Electron Spin and Rotating Vector Fields

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The nature of electron spin has presented an enigma right from the beginning of quantum mechanics. We suggest that a simple realistic picture of a real coherently rotating vector field can account for both the Schrödinger equation and electron spin in a consistent manner. Such a rotating field carries distributed angular momentum and energy in the same way as a circularly polarized electromagnetic wave. We derive the Schrödinger equation from the relativistic Klein-Gordon Equation, where the complex wave function maps onto a fixed-axis real rotating vector. Such a realistic picture can also explain the Stern-Gerlach experiment which first identified electron spin. Remarkably, the predictions of a two-stage Stern-Gerlach experiment within this realistic picture differ from those of the orthodox quantum superposition approach. This two-stage experiment has not actually been done, and could provide insights into the limits of realistic models. This realistic picture also avoids quantum paradoxes and enables realistic explanations for a variety of quantum phenomena.

1 Introduction

Quantum theory originated in the first decade of the 20th century [1], with the photon having a quantized energy $E = h\nu = \hbar\omega$ (following Planck in 1900 and Einstein in 1905), and the quantized electron energy of the Bohr atom in 1913. However, the foundational concepts of the more complete quantum theory were not developed until the 1920s. These key concepts were the matter waves [2] of de Broglie (1923), and the intrinsic electron spin [3], going back to the experiment of Stern and Gerlach in 1922 [4, 5], which was later explained following the 1925 concept of spin by Goudsmit and Uhlenbeck.

De Broglie derived matter waves from special relativity, by assuming that an electron in its rest frame is subject to both $E = mc^2$ and $E = \hbar\omega$. Transforming to a moving reference frame, one has the usual energy-momentum relation for a massive particle,

$$E^2 = (pc)^2 + (mc^2)^2, \tag{1}$$

and the equivalent dispersion relation for the associated wave,

$$\omega^2 = (kc)^2 + \omega_0^2, \tag{2}$$

where $\omega_0 = mc^2/\hbar$ is the characteristic angular frequency of the wave associated with the particle. Thus, a quantum wave has wavelength $\lambda = 2\pi/k = h/p$ in a moving reference frame. It is not always appreciated that quantum waves are intrinsically relativistic and have no classical limit.

Taking Eq. (2) together with a plane wave $\exp[\imath(\mathbf{k}\cdot\mathbf{r} - \omega t)]$ leads directly to a differential equation known as the Klein-Gordon Equation (1926) [6].

$$\partial^2\Psi/\partial t^2 = c^2\nabla^2\Psi - \omega_0^2\Psi \tag{3}$$

This is generally identified as a complex scalar wave, though as we show later, it makes more sense as a real vector wave. However, a much more influential equation was developed by Schrödinger (1927) [7], who derived it from the non-relativistic energy relation $E = p^2/2m + V(\mathbf{r})$, together with $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k}$.

$$\iota\hbar\partial\Psi/\partial t = -(\hbar^2/2m)\nabla^2\Psi + V(\mathbf{r})\Psi \qquad (4)$$

This is still based on relativistic quantum waves, but in the limit where both $p^2/2m$ and $V(\mathbf{r})$ are much less than mc^2, which is generally the case for atomic physics. Its relativistic origin is hidden by the absence of c.

Note that Eq. (4) is a complex equation with complex solutions. In the steady-state case (constant total energy E), the solution is

$$\Psi(\mathbf{r},t) = \psi(\mathbf{r})\exp(-\iota\omega t/\hbar), \qquad (5)$$

where $E = \hbar\omega$, and $\psi(\mathbf{r})$ is the solution to the time-independent Schrödinger equation.

$$E\,\psi(\mathbf{r}) = -(\hbar^2/2m)\boldsymbol{\nabla}^2\psi(\mathbf{r}) + V(\mathbf{r})\,\psi(\mathbf{r}) \qquad (6)$$

Consider a solution $\psi(\mathbf{r}) \propto \exp[\iota\phi(\mathbf{r})]$. A gradient in the phase ϕ corresponds to a current, and in steady state, all currents must form closed loops. Since the phase factor $\exp[\iota\phi(\mathbf{r})]$ is periodic in $\Delta\phi = 2\pi$ radians, the phase change around such a closed loop must be $2\pi n$, where n is an integer, corresponding to n wavelengths around the loop. For a circular loop of circumference $2\pi r$, $\lambda = 2\pi r/n$. Then the angular momentum around the loop is $\mathbf{L} = \mathbf{r} \times \mathbf{p} = r\,(h/\lambda) = n\hbar$. This is true more generally for any closed loop; the "orbital angular momentum" around a loop is quantized in units of $\hbar$. For an electron, this constitutes a circulating electrical current, which produces a magnetic moment.

In addition to orbital angular momentum $L = n\hbar$, an electron always has a spin angular momentum $S = \pm\hbar/2$ (known as "spin one-half"). This also corresponds to a magnetic moment, but there is no circulating current from the Schrödinger equation. The electron's spin can be "flipped", but cannot be increased or decreased. In the Stern-Gerlach experiment (Figure 1) a magnetic field gradient was applied to a beam of univalent atoms. Each atom in that experiment had a single unpaired electron, but no orbital angular momentum. The beam separated into two sub-beams, corresponding to the two discrete values of electron spin. The results of the Stern-Gerlach experiment proved the existence of quantized spin, but did not explain the physical basis or origin of spin.

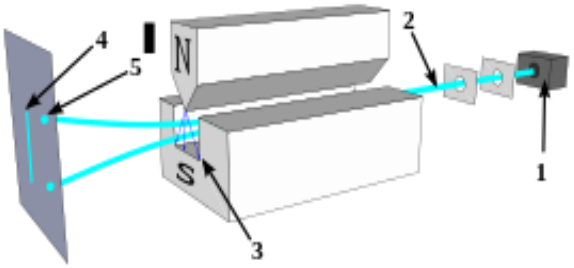

Figure 1: *Conceptual diagram of Stern-Gerlach experiment, where a beam of univalent atoms is separated into two beams in a magnetic field gradient (from [5]).*

The Dirac equation (1928) [8] incorporated spin into a fully relativistic wave equation, but with a complex mathematical formalism allowing for little physical intuition.

The physical interpretations of the de Broglie wave and the spin were never combined in a clear, unified manner. Part of the difficulty was based on "wave-particle duality", whereby it was believed that an electron is both a point particle and a distributed wave. A point particle cannot rotate to carry angular momentum, and, the spin angular momentum was found to be incompatible with any kind of solid-body rotation. In the end, spin was treated as a property completely separate from the de Broglie wave, with only the complex mathematical formalism to guide the physics.

We take the opposite viewpoint: that a simple realistic unified physical picture must guide the foundations of quantum mechanics. Remarkably, we have identified such a simple picture, which was apparently never proposed during the early years of quantum mechanics. The key is the recognition that a real rotating vector field can carry angular momentum, and can also be described mathematically by the complex, scalar Schrödinger equation. Fig. 2 represents a single electron, with a real distributed coherent wave packet having total quantized spin, rather than a probability distribution of point particles. Implications of this picture lead to a novel, realistic approach to quantum measurement, which is amenable to experimental testing. Parts of this analysis have appeared earlier [9-13], but this approach is considered heretical, and has not been permitted in any standard physics journal. We believe that the present paper presents a strong case that the foundations of quantum mechanics can be updated to conform to a consistent, realistic picture of nature.

In Section 2, we review the mathematics of classical rotating vectors, and show that quantized angular momentum follows naturally from the properties of polarized electromagnetic waves.

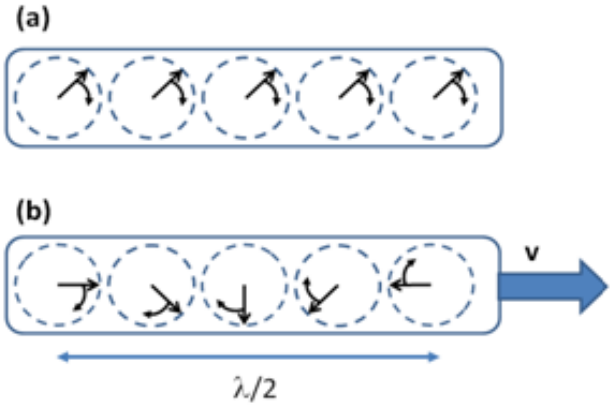

Figure 2: *An alternative physical picture of an electron quantum wave, comprising a rotating vector field carrying spin, distributed over a volume. (a) Electron in rest state, with all regions rotating in phase. (b) A moving electron, with a phase gradient in accordance with the Lorentz transformation.*

In Section 3 we show how a real vector Klein-Gordon equation is the proper equation to describe de Broglie waves of electrons, and how solving this for steady rotation leads to the usual complex scalar Schrödinger equation.

In Section 4 we show how the Stern-Gerlach experiment demonstrating electron spin may be understood with and without quantum superposition. Furthermore, we show how these two approaches can be applied to a two-stage Stern-Gerlach experiment. Remarkably, the predicted results for the two-stage experiment without superposition are strikingly different from those predicted by orthodox quantum mechanics. This experiment could have been done decades ago, and can now be easily done using modern technology.

The further implications of this realistic picture for a variety of problems in quantum theory are discussed in Section 5.

2 Mathematics of Classical Rotating Vector Fields

Steady circular motion [14] is ubiquitous in classical physics. It is present in circular orbits, solid-body rotation, and polarized electromagnetic waves. In each of these cases, the mathematics of the rotating vectors permit significant simplification, by eliminating the time-dependence from the problem.

Consider, for example, a circular orbit of radius r_0 around a fixed point, with the rotation in the x-y plane, at an angular frequency ω in the z-direction. The general solution takes the form

$$\mathbf{r} = r_0 [\hat{\mathbf{x}} \cos(\omega t+\theta_0) \pm \hat{\mathbf{y}} \sin(\omega t+\theta_0)], \quad (7)$$

where the plus sign corresponds to counter-clockwise rotation (from the $+z$-axis) and the minus sign to clockwise rotation. The particle velocity $\mathbf{v} = d\mathbf{r}/dt$ obeys the following relation, taking $\boldsymbol{\omega} = \pm \omega\hat{\mathbf{z}}$:

$$d\mathbf{r}/dt = \boldsymbol{\omega} \times \mathbf{r}, \quad (8)$$

as can be demonstrated by direct substitution. This relation is valid for any uniformly rotating vector around any axis. One can also take a second derivative, which of course is the acceleration:

$$d^2\mathbf{r}/dt^2 = \boldsymbol{\omega} \times(\boldsymbol{\omega} \times \mathbf{r}) = \boldsymbol{\omega} (\boldsymbol{\omega}\cdot\mathbf{r}) - \mathbf{r} (\boldsymbol{\omega}\cdot\boldsymbol{\omega}) = -\omega^2\mathbf{r}, \quad (9)$$

using a standard vector identity. This reproduces the centripetal acceleration associated with circular motion.

While we already know the solution to this real vector differential equation, one can treat this as a complex *scalar* differential equation, with solutions $\tilde{r} = r_0 \exp[\pm\imath(\omega t+\theta_0)]$. This describes vector rotation in the complex plane. To obtain the x- and y-components in real space (Eq. 7), just take the real and imaginary parts of this complex solution.

Another classical example of a rotating vector field is the electric field in the case of circular polarization [15]. The differential equation for an electromagnetic wave in free space, from Maxwell's equations, takes the form

$$\partial^2\mathbf{E}(\mathbf{r},t)/\partial t^2 = c^2 \nabla^2 \mathbf{E}(\mathbf{r},t) \quad (10)$$

If we are looking for rotating steady-state solutions, we can apply the same transformation as in Eq. (9), to eliminate the time dependence and obtain

$$-\omega^2\mathbf{E}(\mathbf{r}) = c^2 \nabla^2 \mathbf{E}(\mathbf{r}). \quad (11)$$

If the wave is propagating in the z-direction, and is uniform in the x- and y-directions, Eq. (11) becomes

$$\partial^2\mathbf{E}(z)/\partial z^2 = -k^2 \mathbf{E}(z), \quad (12)$$

where $k = \omega/c$. We can treat this real vector $\mathbf{E}$ as a complex scalar $\tilde{E}$ to solve for

$$\tilde{E} = E_0 \exp(\pm\imath kz);$$
$$E_x = Re(\tilde{E}) = E_0 \cos(kz);$$
$$E_y = Im(\tilde{E}) = \pm E_0 \sin(kz). \quad (13)$$

We can also reincorporate the time dependence:

$$\tilde{E}(\mathbf{r},t) = E_0 \exp(\imath\omega t) \exp(\pm\imath kz);$$
$$E_x = Re(\tilde{E}) = E_0 \cos(\omega t\pm kz);$$
$$E_y = Im(\tilde{E}) = E_0 \sin(\omega t\pm kz). \quad (14)$$

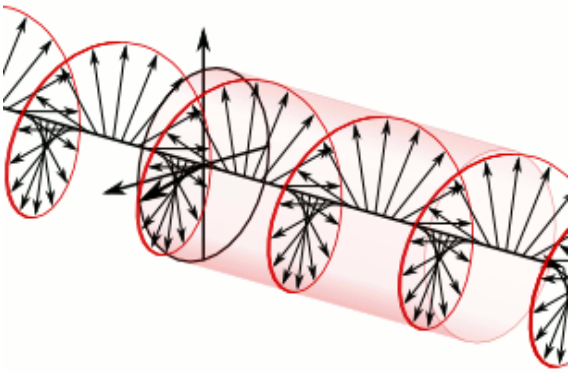

Figure 3: *Snapshot of the rotating electric field vector in a circularly polarized electromagnetic wave. This wave carries not only energy and momentum, but also angular momentum (from [15]).*

This corresponds to the two directions of circular polarization, with helical vectors propagating at the speed of light (see Figure 3). The rotation axis is always in the same direction as the propagation direction.

The reason that we are focusing on circularly polarized electromagnetic waves is that these provide a model for rotating vector fields in quantum theory. A key difference, of course, is that EM waves always move at the speed of light, so that one cannot transform to a rest frame. With de Broglie waves, on the other hand, one *can* transform to the rest frame, as indicated in Figure 2.

It is well known that electromagnetic waves carry energy and momentum distributed across the wave, via the properties of the Poynting vector $\mathbf{E} \times \mathbf{H}$. For example, the energy density is $\mathcal{E} = |\mathbf{E} \times \mathbf{H}|/c = \varepsilon_0 E^2$. It is equally well established, although perhaps not as widely known, that a circularly polarized (CP) wave also carries distributed angular momentum associated with the rotating vector field, following from Maxwell's equations [16,17]. The angular momentum density of a wave with angular frequency ω is $\mathcal{L} = (\mathbf{E} \times \mathbf{A})/\mu_0 c^2 = \varepsilon_0 E^2/\omega$ (where $\mathbf{A}$ is the magnetic vector potential), so that these are related by $\mathcal{L} = \mathcal{E}/\omega$.

Now consider a CP wavepacket where the total angular momentum $\mathbf{L}$ integrated over the volume of the wavepacket is $L = \hbar$. The total energy of this wavepacket must then be $E = \hbar\omega$. This suggests that a photon is properly a CP distributed wavepacket (with spin $\hbar$), rather than a point particle as is often asserted. Further, a CP wavepacket with $L = n\hbar$ corresponds to a coherent n-photon state with $E = n\hbar\omega$, rather than a collection of discrete single-photon states.

More generally, we propose that the primary basis for quantum mechanics is quantization of spin for fundamental entities such as the photon and the electron, as discussed below. Note also that angular momentum is one of the few direct physical quantities that is Lorentz-invariant; the spin of a photon or an electron is the same in any reference frame. Our present model does not explain the mechanism for spin quantization, but embeds it in a model of classical relativistic rotating vector fields.

3 Deriving the Schrödinger Equation from the Vector Klein-Gordon Equation

Let us assume that a de Broglie wave in its rest frame consists of a distributed coherent vector field, rotating with angular velocity $\omega = mc^2/\hbar$ about a fixed axis. This will be described by the Klein-Gordon Equation

$$\partial^2\mathbf{\Psi}/\partial t^2 = c^2\nabla^2\mathbf{\Psi} - \omega_0{}^2\mathbf{\Psi}, \qquad (15)$$

where now $\mathbf{\Psi}(\mathbf{r},t)$ is a real vector rather than a complex scalar. If we substitute $-\omega^2\boldsymbol{\psi}(\mathbf{r})$ for $\partial^2\mathbf{\Psi}/\partial t^2$ in Eq. (15) to eliminate the time dependence (just as in going from Eq. (10) to Eq. (11) above), we have

$$c^2\nabla^2\boldsymbol{\psi} = (\omega_0{}^2 - \omega^2)\boldsymbol{\psi} \qquad (16)$$

Consider first the case of a free electron, with no potential energy $V(\mathbf{r})$. The rest frame corresponds to a uniform field with $\omega = \omega_0$, with solution $\boldsymbol{\psi} = \boldsymbol{\psi_0}$ = constant. Selecting the z-axis as the spin axis, we can select $\boldsymbol{\psi_0}$ in the x-direction. This represents a uniform coherently rotating vector field:

$$\tilde{\psi} = \psi_0 \exp(\pm\imath\omega_0 t);$$
$$\psi_x = Re(\tilde{\psi}) = \psi_0 \cos(\omega_0 t);$$
$$\psi_y = Im(\tilde{\psi}) = \pm\, \psi_0 \sin(\omega_0 t), \qquad (17)$$

where the $\pm$ represent the two senses of circular polarization. Note that $\omega_0/2\pi = mc^2/h = 10^{20}$ Hz for an electron, which is the minimum frequency of an electron wave. The solution for a larger frequency is $\tilde{\psi} = \exp(\imath\mathbf{k}\cdot\mathbf{r})$, where $k = (\omega^2\text{-}\omega_0{}^2)^{0.5}/c$. This corresponds to a phase lag in the direction of motion, with a wavelength $\lambda = 2\pi/k = h/p$, which is what we expect for a de Broglie wave. One can obtain the same result by taking a Lorentz transform of the solution in the rest frame, since Eq. (15) is Lorentz covariant. The phase lag corresponds to a time shift; events that are simultaneous in the rest frame are not simultaneous in a moving reference frame.

Unlike a photon, for which the spin axis is always aligned with the propagation direction, the spin axis for an electron is aligned with a local magnetic field **B**, and may be independent of the direction of motion. Given the magnetic moment $\boldsymbol{\mu}$ of the electron (the Bohr magneton [18]), its energy is shifted by $-\boldsymbol{\mu}\cdot\mathbf{B}$. This is a very small energy shift, given the value of $\mu = e\hbar/2m = 10^{-23}$ J/T. For a field of 1 T, this is equivalent to 10^{-4} eV, as compared to the rest energy of 500 keV, atomic energies of order 1 eV, and thermal energies $kT \sim 25$ meV at room temperature. For spin one-half, only two stable states are available: the ground state with $\boldsymbol{\mu}$ parallel to **B**, and the excited state with $\boldsymbol{\mu}$ anti-parallel to **B**.

In the case where there is a rapidly varying potential, as near an atom, a potential energy $V(\mathbf{r})$ may be included in the rest energy of the electron:

$$\hbar\omega_0 = mc^2 = m_0c^2 + V(\mathbf{r}), \tag{18}$$

where m_0 is the rest mass in the absence of a potential. Potential energies associated with a bound atomic state are negative, thus reducing the rest mass slightly. In the usual case where the potential energy is much smaller than the rest energy, Eq. (16) becomes:

$$\begin{aligned} c^2\nabla^2\boldsymbol{\psi} &= (\omega_0^2 - \omega^2)\boldsymbol{\psi} \\ &= (\omega_0 - \omega)(\omega_0 + \omega)\boldsymbol{\psi} \\ &\approx 2\omega_0(\omega_0 - \omega)\,\boldsymbol{\psi} \end{aligned} \tag{19}$$

This can be rearranged to yield:

$$(-\hbar^2/2m)\nabla^2\boldsymbol{\psi} + [m_0c^2 + V(\mathrm{r})]\,\boldsymbol{\psi} = \mathrm{E}\,\boldsymbol{\psi}, \tag{20}$$

where $E = \hbar\omega$ is the electron energy that includes the full rest energy. If one suppresses the rest energy, one obtains an equation that looks just like the usual time-independent Schrödinger equation. However, Eq. (20) is actually the equation for a real rotating vector field, rather than a complex scalar field. If one rewrites this in terms of the complex equivalent $\tilde{\psi}$ and the offset energy $E' = E - m_0c^2$, one obtains the usual time-independent Schrödinger equation

$$(-\hbar^2/2m)\nabla^2\tilde{\psi} + V(\mathbf{r})\,\tilde{\psi} = E'\tilde{\psi} \tag{21}$$

The solution can be written in the form

$$\tilde{\psi} = \psi_0(\mathbf{r})\exp[\iota\theta(\mathbf{r})], \tag{22}$$

and the time-dependence can be restored to yield

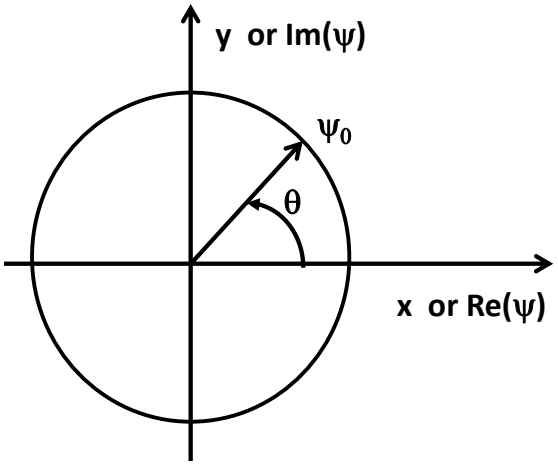

Figure 4: *Solution to Schrodinger equation, representing either a complex number $\tilde{\psi} = \psi_0\exp(\iota\theta)$, rotating in the complex plane, or a real vector field $\boldsymbol{\psi} = \psi_0(\hat{\mathbf{x}}\cos\theta + \hat{\mathbf{y}}\sin\theta)$, rotating (in either sense) about a spin axis.*

$$\begin{aligned} \tilde{\Psi}(\mathbf{r},t) &= \tilde{\psi}(\mathbf{r})\exp(\mp\iota\omega t) \\ &= \psi_0(\mathbf{r})\exp\{\iota[\theta(\mathbf{r}) \mp \omega t]\}, \end{aligned} \tag{23}$$

corresponding in turn to the following differential equation:

$$\pm\iota\hbar\partial\tilde{\Psi}/\partial t = -(\hbar^2/2m)\nabla^2\tilde{\Psi} + [V(\mathbf{r}) + m_0c^2]\tilde{\Psi} \tag{24}$$

We can also write the vector solution, as in Eq. (14):

$$\begin{aligned} \Psi_x &= Re(\tilde{\Psi}) = \psi_0(\mathbf{r})\cos[\theta(\mathbf{r}) \mp \omega t]; \\ \Psi_y &= Im(\tilde{\Psi}) = \psi_0(\mathbf{r})\sin[\theta(\mathbf{r}) \mp \omega t]. \end{aligned} \tag{25}$$

This is a rotating vector pointing in a real direction in space (see Figure 4), but the mathematics are equivalent to a rotation in the complex plane. Both signs ± are needed, as they correspond to both circular polarizations with opposite spins.

The usual time-dependent Schrödinger equation has only one sign, since it is generally believed to correspond to a zero-spin quantum particle. Ironically, a perceived shortcoming of the scalar Klein-Gordon equation is that it has solutions for both positive and negative ω. We view this as reflecting the physical basis of two polarized vector solutions.

4 Quantum Superposition and the Stern-Gerlach Experiment

Let us address briefly the question of the interpretation of the quantum wavefunction [19]. Schrödinger believed that this was a real physical wave, and we agree.

De Broglie considered that the wave guides the motion of a point particle, and that both wave and particle are physical objects; this is the pilot-wave interpretation that was also promoted by Bohm. In the orthodox Copenhagen interpretation, on the other hand, the wavefunction is a mathematical object that represents a statistical distribution of point particles in an ensemble of otherwise identical events. We question the statistical interpretation, and promote a realistic waves-only interpretation without point particles. This electron wave is a rotating vector field that carries spin (like an electromagnetic wave), but apart from spin quantization this is otherwise essentially classical. This realistic picture has experimentally testable implications, as we discuss further below.

We have identified two real rotating solutions of the vector Klein-Gordon equation, which we can call $\Psi_\uparrow$ and $\Psi_\downarrow$, corresponding to spin $+\hbar/2$ and $-\hbar/2$. Since this is a linear equation, any linear combination of these two will also be a solution. For the equivalent complex solutions,

$$\tilde{\Psi} = c_\uparrow \tilde{\Psi}_\uparrow + c_\downarrow \tilde{\Psi}_\downarrow, \qquad (26)$$

where $c_\uparrow$ and $c_\downarrow$ are complex numbers that represent amplitudes and phase shifts. However, not all of these solutions to the equation are physically accessible. In particular, by the Pauli exclusion principle [20], one can have only 1 electron in a given state; neither ½ an electron or 2 electrons in one of the states for the same location is permissible. Furthermore, the only acceptable 2-electron solution is one with opposite spins. We represent these by the points at (1,0), (0,1), and (1,1) on the plot in Figure 5.

However, in the orthodox quantum theory, a single-electron superposition state is also possible [21], such that $|c_\uparrow|^2 + |c_\downarrow|^2 = 1$, as indicated by the circular arc in Figure 5. This is compatible with spin quantization, since $|c_\uparrow|^2$ and $|c_\downarrow|^2$ represent the statistical probabilities of measuring the two spin states in a measurement on one of these superposition states. Furthermore, a 2-electron state is in an entangled superposition of anti-correlated single-electron states, which cannot be fully represented on Figure 5.

In contrast, we would like to suggest that only the two spin-quantized rotating solutions are accessible, and that the superposition states of Eq. (26) do not exist for a single electron. A two-electron state may be a superposition of $\Psi_\uparrow$ and $\Psi_\downarrow$, but it is not entangled. Therefore, we need a different mechanism to explain the Pauli exclusion principle. Our explanation relies on the concept of a soliton [22] in a nonlinear differential equation. A soliton (also called a "solitary wave") is a localized wave packet of a particular amplitude that maintains its integrity as it moves, acting like a particle moving in a linear medium.

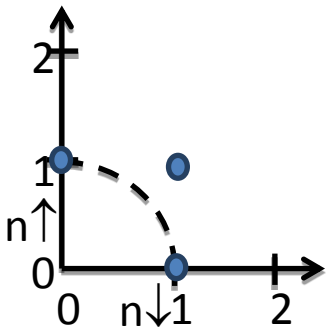

Figure 5: *Physically accessible states in the 2D-space of spin states $\Psi_\uparrow$ and $\Psi_\downarrow$. The dots show the single-electron states $\Psi_\uparrow$ and $\Psi_\downarrow$, and the 2-electron state $\Psi_\uparrow + \Psi_\downarrow$. The dashed circular arc shows superposition states based on Eq. (26), which are not permitted in the soliton-based picture.*

A soliton will not disperse or split apart, and will repel another soliton. This behavior sounds very much like what is required by the Pauli principle. The Schrödinger equation, being linear, cannot provide such solitons, so we suggest that it is incomplete. The complete nonlinear treatment must result in an equation that will generate solitons such that when the spin is $\hbar/2$, the nonlinearities cancel out and the usual linear Schrödinger equation will emerge, describing the wave dynamics. We have not yet identified the appropriate nonlinear equation, but there has been research into the "nonlinear Schrödinger equation" to model soliton-like behavior in optics and other classical systems [23].

The Stern-Gerlach experiment provided the first experimental evidence of electron spin, and its interpretation is based on the orthodox quantum superposition approach. This is a good model system to investigate these issues, and even to do experiments to test these foundations. The experiment actually measures the magnetic moment of atoms rather than single electrons, but the effect is the same. A single electron has spin $\pm\hbar/2$, but the dynamics of a single electron in free space are dominated by its charge; it will tend to follow spiral orbits in a magnetic field, with the effect of the spin being much smaller. In order to measure the spin of an electron, one needs to embed an electron in an appropriate neutral atom. Hydrogen has one electron, but hydrogen forms diatomic molecules where the net spin cancels out. So we must deal with atoms having many electrons.

While all electronic states in atoms have spin angular momentum, electrons tend to fill states of both spins, cancelling out the total spin for an even number of electrons. Some atomic states have zero orbital angular momentum, while others have states with orbital angular momentum $n\hbar$, which corresponds to $\theta(r)$ changing by $2\pi n$

in going around the nucleus. This orbital angular momentum is in the same direction as the spin angular momentum, but the total cancels out for a filled atomic shell. So a filled shell, such as that for a noble gas atom such as Xe, has zero electronic magnetic moment.

Also, atomic nuclei also have spin, but their magnetic moments, on the order of the nuclear magneton $e\hbar/2m_n$, are thousands of times smaller than that of the electron (due to the larger nucleon mass m_n), and are therefore negligible compared to the electronic magnetic moment.

So if one considers a univalent atom, such as an alkali metal or one from the Cu/Ag/Au column in the periodic table, the angular momentum and magnetic moment are essentially equivalent to those of a single electron, but the electric charge can be ignored in the dynamics.

Consider a vacuum chamber with a furnace that heats up such a univalent metal (such as Ag in the original experiment) to create a gas of single atoms. Even without a deliberate magnetic field, the earth's field ~ 50 μT creates the 2 spin states, $\Psi_\uparrow$ and $\Psi_\downarrow$ aligning with the field. One of these is the ground state and the other the excited state, but the energy difference 2μB is much less than kT, so that an equilibrium population will have virtually 50% in each state. If there is a hole in the side of the oven, this creates an atomic beam of atoms, of both spins. As shown in the conceptual diagram of Figure 1, this beam enters a large magnetic field, with a vertical field gradient. This gradient acts to separate the beam into two sub-beams; one going up to smaller fields, and the other going down to larger fields. The magnetic field can be removed, and the atoms continue to follow the separate paths, where the two sub-beams are detected.

The argument for classical magnetic separation reflects the fact that a magnetic material will be attracted to a strong magnetic field, since this reduces the energy. The ground state of an electron spin, where the electron magnetic moment aligns with the external field, is of this type. In contrast, a diamagnetic material (or a superconductor) will repel a strong magnetic field, since this increases the energy. The excited spin state is effectively diamagnetic.

The trends in energy levels can be seen in the energy level diagram of Figure 6. This shows how the two energy levels in the ground and excited states change as they move from the oven into the magnet, and how their energy changes in the magnet due to transverse motion. The initial energy differences in the oven are quite small, and the two mixed sub-beams stream out toward the magnet with thermal kinetic energies ~ kT. The ground-state sub-beam reduces its potential energy as it enters the large magnetic field, thus accelerating its velocity to maintain constant total energy. The excited-state sub- beam

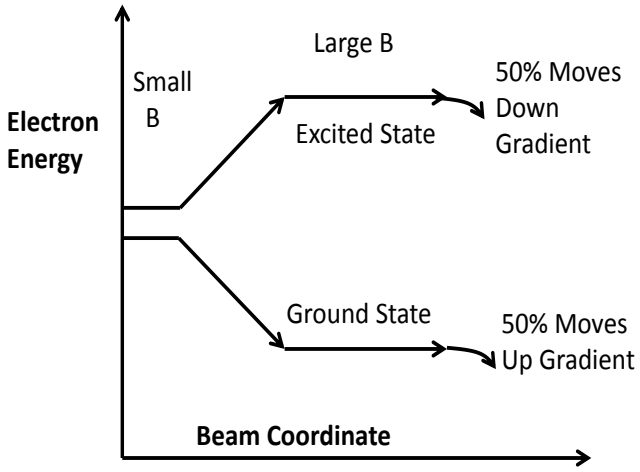

Figure 6: *Trends in ground and excited energy levels in single-stage Stern-Gerlach experiment, moving from the oven (left) to the inhomogeneous magnet (right).*

increases its potential energy as it enters the large magnetic field, requiring the atoms in the beam to slow down. But since the changes in potential energy are much smaller than kT, this change in speed will not be noticed. However, this large magnetic field is vertically inhomogeneous, with increasing field in the vertical direction. The ground-state sub-beam will bend upwards, attracted to the increasing field, while the excited state sub-beam will bend downwards, repelled by the larger field. These changes in direction will be maintained if the fields are reduced, permitting a separation into the two sub-beams.

However, this classical explanation is not quite the one presented in textbooks for the Stern-Gerlach experiment. Instead, it is asserted that this is a prototypical quantum measurement [24]. The initial beam, rather than being a classical mixture of atoms with $\Psi_\uparrow$ and $\Psi_\downarrow$, is a linear superposition of the two as in Eq. (26). Then, the inhomogeneous magnetic field in the magnet acts somehow to decohere this superposition into a statistical mixture, at which point classical magnetic separation can occur. This gives exactly the same final result, so how can we tell which is correct? And if we can't tell, does it matter?

But the single-stage Stern-Gerlach experiment is not the only experiment of this type that is described in textbooks. The two-stage Stern-Gerlach experiment [25] is also described, in a way that suggests that this experiment was also carried out in the early days of quantum mechanics. In the two-stage experiment, one of the two separated beams from the first stage is sent to a second gradient magnet, this one with its transverse field direction rotated

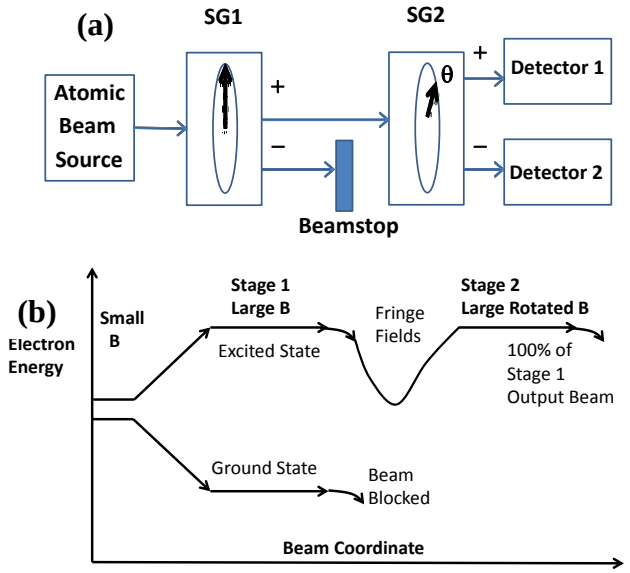

Figure 7: *Two-stage Stern-Gerlach experiment. (a) Experimental diagram. (b) Energy level diagram.*

by an angle θ with respect to that of the first magnet (see Figure 7a).

This is analogous to the double-polarizer optics experiment, with a similar quantum analysis, but the two-stage spin experiment has never actually been reported. Feynman [25] commented that the 2-stage experiment was never done, but others neglect to mention this. In our interpretation of this experiment [26], indicated in Figure 7b, the excited beam from the first stage remains 100% excited in the 2nd stage, regardless of the angle θ. This happens because the spin of the electrons in the excited beam rotate (adiabatically) in the fringe fields to follow the local magnetic field. In contrast, in the orthodox superposition picture, the electrons in the excited beam form a new superposition state

$$\tilde{\Psi}_2 = \cos\theta\, \tilde{\Psi}_\uparrow + \sin\theta\, \tilde{\Psi}_\downarrow, \qquad (27)$$

which gives rise to a 2nd beam splitting that goes as $\cos^2\theta$ and $\sin^2\theta$, similar to that which happens in the optical linear polarizer experiment. This prediction is so strongly believed that a high-quality computer animation has been developed to teach students about this [27]. However, we see little basis for the formation of such a superposition state.

We propose that this 2-stage Stern-Gerlach experiment should be carried out as soon as possible. Instrumentation for the 1-stage Stern-Gerlach experiment designed for student laboratories is available commercially [28], and could be adapted for the 2-stage experiment.

This would make a good student project, and the results could be revolutionary. We would be happy to consult on such an effort.

5 Discussion and Conclusions

We have focused on the importance of spin and the Schrödinger equation to quantum mechanics, but Schrödinger himself is also well known for another aspect – his objection to quantum superposition and entanglement [29,1], which he was the first to name. Einstein was another skeptic of entanglement – he called it "spooky action at a distance" and questioned its consistency with relativity. We suggest that entanglement first entered the theory to explain the Pauli exclusion principle in 1925, although this aspect was not appreciated at the time. In 1935, Schrödinger proposed a thought-experiment (now known as Schrödinger's cat paradox [30]) involving a cat in a quantum superposition of being alive and dead, due to entanglement with a radioactive atom, to illustrate this problem. Also in 1935, Einstein proposed the Einstein-Podolsky-Rosen (EPR) paradox [31] involving complementary measurements on entangled states. Neither Schrödinger nor Einstein ever accepted these quantum deviations from realism as correct, complete, and consistent. We tend to side with Schrödinger and Einstein in this matter.

We have proposed that spin is central to quantum mechanics, and that electrons and photons are soliton-like distributed rotating vector fields with quantized total spin. Soliton-like effects provide a natural explanation for the Pauli exclusion principle, without quantum entanglement or intrinsic uncertainty. An important conceptual feature of this realistic picture is that it has no separation between the quantum and classical worlds. The physical laws and mathematical equations and symmetries are the same on all scales, apart from a characteristic scale of discrete angular momentum given by $\hbar$. But can this really be consistent with physical observations on microscopic and macroscopic scales?

The key question is how one builds up from these fundamental quantized fields. According to the Standard Model of Elementary Particles [32], there are two types of fundamental particles: spin-1/2 fermions like the electron (quarks, neutrinos, muons) and spin-1 bosons like the photon (gluons, W & Z). (We will neglect the Higgs boson, which is proposed to have zero spin.) In a composite particle, the component spins add up, taking account of signs. So for example, a proton or neutron is composed of 3 quarks, with a total spin of ½.

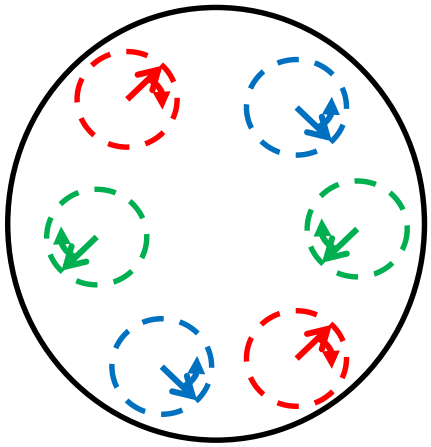

Figure 8: *Realistic picture of a nucleon composed of 3 quarks, each of which is a fermion with spin-1/2. The quarks consist of distributed rotating vector fields (red, green, and blue) confined to the nucleon, but the nucleon itself is not a quantum wave, and is not a fermion.*

In the orthodox picture (known as the "spin-statistics theorem" [33]), a proton or neutron is a fermion just like an electron. Similarly, a composite of an even number of fermions (such as a spin-zero pion or a spin-zero helium atom) would be a boson, acting just like a photon.

In our realistic picture [11], the fundamental particles are true quantum waves; there are no point particles anywhere. Composites of these quantum particles are not waves at all, but merely bags of internally confined wavepackets (see Figure 8). That means that a proton or neutron is just a particle with a scale of 1 fm, while an atom is just a particle with a scale of 1 Å; there are no de Broglie waves for these composite particles, and they are neither fermions nor bosons. We argue that these composite particles follow classical dynamics, but not all classical trajectories are accessible. For example, a molecule has quantized rotational and vibrational states, not because it is a wave, but rather because transitions between states must be mediated by quantized photons.

It is widely believed that quantum diffraction of a particle beam from a crystalline lattice or orifice proves that neutrons, atoms, and molecules are de Broglie waves [34]. However, there is an alternative explanation, even within conventional quantum theory. Van Vliet [35] showed that if one regards the diffracting crystal or mask as a quantum object, then using standard quantum analysis, one obtains the standard diffraction result regardless of the particle or wave nature of the incident particle beam. For example, in neutron diffraction, a neutron may be absorbed by a nucleus and re-emitted, but the momentum transfer from the lattice to the neutron is

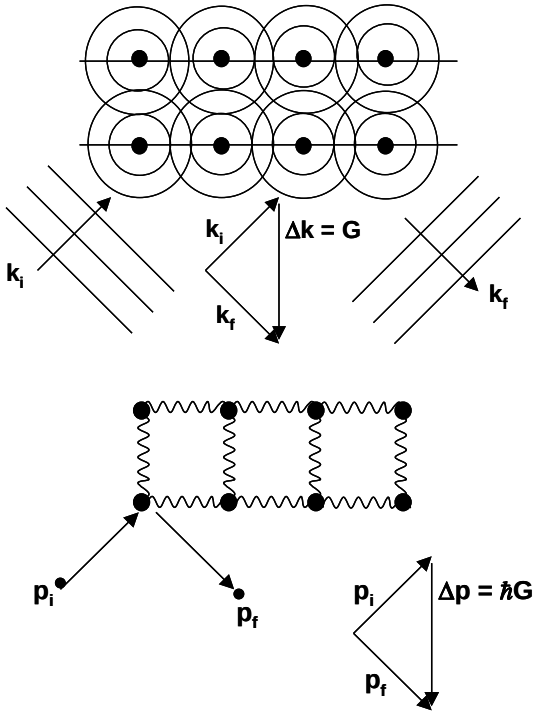

Figure 9: *Conceptual picture of neutron diffraction from a crystal lattice. (Top) Conventional coherent wave picture. (Bottom) Alternative particle picture, where quantized momentum transfer from the lattice, rather than coherent scattering, is responsible for the diffraction.*

quantized at $\mathbf{P} = \hbar\mathbf{G}$ [36], where $\mathbf{G}$ is a reciprocal lattice vector of the crystalline lattice (see Figure 9). This does not require a coherent neutron wave, as would be required for classical wave diffraction. Similarly, a particle beam composed of large molecules such as C_{60} (and even larger) has been shown to diffract on passing through a narrow slit [37]. This has been attributed to a de Broglie wave of C_{60}, but the same quantitative result follows from quantized momentum transfer from the slit to C_{60} molecules after collision.

If a neutron is not a fermion, wouldn't that affect the models of nuclei and neutron stars? Yes, but one can attribute the compressibility properties of nucleons to the underlying fermion properties of the component quarks, rather than to the fermion nature of a nucleon itself. Within this realistic picture, one would expect nucleons in nuclei to be more like atoms in crystals [38], rather than like electrons in metals (which are true waves).

A further question about the quantum nature of composites relates to Bose-Einstein condensates [39], where in analogy with a laser, most or all the atoms or electrons can be in phase in the same ground state. Three types deserve attention: superfluid helium, super-conductors, and dilute atomic gases.

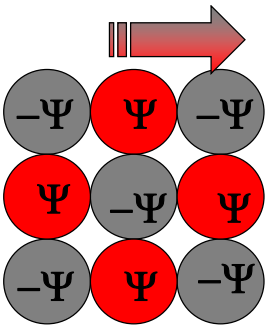

Figure 10: *Checkerboard pattern representing two sublattices of atoms in a real-space model of a superfluid state. Each red atom is surrounded by gray atoms with electrons having the same energy and spin, but with a phase difference of 180 deg., i.e. a change in sign. The entire structure can move together without loss.*

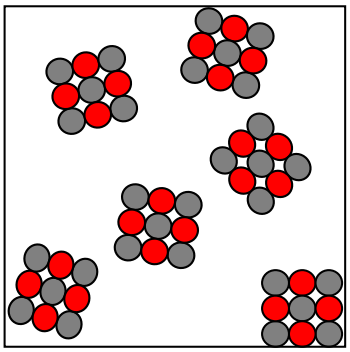

Figure 11: *Proposed alternative explanation for observations of Bose-Einstein Condensation in dilute atomic gases. Below a characteristic temperature, the atoms tend to condense into two-phase clusters similar to that in Figure 10.*

Regarding superfluidity in liquid helium [40], this occurs in the common isotope He-4 at 2 K, while it does not occur in the less-common isotope He-3, at least until temperatures below 2 mK. In both isotopes, the net electron spin is zero, but while the He-4 nucleus has zero spin, the He-3 nucleus has spin-1/2. Therefore, He-4 may be classified as a boson, while He-3 may be classified as a fermion, which would seem to explain the difference. At very low temperatures, two He-3 atoms may form a correlated pair, which again becomes a boson, enabling boson condensation again.

However, an alternative explanation of superfluidity without bosons is shown in Figure 10 [41, 11]. This shows a real-space checkerboard picture (which may be generalized to 3D) of two sublattices of atoms, where each of the red atoms is surrounded by black atoms, and vice versa. Both the red and black atoms have electrons with the same energy and spin direction. However, the electrons in the two sublattices have real quantum rotations that are 180 degrees out of phase, so that there are nodal planes between adjacent electron states. These nodal planes, which represent anti-bonding orbitals, are compatible with the Pauli exclusion principle. This locks individual atoms in place, but this entire structure can move as a rigid block with no dissipation, enabling superfluidity.

This mechanism maintains long-range phase coherence, and should apply for both He-4 and He-3, since they should have the same electron configurations. However, the He-3 nucleus has a magnetic moment due to the uncompensated spin. This magnetic moment, though small, can create an inhomogeneous energy shift (dephasing) of adjacent electrons, destabilizing the structure of Figure 10. At ultra-low temperatures, the nuclear magnetic moments can order ferromagnetically, enabling identical environments for adjacent electrons, thus restoring the order in Figure 10.

In superconductors, the mobile conduction electrons are fermions, but according to the orthodox BCS theory of superconductivity [42], the electrons may pair up with phonons to form "Cooper pairs", which are then bosons, and which can correlate their motion over a coherence length ξ to achieve lossless superconducting behavior. In an alternative real-space realistic picture [43, 44, 45], electron waves diffract from a self-induced coherent phonon field to create localized electron orbitals on the scale of ξ. These localized orbitals with the same energy and spin then organize to form the two correlated sublattices of Figure 11. Again, such a structure prevents local scattering, but maintains long-range phase coherence, and enables lossless electron transport by motion of the entire structure relative to the atomic cores. Remarkably, this theory maps onto the equations for much of the BCS theory. It even reproduces flux quantization in units of $h/2e$, where $2e$ is usually asserted to prove the existence of Cooper pairs. This alternative picture also makes testable predictions that would not be expected from standard theory [46, 47].

Dilute gases of univalent alkali metals, with spins aligned in large magnetic fields at low temperatures (which prevents spin flips and atomic bonding), have been found to undergo a phase transition at ultra-low temperatures. The velocity distribution of the atoms in the gas suddenly narrows, which is attributed to Bose-Einstein condensation [39]. According to orthodox quantum theory, the atoms are far apart, but their extended de Broglie waves tend to lock in phase. In contrast, we suggest that the gas atoms tend to form larger clusters similar to the two-phase condensate in Figure 10 (see Figure 11). Larger clusters have higher mass, and therefore lower thermal velocities, thus fitting the

experimental data without the need for atomic de Broglie waves, which we suggest do not exist. Clusters should also be detectable by other means, which should enable this alternative theory to be tested [41, 11].

Regarding quantum entanglement, there have been numerous experiments based in part on the EPR paradox, measuring the linear polarization of single photons from correlated photon pairs, which have tended to support entanglement over local realism. However, we argue [48] that a single photon should correspond to circular polarization with spin $S = \pm\hbar$, and that a linearly polarized field requires at least 2 oppositely polarized CP photons. Is it conceivable that experiments claiming to detect single photons are really detecting two simultaneous photons? Yes, in fact most photon detectors are avalanche-type event detectors, which cannot distinguish 1 from 2 simultaneous photons. However, certain types of newer superconducting single photon detectors measure absorbed energy, and can therefore count photons. This is another experiment that can help address quantum foundations, which to our knowledge has not yet been done.

Finally, discussions of the foundational interpretation of quantum theory have been going on for almost a century, without clear resolution. One reason has been that there have not been major technological applications that depend on these foundational issues. That has changed recently with quantum computing. In essence, quantum computing promises exponentially massive parallelism in bit processing without massive parallelism in hardware. This follows from entanglement among N qubits, the two-state quantum systems that are the quantum analog of classical bits. Since this effective parallelism goes as 2^N due to entanglement, if $N = 300$, this speedup is greater than the number of atoms in the universe. This would be fantastic if it were true, which is why this has generated so much interest. There are now billions of dollars being invested in quantum computing research, so there should be an answer within a decade as to whether this is possible. We suggest that this issue could be settled more economically by a few simpler experiments testing quantum foundations [26, 49]. However, most of the research is proceedings uncritically. For example, among the several technological approaches to quantum computing, the one using superconducting qubits based on Josephson junctions may be the most prominent. One other research group [50, 51] has re-analyzed many of the Josephson junction systems which claim to provide clear evidence for quantum effects, and has found that virtually all of these can be equally well explained using fully classical dynamics, without superposition and entanglement. However, this important work has been completely ignored by the active researchers in the field.

In conclusion, the central question of quantum mechanics remains what it has been since Schrödinger asked it almost a century ago:

What is the physical meaning of the quantum wavefunction?

The orthodox interpretation of quantum mechanics has denied that this question is meaningful, and has thereby stifled much-needed research into these foundations. We suggest that what is needed is to go back to the beginning, reexamine the assumptions, and identify experimental tests that can illuminate these assumptions. Despite the widespread belief that local realism has been disproven, we maintain that a realistic model remains tenable, and can be tested. In particular, a realistic model of rotating relativistic vector fields seem eminently suitable as a basis for quantum waves with spin, but this was apparently never examined in the early days. Our model is not yet complete; it requires a nonlinear self-interaction to create discrete soliton-like objects with quantized total spin. But it already suggests experimental tests that may illuminate quantum foundations. For example, a two-stage Stern-Gerlach experiment can address quantum superposition, and other laboratory experiments can address issues of quantum entanglement and boson condensation. The new field of quantum computing may also provide insights into these questions, provided that physicists keep an open mind. The next decade should be interesting, and may provide a resolution of the central question. We suggest that the future will restore local realism to an honored role in the physical universe, which both Schrödinger and Einstein might have approved.

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