polarized. The photon is reflected at PBS2 and travels to the MZ. The signal photon travels through the MZ and is detected in either D4 or D5. The V polarized idler photon of the pair reaches OC2 and is ultimately detected in D2.

As is shown in the Appendix 2, due to "one-photon" interference, there is only one possible path that allows the signal photon to be detected in D5. To be detected in D5, the signal photon must have passed through both ABS1a and ABS1b without reflection. It then must have travelled the short path through the MZ to D5. Starting from PBS1, the time required for the signal

photon to travel this path is equal to τ3. Two-photon interference does not occur in this instance.

In case II, the probability that the idler photon is detected in D2 and then the signal photon is detected in D5 with time difference between detections equal to τ is non-zero:

$$P[D2,D5;\Delta=\tau] = \left|pa[D2,D5;\Delta=\tau]\right|^2 = \left|-r_2 t_1^2 t_3 r_4\right|^2 = R_2 T_1^2 T_3 R_4$$

The only way that this result can be realized is if the idler photon travelled from PBS1, initially reflected from ABS2 without entering OC2, and was then detected in D2. Starting from PBS1, the time required for the idler photon to travel this path is equal to τ2. Note that (τ3 − τ2) = τ.

In case II, the detection of the idler photon of a down-converted pair in D2 and signal photon of the pair in D5 with time difference equal to τ is definitive evidence that the idler photon initially reflected from ABS2 without entering OC2. This result is forbidden in case I.

There are also instances in which the idler photon initially reflects from ABS2 without entering OC2 during events in which the signal photon is detected in D4. However, these events are not definitive, because P[D2,D4;Δ=τ] involves a composite of multiple times of detection rather than just a single time of detection.


5. Conclusion

Case I is selected by turning the PC off, allowing signal photons to reach D3. In this case, all idler photons will be initially transmitted through ABS2 and will always make at least one cycle through OC2. None of the idler photon will be initially reflected at ABS2.

Case II is selected by turning the PC on, allowing signal photons to travel through the MZ and reach either D4 or D5. In this case, some of the idler photons will be initially reflected at ABS2 and will not enter OC2.

These same experimental results occur even if the distance between optical circulator OC1 and Pockels cell PC is increased so that τ3 >> τ2. At this large separation, the detection of the idler photon of a down-converted pair happens long before both the case selection by the PC and the detection of the signal photon of the pair. The behavior of the idler photon is controlled by events that do not occur until long after the idler photon is annihilated in detector D2.

The fate of the idler photon (the effect) remains indeterminate, until the case selection and detection of the signal photon (the cause). The experimenter chooses which fate was

experienced by the idler photon in the past by choosing the method used to detect the signal photon in the present. Thus, the normal temporal order of causality is reversed.

The fact that the experimenter can choose the fate of the idler photon proves that the experimenter actually creates a past event, rather than simply reveals a past event.

The same experimental results also occur if the distance between PBS1 and OC2 is increased so that the detection of the idler photon, and the case selection and detection of the signal photon are space-like separated events [4].

Appendix 1: Case I [5]

The experimental distances are adjusted so that, starting from PBS1, the time required for a signal photon to pass through both ABS1a and ABS1b without reflection, pass through PBS2, and then reach D3 is equal to $\tau 3$.

Starting from PBS1, the time required for an idler photon to travel to ABS2, reflect from ABS2 without entering OC2, and then reach D2 is equal to $\tau 2$. Note: $\tau 3 > \tau 2$ and $(\tau 3 - \tau 2) = \tau$.

a) For detections in D2 and D3 with time difference, $\Delta = \tau$:
$$pa[D2,D3;\Delta=\tau] = [(ir_2)(t_1^2)]_{(\tau 2,\tau 3)} + [(-it_2^2)(t_1^2 r_1^2)]_{(\tau 2+X,\tau 3+X)}$$
$$+ [(-it_2^2 r_2)(t_1^2 r_1^4)]_{(\tau 2+2X,\tau 3+2X)} + \ldots =$$
$$[ir_2 t_1^2] - [(it_2^2 t_1^2 r_1^2)/(1-r_2 r_1^2)]$$
With $r_2 = r_1^2$, and using $t_2^2 = (1-r_2^2)$:
$$pa[D2,D3;\Delta=\tau] = [ir_2 t_1^2] - [(it_2^2 t_1^2 r_2)/(1-r_2^2)] = 0$$
$$P[D2,D3;\Delta=\tau] = \left| pa[D2,D3;\Delta=\tau] \right|^2 = 0$$

b) For detections in D2 and D3 with time difference, $\Delta > \tau$:
$$pa[D2,D3;\Delta=\tau+X] = [(ir_2)(t_1^2 r_1^2)]_{(\tau 2,\tau 3+X)} + [(-it_2^2)(t_1^2 r_1^4)]_{(\tau 2+X,\tau 3+2X)}$$
$$+ [(-it_2^2 r_2)(t_1^2 r_1^6)]_{(\tau 2+2X,\tau 3+3X)} + \ldots =$$
$$pa[D2,D3;\Delta=\tau] \cdot r_1^2 = 0$$
$$P[D2,D3;\Delta=\tau+X] = \left| pa[D2,D3;\Delta=\tau+X] \right|^2 = 0$$
$$pa[D2,D3;\Delta=\tau+NX] = pa[D2,D3;\Delta=\tau] \cdot r_1^{2N} = 0 , \text{ (integer N>0)}$$
$$P[D2,D3;\Delta=\tau+NX] = \left| pa[D2,D3;\Delta=\tau+NX] \right|^2 = 0$$

c) For detections in D2 and D3 with time difference, $\Delta < \tau$:
$$pa[D2,D3;\Delta=\tau-X] = [(-it_2^2)(t_1^2)]_{(\tau 2+X,\tau 3)} + [(-it_2^2 r_2)(t_1^2 r_1^2)]_{(\tau 2+2X,\tau 3+X)}$$
$$+ [(-it_2^2 r_2^2)(t_1^2 r_1^4)]_{(\tau 2+3X,\tau 3+2X)} + \ldots =$$
$$[(-it_2^2 t_1^2)/(1-r_2 r_1^2)] = -it_1^2$$
$$P[D2,D3;\Delta=\tau-X] = \left| pa[D2,D3;\Delta=\tau-X] \right|^2 = T_1^2$$
$$pa[D2,D3;\Delta=\tau-NX] = pa[D2,D3;\Delta=\tau-X] \cdot r_2^{(N-1)} = -it_1^2 r_2^{(N-1)}$$
$$P[D2,D3;\Delta=\tau-NX] = \left| pa[D2,D3;\Delta=\tau-NX] \right|^2 = T_1^2 R_2^{(N-1)} = T_1^2 R_1^{2(N-1)}$$

$$P[D2,D3] = T_1^2 \cdot (1 + R_1^2 + R_1^4 + \ldots) = T_1^2/(1-R_1^2) = T_1/(1+R_1)$$

In case I, due to "two-photon" interference between the signal and idler photons, $P[D2,D3;\Delta] = 0$ for all time differences ($\Delta$) that are greater than or equal to $\tau$. $P[D2,D3;\Delta]$ is non-zero only for time differences $\Delta < \tau$. Note that for non-zero probability, the minimum time required for the idler photon to reach D2 is $(\tau 2+X)$. Thus, the idler photon makes at least one cycle through OC2.

Appendix 2: Case II [5]

Starting from PBS1, the time required for a signal photon to pass through both ABS1a and ABS1b without reflection, reflect from PBS2, then travel via the short path through the MZ and reach either D4 or D5 is equal to $\tau 3$.

Starting from PBS1, the time required for an idler photon to travel to ABS2, reflect from ABS2 without entering OC2, and then reach D2 is equal to $\tau 2$. Note: $\tau 3 > \tau 2$ and $(\tau 3 - \tau 2) = \tau$.

a) For detections in D2 and D5 with time difference, $\Delta = \tau$:
$$pa[D2,D5;\Delta=\tau] = [(ir_2)(it_1^2 t_3 r_4)]_{(\tau 2, \tau 3)}$$
$$+ \{(-it_2^2)[(it_1^2 r_1^2 t_3 r_4)+(-it_1^2 r_3 r_4)]\}_{(\tau 2+X, \tau 3+X)}$$
$$+ \{(-it_2^2 r_2)[(it_1^2 r_1^4 t_3 r_4)+(-it_1^2 r_1^2 r_3 r_4)]\}_{(\tau 2+2X, \tau 3+2X)} + \ldots$$
With $r_1^2 t_3 r_4 = r_3 t_4$ :
$$(it_1^2 r_1^{(M+2)} t_3 r_4) - (it_1^2 r_1^M r_3 t_4) = 0$$

Due to one-photon "destructive" interference of the signal photon at ABS4, only the first term in $pa[D2,D5;\Delta=\tau]$ remains:
$$pa[D2,D5;\Delta=\tau] = -r_2 t_1^2 t_3 r_4$$
$$P[D2,D5;\Delta=\tau] = \left|pa[D2,D5;\Delta=\tau]\right|^2 = R_2\, T_1^2\, T_3\, R_4$$

Note that $P[D2,D5;\Delta=\tau]$ is non-zero, and the time required for the idler photon to reach D2 is $\tau 2$. Thus, the idler photon does not enter OC2.

b) For detections in D2 and D5 with time difference, $\Delta > \tau$:
$$pa[D2,D5;\Delta=\tau+X] = \{(ir_2)[(it_1^2 r_1^2 t_3 r_4)+(-it_1^2 r_3 t_4)]\}_{(\tau 2, \tau 3+X)}$$
$$+ \{(-it_2^2)[(it_1^2 r_1^4 t_3 r_4)+(-it_1^2 r_1^2 r_3 t_4)]\}_{(\tau 2+X, \tau 3+2X)} + \ldots = 0$$
$$P[D2,D5;\Delta=\tau+X] = \left|pa[D2,D5;\Delta=\tau+X]\right|^2 = 0$$
$$pa[D2,D5;\Delta=\tau+NX] = pa[D2,D5;\Delta=\tau+X] \cdot r_1^{2(N-1)} = 0 \text{ , (integer } N>0)$$
$$P[D2,D5;\Delta=\tau+NX] = \left|pa[D2,D5;\Delta=\tau+NX]\right|^2 = 0$$

c) For detections in D2 and D5 with time difference, $\Delta < \tau$:
$$pa[D2,D5;\Delta=\tau-X] = [(-it_2^2)(it_1^2 t_3 r_4)]_{(\tau 2+X, \tau 3)}$$
$$+ \{(-it_2^2 r_2)[(it_1^2 r_1^2 t_3 r_4)+(-it_1^2 r_3 t_4)]\}_{(\tau 2+2X, \tau 3+X)} + \ldots = t_2^2 t_1^2 t_3 r_4$$
$$P[D2,D5;\Delta=\tau-X] = \left|pa[D2,D5;\Delta=\tau-X]\right|^2 = T_2^2\, T_1^2\, T_3\, R_4$$
$$pa[D2,D5;\Delta=\tau-NX] = pa[D2,D5;\Delta=\tau-X] \cdot r_2^{(N-1)} = t_2^2 r_2^{(N-1)} t_1^2 t_3 r_4$$
$$P[D2,D5;\Delta=\tau-NX] = \left|pa[D2,D5;\Delta=\tau-NX]\right|^2 = T_2^2\, R_2^{(N-1)}\, T_1^2\, T_3\, R_4$$

$$P[D2,D5] = [R_2\, T_1^2\, T_3\, R_4] + [(T_2^2\, T_1^2\, T_3\, R_4)/(1-R_2)] = T_1^2\, T_3\, R_4$$

d) For detections in D2 and D4 with time difference, $\Delta = \tau$:
$$pa[D2,D4;\Delta=\tau] = [(ir_2)(t_1^2 t_3 t_4)]_{(\tau 2, \tau 3)}$$
$$+ \{(-it_2^2)[(t_1^2 r_1^2 t_3 t_4)+(t_1^2 r_3 t_4)]\}_{(\tau 2+X, \tau 3+X)}$$
$$+ \{(-it_2^2 r_2)[(t_1^2 r_1^4 t_3 t_4)+(t_1^2 r_1^2 r_3 t_4)]\}_{(\tau 2+2X, \tau 3+2X)} + \ldots =$$
$$[ir_2 t_1^2 t_3 t_4] - [(it_2^2 t_1^2 r_1^2 t_3 t_4)/(1-r_2 r_1^2)] - [(it_2^2 t_1^2 r_3 t_4)/(1-r_2 r_1^2)]$$

With $r_2 = r_1^2$, and using $t_2^2 = (1-r_2^2)$:

$pa[D2,D4;\Delta=\tau] = -it_1^2 r_3 r_4$

$P[D2,D4;\Delta=\tau] = \left|pa[D2,D4;\Delta=\tau]\right|^2 = T_1^2 R_3 R_4$

e) For detections in D2 and D4 with time difference, $\Delta > \tau$:

$pa[D2,D4;\Delta=\tau+X] = \{(ir_2)[(t_1^2 r_1^2 t_3 t_4)+(t_1^2 r_3 r_4)]\}_{(\tau 2,\tau 3+X)}$

$+ \{(-it_2^2)[(t_1^2 r_1^4 t_3 t_4)+(t_1^2 r_1^2 r_3 r_4)]\}_{(\tau 2+X,\tau 3+2X)}$

$+ \{(-it_2^2 r_2)[(t_1^2 r_1^6 t_3 t_4)+(t_1^2 r_1^4 r_3 r_4)]\}_{(\tau 2+2X,\tau 3+3X)} + \ldots = 0$

$P[D2,D4;\Delta=\tau+X] = \left|pa[D2,D4;\Delta=\tau+X]\right|^2 = 0$

$pa[D2,D4;\Delta=\tau+NX] = pa[D2,D4;\Delta=\tau+X] \cdot r_1^{2(N-1)} = 0$ , (integer N>0)

$P[D2,D4;\Delta=\tau+NX] = \left|pa[D2,D4;\Delta=\tau+NX]\right|^2 = 0$

f) For detections in D2 and D4 with time difference, $\Delta < \tau$:

$pa[D2,D4;\Delta=\tau-X] = [(-it_2^2)(t_1^2 t_3 t_4)]_{(\tau 2+X,\tau 3)}$

$+ \{(-it_2^2 r_2)[(t_1^2 r_1^2 t_3 t_4)+(t_1^2 r_3 r_4)]\}_{(\tau 2+2X,\tau 3+X)} + \ldots =$

$-it_1^2(t_3 t_4+r_2 r_3 r_4)$

$P[D2,D4;\Delta=\tau-X] = \left|pa[D2,D4;\Delta=\tau-X]\right|^2 = T_1^2[T_3 T_4+R_2 R_3 R_4+2R_3 T_4]$

$pa[D2,D4;\Delta=\tau-NX] = pa[D2,D4;\Delta=\tau-X] \cdot r_2^{(N-1)} = -ir_2^{(N-1)} t_1^2[t_3 t_4+r_2 r_3 r_4]$

$P[D2,D4;\Delta=\tau-NX] = \left|pa[D2,D4;\Delta=\tau-NX]\right|^2 = R_2^{(N-1)} T_1^2[T_3 T_4+R_2 R_3 R_4+2R_3 T_4]$

$P[D2,D4] = \{T_1^2 R_3 R_4\} + \{T_1^2(T_3 T_4+R_2 R_3 R_4+2R_3 T_4)/(1-R_2)\} =$

$[T_1^2(T_3 T_4+R_2 R_3 R_4+2R_3 T_4+T_2 R_3 R_4)]/T_2 =$

$[T_1^2(T_4+R_3)]/(1-R_2) = [T_1(T_4+R_3)]/(1+R_1)$


Appendix 3: Detection in D1 [5]

Events in which the signal photon is detected in D1 are included for completeness.

Starting from PBS1, the time required for a signal photon to travel to ABS1a, reflect from ABS1a without entering OC1, and then reach D1 is equal to $\tau 1$. Note that $\tau 1 \gg X$.

Starting from PBS1, the time required for an idler photon to travel to ABS2, reflect from ABS2 without entering OC2, and then reach D2 is equal to $\tau 2$. Note that $\tau 2 > \tau 1$ and $(\tau 2 - \tau 1) = \Theta$.

a) For detection in D1 and D2 with time difference, $\Delta = \Theta$:

$pa[D1,D2;\Delta=\Theta] = [(ir_1)(ir_2)]_{(\tau 1,\tau 2)} + [(-it_1^2 r_1)(-it_2^2)]_{(\tau 1+X,\tau 2+X)}$

$+ \{(-it_1^2 r_1^3)(-it_2^2 r_2)]_{(\tau 1+2X,\tau 2+2X)} + \ldots =$

$-[r_1 r_2]-[(r_1 t_1^2 t_2^2)/(1-r_1^2 r_2)]$

With $r_2 = r_1^2$, and using $t_2^2 = (1-r_2^2)$:

$pa[D1,D2;\Delta=\Theta] = -r_1$

$P[D1,D2;\Delta=\Theta] = \left|pa[D1,D2;\Delta=\Theta]\right|^2 = R_1$

b) For detection in D1 and D2 with time difference, $\Delta > \Theta$:

$pa[D1,D2;\Delta=\Theta+X]=[(ir_1)(-it_2{}^2)]_{(\tau1,\tau2+X)} + [(-it_1{}^2r_1)(-it_2{}^2r_2)]_{(\tau1+X,\tau2+2X)}$
$\qquad\qquad + [(-it_1{}^2r_1{}^3)(-it_2{}^2r_2{}^2)]_{(\tau1+2X,\tau2+3X)} + ... =$
$\qquad\qquad [r_1t_2{}^2]-[(r_1t_1{}^2t_2{}^2r_2)/(1-r_1{}^2r_2)] = r_1t_1{}^2$

$P[D1,D2;\Delta=\Theta+X] = |pa[D1,D2;\Delta=\Theta+X]|^2 = R_1T_1{}^2$

$pa[D1,D2;\Delta=\Theta+NX]=pa[D1,D2;\Delta=\Theta+X]\cdot r_2{}^{(N-1)}=r_1t_1{}^2r_2{}^{(N-1)}$ , (integer N>0)

$P[D1,D2;\Delta=\Theta+NX] = |pa[D1,D2;\Delta=\Theta+NX]|^2 = R_1T_1{}^2R_2{}^{(N-1)} = (R_1T_1{}^2)R_1{}^{2(N-1)}$

$P[D1,D2;\Delta>\Theta] = R_1T_1{}^2 \cdot (1 + R_1{}^2 + R_1{}^4 + ...) = R_1T_1/(1+R_1)$

c) For detection in D1 and D2 with time difference, $\Delta < \Theta$:

$pa[D1,D2;\Delta=\Theta-X]=[(-it_1{}^2r_1)(ir_2)]_{(\tau1+X,\tau2)} + [(-it_1{}^2r_1{}^3)(-it_2{}^2)]_{(\tau1+2X,\tau2+X)}$
$\qquad\qquad + [(-it_1{}^2r_1{}^5)(-it_2{}^2r_2)]_{(\tau1+3X,\tau2+2X)} + ... =$
$\qquad\qquad [r_1t_1{}^2r_2]-[(r_1{}^3t_1{}^2t_2{}^2)/(1-r_1{}^2r_2)] = 0$

$P[D1,D2;\Delta=\Theta-X] = |pa[D1,D2;\Delta=\Theta-X]|^2 = 0$

$pa[D1,D2;\Delta=\Theta-NX] = pa[D1,D2;\Delta=\Theta-X] \cdot r_1{}^{2(N-1)} = 0$

$P[D1,D2;\Delta=\Theta-NX] = |pa[D1,D2;\Delta=\Theta-NX]|^2 = 0$


$P[D1,D2] = R_1 + [R_1T_1/(1+R_1)]$


d) Summation:

$P[Case\ I] = P[D1,D2] + P[D2,D3] =$
$\qquad\qquad \{R_1 + [R_1T_1/(1+R_1)]\} + \{T_1/(1+R_1)\} = R_1 + T_1 = 1$


Using $R_2 = R_1{}^2$ and $R_1{}^2T_3R_4 = R_3T_4$:

$P[Case\ II] = P[D1,D2] + P[D2,D4] + P[D2,D5] =$
$\quad \{R_1 + [R_1T_1/(1+R_1)]\} + \{[T_1{}^2(T_4+R_3)]/(1-R_2)\} + \{T_1{}^2 T_3 R_4\} =$
$\quad \{R_1 + [R_1T_1/(1+R_1)]\} + \{[T_1(T_4+R_3+T_3R_4-R_3T_4)]/(1+R_1)\} = 1$

References

[1] J. A. Wheeler, <u>Mathematical Foundations of Quantum Theory</u>,
    A. R. Marlow (ed.), New York: Academic Press (1978).

[2] J. D. Franson, "Bell Inequality for Position and Time",
    <u>Physical Review Letters</u>, 62(19), 2205 (May 1989).

[3] C. K. Hong, Z. Y. Ou, and L. Mandel, "Measurement of
    Subpicosecond Time Intervals between Two Photons by
    Interference", <u>Physical Review Letters</u>, 59(18), 2044
    (Nov. 1987).

[4] T. Norsen, "Bell Locality and the Nonlocal Character of
    Nature", quant-ph/0601205, <u>Foundations of Physics Letters</u>,
    19(7), 633-655 (Dec. 2006).

[5] G. Weihs and A. Zeilinger, "Photon Statistics at Beam
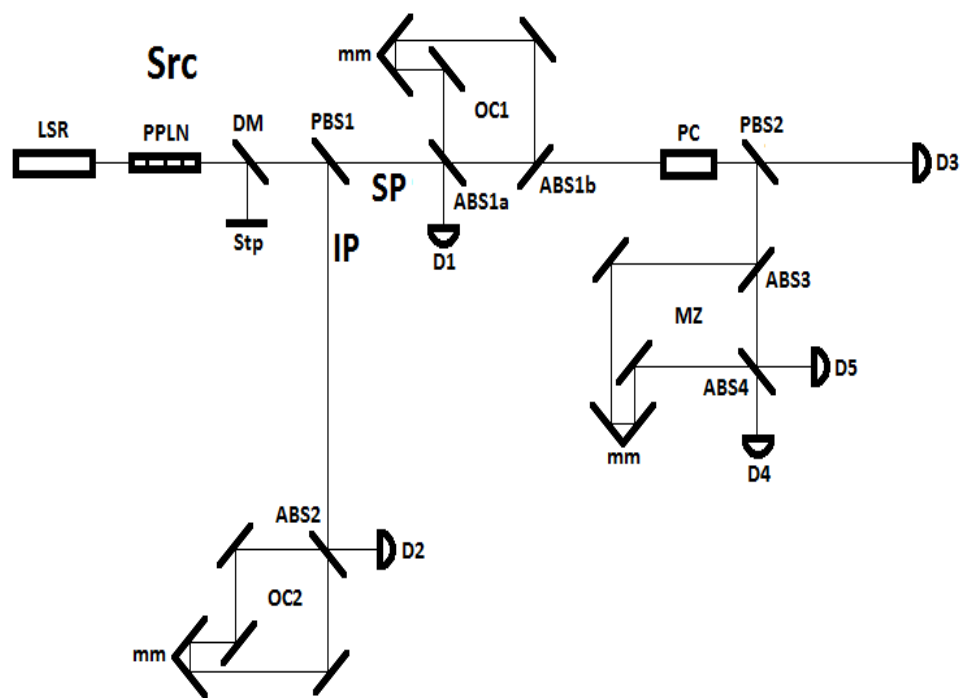    Splitters", <u>Coherence and Statistics of Photons and Atoms</u>,
    J. Perina (ed.), John Wiley & Sons, Inc. (2001).

**Figure 1: Experimental Setup**