A TEMPORAL HYDROGEN ATOM AND ELECTROMAGNETIC FIELD

Vu B Ho

Advanced Study, 9 Adela Court, Mulgrave, Victoria 3170, Australia

Email: vubho@bigpond.net.au

Abstract: In this work, we discuss the possibility to formulate a temporal electromagnetic field that can be produced by a temporal system, such as a temporal hydrogen atom in the temporal continuum. We also show that a temporal electromagnetic field can manifest itself not only in the temporal continuum but also in the spatial space.

In our work on a temporal dynamics [1], we generalised to formulate a 3-dimensional temporal dynamics that involves the second rate of change of time with respect to distance. Mathematically, spacetime can be assumed to be a six-dimensional metrical continuum, which is a union of a 3-dimensional spatial manifold and a 3-dimensional temporal manifold. The spatial manifold is a simply connected Euclidean space R^3 and the temporal manifold is also a simply connected Euclidean manifold R^3. The points of this spacetime are expressed as $(x^1, x^2, x^3, x^4, x^5, x^6)$, where (x^4, x^5, x^6) representing (t^1, t^2, t^3), and the square of the infinitesimal spacetime length is of a quadratic form $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. In this work, however, as in Newtonian physics, we will consider spacetime as two separate Euclidean manifolds which exist together, even though the spatial and temporal manifolds can be shown to be connected dynamically [1]. In this case, the quadratic forms for the infinitesimal spatial arc length and the temporal arc length are reduced respectively to the forms $ds^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2$ and $d\tau^2 = (dt_1)^2 + (dt_2)^2 + (dt_3)^2$. In Newtonian physics, the dynamics of a particle is a description of the rate of change of its position in space with respect to time according to Newton's laws of motion, where time is assumed to flow at a constant rate and is considered to be a one-dimensional continuum. The Newtonian formulation can be generalised by considering the dynamics of a particle as a description of the mutual rates of change of the position and the time of a particle with respect to one another, where not only space but time is also considered to be a 3-dimensional manifold. Mathematically, a temporal manifold can be considered as a 3-dimensional Euclidean continuum in which the temporal arc length τ of a temporal curve $\mathbf{t}(\tau)$ can be identified with the one-dimensional time in Newtonian physics. In this case we can also define the unit tangent vector $\mathbf{t}_T(\tau)$, the unit principal normal vector $\mathbf{p}(\tau)$ and the unit binormal vector $\mathbf{b}(\tau)$, defined by the relation $\mathbf{b}(\tau) = \mathbf{t}_T(\tau) \times \mathbf{p}(\tau)$, so that they satisfy the Frenet equations

$$\frac{d\mathbf{t}_T}{d\tau} = \kappa\mathbf{p}, \qquad \frac{d\mathbf{p}}{d\tau} = -\kappa\mathbf{t}_T + \varrho\mathbf{b}, \qquad \frac{d\mathbf{b}}{d\tau} = -\varrho\mathbf{p} \tag{1}$$

where $\kappa(\tau)$ and $\varrho(\tau)$ are the curvature and the torsion, respectively, and $d\tau = \sqrt{d\mathbf{t}.d\mathbf{t}}$ is the linear element of a temporal curve. When we only consider the motion of a particle in a temporal plane, the equations (1) reduce to

$$\frac{d\mathbf{t}_T}{d\tau} = \kappa\mathbf{p}, \qquad \frac{d\mathbf{p}}{d\tau} = -\kappa\mathbf{t}_T \tag{2}$$

By differentiation we obtain the following system of differential equations

$$\frac{d^2\mathbf{t}_T}{d\tau^2} - \frac{d(\ln\kappa)}{d\tau}\frac{d\mathbf{t}_T}{d\tau} + \kappa^2\mathbf{t}_T = 0 \tag{3}$$

$$\frac{d^2\mathbf{p}}{d\tau^2} - \frac{d(\ln\kappa)}{d\tau}\frac{d\mathbf{p}}{d\tau} + \kappa^2\mathbf{p} = 0 \tag{4}$$

If the curvature $\kappa(\tau)$ is assumed to vary slowly along the curve $\mathbf{t}(\tau)$, so that the condition $d(\ln\kappa)/d\tau = 0$ can be imposed, then $\mathbf{t}_T(\tau)$ and $\mathbf{p}(\tau)$ may be regarded as being oscillating with a period T, whose relationship to the curvature is found from the differential equation (3) or (4) as

$$\kappa = \frac{2\pi}{T} \tag{5}$$

This result shows that the temporal curvature κ is actually the angular frequency ω. In principle, the structure of the 3-dimensional spatial manifold and the 3-dimensional temporal manifold are identical, because without matter they are just an unconceivable 3-dimensional Euclidean continuum. In order to incorporate this elementary differential geometry into quantum mechanics, we identify the angular frequency defined in Equation (5) with the angular frequency in Planck's quantum of energy $E = \hbar\omega$ of a particle. With this assumption, the energy of the particle and the curvature κ are related through the relation

$$E = \hbar\kappa \tag{6}$$

We have shown that a quantum theory can be introduced into the temporal manifold via Feynman's path integral method, which is shown to be endorsed by the principle of least action in the temporal manifold [2]. For completeness, we recapture the main idea as follows. According to the canonical formulation of classical physics, the particle dynamics is governed by the action principle $\delta S = \delta\int pds = \delta\int Ed\tau = 0$. Using the relation $E = \hbar\kappa$ and the well-known expression for the curvature of a temporal path $f(x)$ in a temporal plane, $\kappa = f''/(1+f'^2)^{3/2}$, the action integral S takes the form

$$S = \int \hbar\kappa d\tau = \int \frac{\hbar f''}{1+f'^2}dx \tag{7}$$

It is shown in the calculus of variations that to extremise the integral $S = \int L(f, f', f'', x)dx$, the function $f(x)$ must satisfy the differential equation

$$\frac{\partial L}{\partial f} - \frac{d}{dx}\frac{\partial L}{\partial f'} + \frac{d^2}{dx^2}\frac{\partial L}{\partial f''} = 0 \qquad (8)$$

However, with the functional of the form given in Equation (7), $L = \hbar f''/(1 + f'^2)$, it is straightforward to verify that the differential equation (8) is satisfied by any function $f(x)$. This result may be considered as a foundation for the Feynman's path integral formulation of quantum mechanics, which uses all classical trajectories of a particle in order to calculate the transition amplitude of a quantum mechanical system. Since any path can be taken by a particle moving in a plane, if the orbits of the particle are closed, it is possible to represent each class of paths of the fundamental homotopy group of the particle by a circular path, since topologically, any path in the same equivalence class can be deformed continuously into a circular path. This assumption then leads immediately to the Bohr quantum condition

$$\oint E d\tau = \hbar \oint \kappa d\tau = \hbar \oint \frac{d\tau}{t} = \hbar \oint d\theta = nh \qquad (9)$$

The Bohr quantum condition possesses a topological character in the sense that the principal quantum number n is identified with the winding number, which is used to represent the fundamental homotopy group of temporal paths of the temporal electron of a temporal hydrogen atom.

The above discussions show that a quantum mechanics can be introduced into the temporal manifold via Feynman's path integral method. However, a wave mechanics, which is similar to Schrödinger wave mechanics for the spatial manifold, can be established using a conservation law of energy in the temporal continuum [1]. In normal physics, in order for the dynamics of a physical object to be established, we need a change of position in space and a flow of time. Space is assumed to be a 3-dimensional continuum and time is assumed to be a one-dimensional continuum. However, when we establish a temporal dynamics in a 3-dimensional temporal continuum, our view of a dynamics is that not only a change of space needs a flow of time but also a change of time needs an expansion of space. Space and time have the same level of existence. This mutual characteristic of space and time is essential, which leads to our assumption that there is always a temporal dynamics that is associated with a spatial dynamics. Consider a normal hydrogen-like atom, which is a spatial object. If the normal dynamics of the atom is due to the motion of the electron in space, then, according to our point of view, there should also be a temporal dynamics which is related to it. Because a temporal dynamics is associated with temporal inertial masses and temporal potential energies, therefore it is possible for the temporal matter to form structures similar to those that can be formed from normal matter, and the simplest is the structure of a hydrogen-like atom. If a temporal atom can be formed then a question that can be raised is whether these atoms can form temporal molecules and, in turns, the temporal molecules can form temporal biological structures. In this work we show that a temporal hydrogen-like atom in a 3-dimensional temporal manifold can be formulated and described by a wave equation that has an identical form to the Schrödinger wave equation in spatial wave mechanics. In wave mechanics, the Schrödinger equation for a spatial hydrogen-like atom can be derived from the law of conservation of energy using the concept of work done and a quantisation procedure. In classical mechanics, the work done by a force $\mathbf{F}$ is defined by the integral

$$W = \int_{r_1}^{r_2} \mathbf{F}.d\mathbf{r}. \tag{10}$$

For an inverse square field with a force determined by the relation $F = A/r^2$, where A is a constant, the work done on a particle along its radial motion becomes

$$W = \int_{r_1}^{r_2} \frac{A}{r^2} dr. \tag{11}$$

Using Newton's law of motion, we obtain

$$\int_{r_1}^{r_2} m\frac{d^2r}{dt^2} dr = \int_{r_1}^{r_2} \frac{A}{r^2} dr \tag{12}$$

From Equation (12) we can derive the following form of the conservation law of energy

$$\frac{1}{2m}\mathbf{p}^2 + \frac{A}{r} = E \tag{13}$$

where the spatial momentum $\mathbf{p}$ is defined by $\mathbf{p} = m\,d\mathbf{r}/dt$ and E is the total energy of the particle. Using the standard procedure in quantum mechanics to replace $E = i\hbar\,\partial/\partial t$ and $\mathbf{p} = -i\hbar\boldsymbol{\nabla}$, the corresponding wave equation to Equation (13) is found as

$$-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\Psi + \frac{A}{r}\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{14}$$

The well-known Schrödinger wave equation for the spatial hydrogen-like atom is then obtained by replacing $A = -kq^2$ in Equation (14), with q is the elementary charge. As discussed in our previous work on the temporal dynamics [1], from the similarity between the spatial dynamics and the temporal dynamics of a particle, we may assume the temporal work done along the radial time to take the form

$$W = \int_{t_1}^{t_2} \frac{B}{t^2} dt, \tag{15}$$

where B is a constant. In fact, the form of the work done given by Equation (15) can be realised, for example, by Planck's quantum of energy $\Delta E = h/T$ in quantum physics because it can be rewritten in this form as follows

$$W = \frac{h}{T} = \int_{T}^{\infty} \frac{h}{t^2} dt. \tag{16}$$

Using the temporal dynamical law given in Equation (A2) in the appendix and integrating with respect to time, we obtain

$$\int_{t_1}^{t_2} cD\frac{d^2t}{dr^2}dt = \int_{t_1}^{t_2} \frac{B}{t^2}dt \tag{17}$$

where c is a dimensional constant which has the dimension of a speed. To be specific, in the following we will assume $B = h$, where h is the Planck constant. From Equation (17) we obtain the following conservation law of energy

$$\frac{1}{2}\frac{c}{D}\mathbf{p}_T{}^2 + \frac{h}{t} = E \tag{18}$$

where $\mathbf{p}_T$ is the temporal momentum as defined in Equation (A4) in the appendix, and E is the total energy. If we also apply the same standard procedure in quantum mechanics to replace $E = i\hbar_T\,\partial/\partial r$ and $\mathbf{p}_\mathrm{T} = -i\hbar_T\boldsymbol{\nabla}_\mathrm{T}$, where $\boldsymbol{\nabla}_\mathrm{T}$ operates with respect to time and $\partial/\partial r$ is operated with respect to space. The constant $\hbar_T$ is a new constant that needs to be specified. By dimensions, it is possible to assume that the constant $\hbar_T$ is related to Planck's constant by the relation $\hbar_T = c\hbar$. Furthermore, it should also be mentioned here that the temporal potential of the form h/t can be derived from a temporal theory of general relativity, similar to the case when the Newtonian gravitational potential k/r can be derived from field equations of Einstein's general theory of relativity [3]. The corresponding wave equation to Equation (18) is given by

$$-\frac{c\hbar_T{}^2}{2D}\boldsymbol{\nabla}_\mathrm{T}{}^2\Psi + \frac{h}{t}\Psi = i\hbar_T\frac{\partial\Psi}{\partial r} \tag{19}$$

It is observed that if we compare Equation (19) to the Schrödinger wave equation for a spatial hydrogen-like atom then the Planck constant h should be seen as a composite constant in the sense that it may take the form $h = kd^2$, where d is understood to be a physical entity that plays the role of the elementary charge q. For the purpose of constructing a temporal hydrogen-like atom, in the following we will discuss the case when solutions to Equation (19) are written in the form

$$\Psi(\mathbf{t}, r) = \psi(\mathbf{t})\exp\left(-\frac{iEr}{\hbar_T}\right) \tag{20}$$

With solutions in the form given by Equation (20), a space-independent Schrödinger-like equation can be derived as

$$-\frac{c\hbar_T{}^2}{2D}\nabla_\mathrm{T}{}^2\psi(\mathbf{t}) + \frac{h}{t}\psi(\mathbf{t}) = E\psi(\mathbf{t}) \tag{21}$$

In the 3-dimensional temporal continuum, since the potential h/t is spherically symmetric, Equation (21) can be written in the spherical temporal polar coordinates as

$$-\frac{c{\hbar_T}^2}{2D}\left(\frac{1}{t^2}\frac{\partial}{\partial t}\left(t^2\frac{\partial}{\partial t}\right)-\frac{\mathbf{L}^2}{{\hbar_T}^2 t^2}\right)\psi(\mathbf{t})+\frac{h}{t}\psi(\mathbf{t})=E\psi(\mathbf{t}) \tag{22}$$

where the temporal orbital angular momentum operator $\mathbf{L}^2$ is given by

$$\mathbf{L}^2=-{\hbar_T}^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial^2\phi}\right) \tag{23}$$

Solutions of Equation (22) can be found using the separable form

$$\psi_{El}(\mathbf{t})=R_{El}(t)Y_{lm}(\theta,\phi) \tag{24}$$

where R_{El} is a temporal radial function and Y_{lm} is the temporal spherical harmonic. With the form of solutions given by Equation (24), Equation (22) is reduced to the system of equations

$$\mathbf{L}^2 Y_{lm}(\theta,\phi)=l(l+1){\hbar_T}^2 Y_{lm}(\theta,\phi) \tag{25}$$

$$\left(-\frac{c{\hbar_T}^2}{2D}\left(\frac{d^2}{dt^2}+\frac{2}{t}\frac{d}{dt}\right)+\frac{l(l+1)c{\hbar_T}^2}{2Dt^2}+\frac{h}{t}\right)R_{El}(t)=ER_{El}(t) \tag{26}$$

The temporal radial eigenfunctions of the bound states can be found as

$$R_{nl}(t)=-Ce^{-\rho/2}\rho^l L_{n+l}^{2l+1}(\rho) \tag{27}$$

where C is a constant, $\rho=\left(-8DE/c{\hbar_T}^2\right)^{1/2}t$ and $L_{n+l}^{2l+1}(\rho)$ is the associated Laguerre polynomial with the bound state energy eigenvalues given by

$$E_n=-\frac{Dh^2}{2c{\hbar_T}^2}\frac{1}{n^2} \tag{28}$$

Depending on the sign of the temporal inertial mass D, we have the following situations. If $D<0$ then $E_n>0$, and from Equation (18) it is seen that solutions of the wave equation derived from Equation (22) are valid only when $t>h/E_n$. This result is consistent with the fact that in order for the particle to move forward in time the temporal inertial mass D must be negative [1]. It is also noted that due to the temporal nature of the solutions, an object with a temporal inertial mass can appear and disappear at a particular spatial position and the process can repeat itself depending on the energy level of the object. On the other hand, if $D>0$ then $E_n<0$. In this case the solutions of the wave equation are valid only for the negative time $t<h/E_n$.

Using the relations $h=k_T d^2$ and $h_T=kq^2$, and if we choose the constant k_T so that the value of d is the same as that of the charge q, then Equation (28) is reduced to

$$E_n = -\frac{D{k_T}^2}{2ck^2}\frac{1}{n^2} \tag{29}$$

It is also observed that if the charge q is the source of an electromagnetic field in classical physics in the spatial manifold then it is anticipated that the temporal charge d should be the source of a physical field whose formulation is similar to the spatial electromagnetic field. A relationship between the temporal charge d and a temporal electric field $\mathbf{E}_T$ can be established through Gauss's law and a system of field equations for temporal electromagnetic fields, similar to Maxwell's equations of electromagnetism in space, can be formulated. In spatial electromagnetism, the electric field $\mathbf{E}$ produced by an electric charge q can be introduced via Gauss's law as follows

$$\oint_S \mathbf{E}.d\mathbf{S} = \int_V \boldsymbol{\nabla}.\mathbf{E}dV = \int_V \frac{\rho}{\epsilon}dV = \frac{q}{\epsilon} \tag{30}$$

where S is a spatial surface which encloses the spatial volume V which contains the charge q, ρ is charge density and ϵ is the permittivity. In temporal electromagnetism, the temporal electric field $\mathbf{E}_T$ can be introduced in the same manner as follows

$$\oint_S \mathbf{E}_T.d\mathbf{S} = \int_V \boldsymbol{\nabla}_T.\mathbf{E}_T dV = \int_V \frac{\rho_T}{\varepsilon_T}dV = \frac{d}{\epsilon_T} \tag{31}$$

where S is a temporal surface which encloses the temporal volume V which contains the temporal charge d, ρ_T is temporal charge density and ϵ_T is the temporal permittivity and $\boldsymbol{\nabla}_T$ is defined as a temporal operator. With those similarities discussed above, we now suggest that a temporal electromagnetic field is governed by a system of field equations that is similar to Maxwell's equations. By introducing a temporal magnetic field $\mathbf{B}_T$ we arrive at the following system of equations

$$\boldsymbol{\nabla}_\mathrm{T}.\mathbf{E}_T = \frac{\rho_T}{\epsilon_T} \tag{32}$$

$$\boldsymbol{\nabla}_\mathrm{T}.\mathbf{B}_T = 0 \tag{33}$$

$$\boldsymbol{\nabla}_\mathrm{T} \times \mathbf{E}_T + \frac{\partial \mathbf{B}_T}{\partial r} = 0 \tag{34}$$

$$\boldsymbol{\nabla}_\mathrm{T} \times \mathbf{B}_T - \epsilon_T \mu_T \frac{\partial \mathbf{E}_T}{\partial r} = \mu_T \mathbf{j}_T \tag{35}$$

where μ_T and ϵ_T are dimensional constants which need to be determined. We also assume that the temporal electric charge density ρ_T and the electric current density $\mathbf{j}_T$ satisfy the following equation of continuity

$$\boldsymbol{\nabla}_\mathrm{T}.\mathbf{j}_T + \frac{\partial \rho_T}{\partial r} = 0 \tag{36}$$

From the temporal field equations given in Equations (32-35), the following wave equations follow

$$\boldsymbol{\nabla}_T{}^2\mathbf{E}_T - \epsilon_T\mu_T\frac{\partial^2\mathbf{E}_T}{\partial r^2} = \boldsymbol{\nabla}_\mathrm{T}\left(\frac{\rho_T}{\epsilon_T}\right) + \mu_T\frac{\partial\mathbf{J}_T}{\partial r} \tag{37}$$

$$\boldsymbol{\nabla}_T{}^2\mathbf{B}_T - \epsilon_T\mu_T\frac{\partial^2\mathbf{B}_T}{\partial r^2} = -\mu_T\boldsymbol{\nabla}_T \times \mathbf{J}_T \tag{38}$$

In free temporal continuum, the source-free wave equations for $\mathbf{E}_T$ and $\mathbf{B}_T$ take the forms

$$\boldsymbol{\nabla}_T{}^2\mathbf{E}_T - \epsilon_T\mu_T\frac{\partial^2\mathbf{E}_T}{\partial r^2} = 0 \tag{39}$$

$$\boldsymbol{\nabla}_T{}^2\mathbf{B}_T - \epsilon_T\mu_T\frac{\partial^2\mathbf{B}_T}{\partial r^2} = 0 \tag{40}$$

If the temporal electromagnetic waves are plane surfaces perpendicular to the direction of the radial time τ along the radial direction r, then general solutions for E_τ and B_τ can be written as follows

$$E_\tau = F(r + c\tau) + G(r - c\tau) \tag{41}$$

$$B_\tau = R(r + c\tau) + S(r - c\tau) \tag{42}$$

where F, G, R and S are arbitrary functions of r and t, and $c = \sqrt{\epsilon_T\mu_T}$. It is interesting to observe that in this particular case a temporal electromagnetic field manifests itself indistinguishably from that of an electromagnetic field in the normal spatial continuum.

Appendix

Consider a particle of inertial mass m that occupies a position in space. In a coordinate system S, the position of the particle at the time τ is determined by the position vector $\mathbf{r}(\tau) = x_1(\tau)\mathbf{i} + x_2(\tau)\mathbf{j} + x_3(\tau)\mathbf{k}$. We have assumed that the Newtonian time is the temporal arc length τ. In classical physics, the classical dynamics of the particle is governed by Newton's laws of motion. We will term Newton's laws as spatial laws. These laws are stated as follows [4]:

- First spatial law: In an inertial reference frame, unless acted upon by a force, an object either remains at rest or continues to move at a constant velocity.
- Second spatial law:

$$m\frac{d^2\mathbf{r}}{d\tau^2} = \mathbf{F}. \tag{A1}$$

This law is used to determine the spatial trajectory of the particle in space with respect to time.

- Third spatial law: for every action, there is an equal and opposite reaction.

These spatial laws determine the dynamics of a particle in space with the assumption that time is 1-dimensional, universal and flowing at a constant rate.

As in classical dynamics, in order for a particle to change its position it needs a flow of time, so, similarly, we may assume that in order for a particle to change its time it would need an expansion of space. We consider the motion of a particle in space as its local spatial expansion. This assumption then allows us to define the rate of change of time with respect to space. From this mutual symmetry between space and time, a temporal dynamics, which is identical to Newtonian dynamics, can be assumed. Consider a particle of a temporal mass D that occupies a time in the 3-dimensional temporal manifold. In the coordinate system S, the time of the particle at the position specified by the spatial vector $\mathbf{r}$ is determined by the temporal vector $\mathbf{t}(s) = t_1(s)\mathbf{i} + t_2(s)\mathbf{j} + t_3(s)\mathbf{k}$, where s is the spatial arc length in the 3-dimensional spatial manifold and $ds^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2$. We assume the temporal dynamics of the particle is governed by dynamical laws which are similar to Newton's laws of motion in space. In the following we will term these laws as temporal laws. These laws are stated as follows:

- First temporal law: In an inertial reference frame, unless acted upon by a force, the time of an object either does not flow or flows at a constant rate. This is a generalisation of Newtonian concept of time, which is considered to be universal and flowing at a constant rate independent of the state of motion of the particle.
- Second temporal law:

$$D\frac{d^2\mathbf{t}}{ds^2} = \mathbf{F}. \tag{A2}$$

The constant D is a dimensional constant which plays the role of the inertial mass m of the particle in space. We can choose a unit for D so that the force $\mathbf{F}$ remains a force. This law is used to determine the temporal trajectory of the particle in the time manifold with respect to space.

- Third temporal law: for every action, there is an equal and opposite reaction.

With the view that time is a 3-dimensional manifold, it follows that time flow is a complex description with regards to a physical process. Time is not simply specified as past, present and future, but also dependent on its direction of flow. Only when the direction of flow of time can be specified then the state and the dynamics of a particle can be determined completely. For example, if time is a 3-dimensional continuum whose topology is Euclidean R^3 then the time of a particle with a temporal distance of unit length from the origin of a reference system is a temporal sphere of unit radius. The 3-dimensional temporal manifold can be reduced to 1-dimensional continuum by considering the 3-dimensional temporal manifold as a compactified manifold of the form $\mathrm{R} \times S^2$, where S^2 is a 2-dimensional compact manifold whose size is much smaller than any length. If we only consider forces that act along a radial spatial direction, such as the force of gravity and Coulomb force then we can assume $ds = dr$ and Equation (A2) can be re-written in the form

$$\frac{d}{dr}\left(D\frac{d\mathbf{t}}{dr}\right) = \mathbf{F} \tag{A3}$$

We define the temporal momentum $\mathbf{p}_T$ of a particle as follows

$$\mathbf{p}_T = D\frac{d\mathbf{t}}{dr} \tag{A4}$$

References

[1] Vu B Ho, *A Temporal Dynamics: A Generalised Newtonian and Wave Mechanics* (Preprint, ResearchGate, 2016), viXra 1708.0198v2.

[2] Vu B Ho, *On the Principle of Least Action* (Preprint, ResearchGate, 2016), viXra 1708.0226v1.

[3] Vu B Ho, *A theory of temporal relativity* (Preprint, ResearchGate, 2016), viXra 1708.0196v1.

[4] H. Goldstein, *Classical Mechanics* (Addison-Wesley Inc., Sydney, 1980).