The Mother of All Field Equations

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Abstract

The first order quaternionic partial differential equation can play as the mother of all field equations. Second order partial differential equations describe the interaction between point-like artifacts and fields. A direct relationship exists between the first order quaternionic partial differential equation and integral balance equations.

1 Quaternions

Quaternions constitute nature's natural number system. Reality applies a read only repository that stores the dynamic geometric data of its inhabitants and that repository can only cope with elements of a division ring. The only suitable division rings are the real numbers, the complex numbers, and the quaternions. Quaternions form the most elaborate division ring. The repository can also store quaternionic continuums that are defined by quaternionic functions. Quaternionic differential equations can describe the dynamic behavior of quaternionic continuums. Quaternions are ideally suited for the storage of discrete dynamic geometric data. The real part of the quaternion can represent a timestamp, and the imaginary part can represent a three-dimensional spatial location. Quaternionic fields combine a scalar field and a vector field.

Quaternionic second order partial differential equations can describe the interaction between point-like artifacts and quaternionic continuums. In a quaternionic model of the universe, these point-like artifacts constitute the objects that occur in the model.

The quaternionic first order partial differential equation appears to be the mother of all field equations. It applies the quaternionic nabla, and this differential operator behaves as a quaternionic multiplier. Thus, the quaternionic multiplication rule acts as the template for the quaternionic first order partial differential equation.

$$c = c_r + \mathbf{c} = ab \tag{1}$$

$$\equiv (a_r + \mathbf{a})\,(b_r + \mathbf{b}) = a_r b_r - \langle \mathbf{a},\mathbf{b} \rangle + \mathbf{a} b_r + a_r \mathbf{b} \pm \mathbf{a} \times \mathbf{b}$$

Here the real part gets subscript $_r$, and the imaginary part appears in bold face.

The right side covers five different terms.
$\langle \mathbf{a},\mathbf{b} \rangle$ is the inner product.
$\mathbf{a} \times \mathbf{b}$ is the external product.
$\pm$ indicates the choice between right and left handedness.

2 The quaternionic nabla

Partial quaternionic differential equations that apply the quaternionic nabla ∇ describe the interaction between a field and a point-like artifact.

$$\nabla \equiv \{\partial/\partial\tau, \partial/\partial x, \partial/\partial y, \partial/\partial z\} \tag{1}$$

$$\boldsymbol{\nabla} \equiv \{\partial/\partial x, \partial/\partial y, \partial/\partial z\} \quad (2)$$

$$\nabla_r \equiv \partial/\partial\tau \quad (3)$$

τ is progression or proper time.

In the quaternionic differential calculus, differentiation with the quaternionic nabla is a quaternionic multiplication operation:

$$\Phi = \phi_r + \boldsymbol{\Phi} = \nabla\psi \quad (4)$$

$$\equiv (\nabla_r + \boldsymbol{\nabla})\,(\psi_r + \boldsymbol{\psi}) = \nabla_r\psi_r - \langle\boldsymbol{\nabla}, \boldsymbol{\psi}\rangle + \boldsymbol{\nabla}\psi_r + \nabla_r\,\boldsymbol{\psi} \pm \boldsymbol{\nabla}\times\boldsymbol{\psi}$$

$$\phi_r = \nabla_r\psi_r - \langle\boldsymbol{\nabla}, \boldsymbol{\psi}\rangle \quad (5)$$

$$\boldsymbol{\Phi} = \boldsymbol{\nabla}\psi_r + \nabla_r\,\boldsymbol{\psi} \pm \boldsymbol{\nabla}\times\boldsymbol{\psi} \quad (6)$$

$\langle\boldsymbol{\nabla}, \boldsymbol{\psi}\rangle$ is the divergence of $\boldsymbol{\psi}$

$\boldsymbol{\nabla}\psi_r$ is the gradient of ψ_r

$\boldsymbol{\nabla}\times\boldsymbol{\psi}$ is the curl of $\boldsymbol{\psi}$

Some of the terms get new symbols

$$\mathbf{E} = -\boldsymbol{\nabla}\psi_r - \nabla_r\boldsymbol{\psi} \quad (7)$$

$$\mathbf{B} = \boldsymbol{\nabla}\times\boldsymbol{\psi} \quad (8)$$

3 Higher order differentiation

Double differentiation leads to the second order partial differential equation:

$$\rho = \nabla^*\phi = (\nabla_r - \boldsymbol{\nabla})\,(\nabla_r + \boldsymbol{\nabla})\,(\psi_r + \boldsymbol{\psi}) = (\nabla_r\nabla_r + \langle\boldsymbol{\nabla}, \boldsymbol{\nabla}\rangle)\,(\psi_r + \boldsymbol{\psi}) \quad (1)$$

$$= \rho_r + \mathbf{J}$$

This equation splits into two first order partial differential equations $\Phi = \nabla\psi$ and $\rho = \nabla^*\phi$.

$$\rho_r = \langle\boldsymbol{\nabla}, \mathbf{E}\rangle \quad (2)$$

$$\mathbf{J} = \boldsymbol{\nabla}\times\mathbf{B} - \nabla_r\mathbf{E} \quad (3)$$

$$\nabla_r\mathbf{B} = -\boldsymbol{\nabla}\times\mathbf{E} \quad (4)$$

Two quite similar second order partial differential operators exist. The first appears above.

$$(\nabla_r\nabla_r + \langle\boldsymbol{\nabla}, \boldsymbol{\nabla}\rangle)\,\psi = \rho \quad (5)$$

This equation is still nameless.

The second is the quaternionic equivalent of d'Alembert's operator $(\nabla_r\nabla_r - \langle\nabla, \nabla\rangle)$. It defines the quaternionic equivalent of the well-known wave equation.

$$(\nabla_r\nabla_r - \langle\boldsymbol{\nabla}, \boldsymbol{\nabla}\rangle)\, \psi = \varphi \tag{6}$$

Both second order partial differential operators are Hermitian differential operators.

3.1 Solutions

The homogeneous second order partial differential equations offer solutions that occur when actuators trigger them.

3.1.1 Waves

$$f\,(\tau, \mathbf{x}) = a \exp\,(i\,\omega\,(c\tau\text{-}|\mathbf{x\text{-}x'}\,|));\ c{=}\pm 1 \tag{1}$$

solves

$$\nabla_r\nabla_r\, f = \langle\boldsymbol{\nabla}, \boldsymbol{\nabla}\rangle\, f = -\omega^2\, f \tag{2}$$

3.1.2 Warps

$$\psi = g(x\,\mathbf{i}\pm\tau) \tag{3}$$

3.1.3 Clamps

$$\psi = g(r\,\mathbf{i}\pm\tau)/r \tag{4}$$

Clamps and warps are shock fronts. They only occur in odd dimensions.

All solutions have advanced and retarded components.

Clamps integrate into the Green's function. They quickly fade away. For that reason, they temporarily deform the carrier.

4 Balance equations

The first order partial differential equation is a continuity equation. It has a direct relation to integral balance equations.

Concerning a local part of a closed boundary that is oriented perpendicular to vector ***n*** the partial differentials relate as

$$\boldsymbol{\nabla}\psi = \boldsymbol{\nabla}\psi_r - \langle\boldsymbol{\nabla}, \boldsymbol{\psi}\,\rangle \pm \boldsymbol{\nabla}\times\boldsymbol{\psi} \Leftrightarrow \mathbf{n}\,\psi_r - \langle\mathbf{n}, \boldsymbol{\psi}\,\rangle \pm \mathbf{n}\times\,\boldsymbol{\psi} \tag{1}$$

This correspondence is exploited in the generalized Stokes theorem

$$\iiint_V \boldsymbol{\nabla}\psi = \oiint_S \mathbf{n}\psi \tag{2}$$

Gradient theorem

$$\iiint_V \boldsymbol{\nabla}\psi_r = \oiint_S \mathbf{n}\psi_r \tag{3}$$

Divergence theorem (Gauss theorem)

$$\iiint_V \langle\boldsymbol{\nabla}, \boldsymbol{\psi}\,\rangle = \oiint_S \langle\mathbf{n}, \boldsymbol{\psi}\,\rangle \tag{4}$$

Curl theorem (Stokes theorem)

$$\iiint_V \nabla\times\boldsymbol{\psi} = \oiint_S \mathbf{n}\times\boldsymbol{\psi} \tag{5}$$

4.1 Two-dimensional balance equations

$$\nabla_r \mathbf{B} = -\boldsymbol{\nabla}\times\mathbf{E} \tag{1}$$

$$\iint_S \langle \nabla_r \mathrm{B}, \mathrm{dA}\rangle = -\iint_S \langle \boldsymbol{\nabla}\times\mathbf{E}, \mathrm{d}\mathbf{A}\rangle = \oint \langle \mathbf{E}, \mathrm{d}\ell\rangle \tag{2}$$

$$\mathbf{J} = \boldsymbol{\nabla}\times \mathbf{B} - \nabla_r\mathbf{E} \tag{3}$$

$$\iint_S \langle \mathbf{J} + \nabla_r\mathbf{E}, \mathrm{d}\mathbf{A}\rangle = \oint \langle \mathbf{B}, \mathrm{d}\boldsymbol{\ell}\rangle \tag{4}$$

5 Material penetrating field

Basic fields can penetrate homogeneous regions of condensed matter. Within these regions, the fields get crumpled. Consequently, the average speed of warps, clamps, and waves diminish, or these vibrations just get dampened away.

The basic field that we consider here is a smoothed version ψ of the original field Ψ that penetrates the material.

$$\Phi = \phi_r + \boldsymbol{\Phi} = \nabla\psi \equiv (\nabla_r + \boldsymbol{\nabla})\,(\psi_r + \boldsymbol{\psi}) = \tag{1}$$

$$\nabla_r\psi_r - \langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle + \boldsymbol{\nabla}\psi_r + \nabla_r\,\boldsymbol{\psi} \pm \boldsymbol{\nabla}\times\boldsymbol{\psi} = \nabla_r\psi_r - \langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle \pm \mathbf{B} - \mathbf{E}$$

The penetration adds a polarization $\mathbf{P}$ to the smoothed vector field $\mathfrak{E}$, which is reduced with the permittivity factor ϵ.

$$\mathbf{D} = \epsilon\,\mathfrak{E} + \mathbf{P} \tag{2}$$

The penetration adds a magnetization $\mathbf{M}$ to the smoothed vector field $\mathfrak{B}$, which is reduced with permeability factor μ.

$$\mathbf{H} = \mu\,\mathfrak{B} + \mathbf{M} \tag{3}$$

This results in corrections in the $\mathfrak{E}$ and the $\mathfrak{B}$ field and the average speed v of warps and waves reduces from 1 to

$$\mathrm{v}=1/\sqrt{(\epsilon\mu)} \tag{4}$$

6 Green’s Function

The Green’s function describes the interaction between a field and a point-like artifact.

We can use the gradient of the inverse of the spatial distance $|\boldsymbol{q}-\boldsymbol{c}|$.

$$\boldsymbol{\nabla}\frac{1}{|\boldsymbol{q}-\boldsymbol{c}|} = -\frac{\boldsymbol{q}-\boldsymbol{c}}{|\boldsymbol{q}-\boldsymbol{c}|^3} \tag{1}$$

The divergence of this gradient is a Dirac delta function.

$$\delta(\boldsymbol{q}-\boldsymbol{c}) = -\frac{1}{4\pi}\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\frac{1}{|\boldsymbol{q}-\boldsymbol{c}|}\rangle = -\frac{1}{4\pi}\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\frac{1}{|\boldsymbol{q}-\boldsymbol{c}|} \tag{2}$$

This equation means that:

$$\phi(\boldsymbol{c}) = \iiint_V \phi(\boldsymbol{q})\delta(\boldsymbol{q}-\boldsymbol{c}) = -\frac{1}{4\pi}\iiint_V \phi(\boldsymbol{q})\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\frac{1}{|\boldsymbol{q}-\boldsymbol{c}|} \tag{3}$$

As alternative, we can also use the Green's function $G(\boldsymbol{q})$ of the partial differential equation.

$$\phi(\boldsymbol{c}) = \iiint_V \phi(\boldsymbol{q})G(\boldsymbol{q}-\boldsymbol{c}) \tag{4}$$

For the Laplacian $\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle$ this obviously means:

$$\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\mathfrak{F} = \phi(\boldsymbol{q}) \tag{5}$$

$$G(\boldsymbol{q}-\boldsymbol{c}) = \frac{1}{|\boldsymbol{q}-\boldsymbol{c}|} \tag{6}$$

However, when added to the Green's function, every solution f of the homogeneous equation

$$\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle f = 0 \tag{7}$$

is also a solution of the Laplace equation.

$$\phi(\boldsymbol{c}) = \iiint_V \frac{\phi(\boldsymbol{q})}{|\boldsymbol{q}-\boldsymbol{c}|} \tag{8}$$

Function $\phi(\boldsymbol{c})$ can be interpreted as the potential that is raised by charge distribution $\phi(\boldsymbol{q})$.

In pure spherical conditions, the Laplacian reduces to:

$$\langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \mathfrak{F}(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \mathfrak{F}(r)}{\partial r} \right) \tag{9}$$

For the following ***test function*** $\mathfrak{T}(r)$ this means [3]:

$$\mathfrak{T}(r) = \frac{Q}{4\pi} \frac{ERF\left({}^{r}\!/_{\sigma\sqrt{2}} \right)}{r} \tag{10}$$

$$\rho(r) = \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \mathfrak{F}(r) = \frac{Q}{\left(\sigma\sqrt{2\pi}\right)^3} \exp\left(-\frac{r^2}{2\sigma^2} \right) \tag{11}$$

References

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