《Universal and Unified Field Theory》
1. Universal Topology of Event Framework and Field Equations

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Abstract: The workings of ***Universal Topology*** unfolds a duality of our natural world at the following remarks:

a. Dual complex manifolds and the interactive world planes beyond a single spacetime manifold,

b. Two pairs of the scalar potentials for field entanglements complementarily, reciprocally and interdependently,

c. A mathematical framework of the dual variances to clone the event operations as an inevitable feature of reality,

d. ***Law of Event Evolutions*** carrying out ***World Equations***, and ***Horizon*** hierarchy,

e. A series of ***Universal Field Equations***, foundational and general to all dynamic fields of natural evolutions.

Upon this foundation, our *Universal Topology and Framework* are ready to give rise to the unified classical and contemporary physics ...

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INTRODUCTION

The classical theories, based on the observation at many observable collapsed states, have resulted in their theoretical models toward the decoherence interpretations or physical existence only [1]. After an observation is made, each element of the superposition becomes the combined subject–object and any object with the two "relative states" is "collapsed" at its state with the same collapsed outcome. As a single manifold, reality has always been viewed or isolated as an unfolding history. Schrödinger's cat, for example, is one of the well-known paradox as a thought experiment [2] of the classic concepts.

To extend our fundamental physics into a duality of a oneness nature, *Universal Topology* views historical and real-life reality as a many-branched tree, wherein every possible quantum outcome is realized or rising from horizons. Naturally, many-worlds [3,4], multiverse [12], and dark energy [5] has become the main mainstream of philosophical interpretations. Although these models describes indirectly to the observable states, they are developed as the most accepted hypothesis today [6]. These theories or interpretations imply the common ideas that, at minimum, there exists a pair of the fields: one for our physical world and the other for its reciprocal other world or virtual world [11,13].

More precisely in principle, an object possesses a pair of the fields and requires a duality of manifolds for their life entanglement. Because each object possesses a pair of the virtual and physical fields, an interruption between two objects involves two pairs of the fields, which constitute cross-entangling simultaneously and reciprocally [7,8,11]. In mathematics, this means that, instead of a single manifold, a oneness of the real world of our universe must be modeled by a duality of the *World Planes*, or the *Dual Manifolds* of *Universal Topology*.

Therefore, it provides the context for our main philosophical interpretation to extend our fundamental physics into a duality of a oneness of natural world [13, 14]. As a conceptual simplicity, our entire theory of the nature is based on the principle: *Dual Manifolds*, instead of one single manifold. In fact, the two metric signatures $(+---)$ or $(-+++)$ of *Minkowski* spacetime have been discovered since 1908 [15].

In our universe, a duality of the two-sidedness lies at the heart of all events or instances as they are interrelate, opposite or contrary to one another, each dissolving into the other in alternating streams that operates a life of creation, generation, or actions complementarily, reciprocally and interdependently. The nature consistently emerges as or dynamically entangles with a set of the fields that communicates and projects their interoperable values to its surrounding environment, alternatively arisen by or acting on its opponent through the reciprocal interactions.

I. UNIVERSAL TOPOLOGY AND WORLD PLANES

As the nature duality, our world always manifests a mirrored pair in the imaginary part or a conjugate pair of the complex manifolds, such that the physical nature of *P* functions is associated with its virtual nature of *V* functions to constitute a duality of the real world functions. Among them, the most fundamental dynamics are our dark resources of the universal energies, known as *Yin* "−" and *Yang* "+" dark objects, with neutral balance "0" as if there were nothing. Each type of the dark objects $(+,0,-)$ appearing as energy fields has their own domain of the relational manifolds such that one defines a Y^- (Yin) manifold while the other the Y^+ (Yang) manifold, respectively. They jointly present the two-sidedness of any events, operations, transportations, and entanglements, each dissolving into the other in the alternating streams that generates the life of entanglements, conceals the inanimacy of resources, and operates the event actions.

Because each manifold has unique representations, worlds do not exactly coincide and require transportations to pass from one to the other through commonly shared natural foundations. Therefore, our universe manifests as an associative framework of objects, crossing neighboring worlds of manifolds, illustrated as the three dimensions as the mutually orthogonal units: a coordinate manifold of physical world $P(\mathbf{r},\lambda)$, a coordinate manifold of virtual world $V(\mathbf{k},\lambda)$, and a coordinate manifold of global function $G(\lambda)$, of *Word Events* λ, shown in Figure 1a.

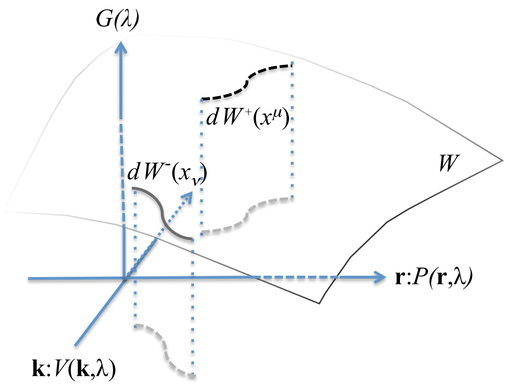


Figure 1a: Worlds of Universal Topology

where $P(\mathbf{r},\lambda)$ is parameterized by the coordinates of spatial vector $\mathbf{r}(\lambda)=\mathbf{r}(x_1,x_2,x_3)$, and $V(\mathbf{k},\lambda)$ is parameterized by the coordinates of *timestate* vector $\mathbf{k}(\lambda)=\mathbf{k}(x^0,x^{-1},x^{-2},\cdots)$. The global functions in $G(\lambda)$ axis is a collection of common objects and states of events λ, with unique functions applicable to both virtual and physical spaces of the world *W*. In other words, a universe manifold is visualized as a transitional region among the associated manifolds of the worlds, which globally forms the topological hierarchy of a universe. A curve in this three-dimensional manifold $\{\mathbf{r},\mathbf{k},G(\lambda)\}$ is called a *Universe Line*, corresponding to intersection of world planes from the two-dimensions of virtual and physical regimes of YinYang (Y^-Y^+) manifolds.

As a two-dimensional plane, the virtual positions of $\pm i\mathbf{k}$ naturally form a duality of the conjugate manifolds: $Y^-\{\mathbf{r}+i\mathbf{k}\}$ and $Y^+\{\mathbf{r}-i\mathbf{k}\}$. Each of the system constitutes its world plane $W^\pm$ distinctively, forms a duality of the universal topology $W^\mp = P \pm iV$ cohesively, and maintains its own sub-coordinate system $\{\mathbf{r}\}$ or $\{\mathbf{k}\}$ respectively. Because of the two dimensions of the world planes $\{\mathbf{r}\pm i\mathbf{k}\}$, each transcends its event operations further down to its sub-coordinate system with extra degrees of freedoms for either physical dimensions $\mathbf{r}(\lambda)$ or virtual dimensions $\mathbf{k}(\lambda)$. For example, in the scope of space and time duality at event $\lambda = t$, the compound dimensions become the tetrad-coordinates, known as the following spacetime manifolds:

$$x_m \in \check{x}\{x_0, x_1, x_2, x_3\} \subset Y^-\{\mathbf{r}+i\mathbf{k}\} \quad : x_0 = ict, \check{x} \in Y^- \tag{1.1}$$

$$x^\mu \in \hat{x}\{x^0, x^1, x^2, x^3\} \subset Y^+\{\mathbf{r}-i\mathbf{k}\} \quad : x^0 = -x_0, \hat{x} \in Y^+ \tag{1.2}$$

As a consequence, a manifold appears as or is combined into the higher dimensional coordinates, which results in the spacetime manifolds in the four-dimensional spaces.

In complex analysis, events of world planes $W^\pm$ are holomorphic functions, representing a duality of complex-conjugate functions $W^\pm$ of one or more complex variables $\check{x}$ and $\hat{x}$ in neighborhood spaces of every point in its universe regime of an open set Ʊ.

$$G(W^+, W^-, \lambda) = G(\hat{x}, \check{x}, \lambda) \quad : W^\pm \in Y^\pm \subset Ʊ \tag{1.3}$$

$$W^+(\hat{x}, \lambda) = P(\hat{x}, \lambda) - iV(\hat{x}, \lambda), \tag{1.3}$$

$$W^-(\check{x}, \lambda) = P(\check{x}, \lambda) + iV(\check{x}, \lambda) \tag{1.4}$$

These formulae are called the Y^-Y^+ *Topology of Universe*. Composed into a Y^- component, the world W^- is in the manifold of yin supremacy which dominants the processes of reproductions or animations. Likewise, composed into a Y^+ component, the world W^+ is in the manifold of yang supremacy which dominants the processes of creations or annihilations.

Together, the two world planes $\{\mathbf{r}\pm i\mathbf{k}\}$ compose the *two-dimensional* dynamics of *Boost*, a residual generators, and *Spiral*, a rotational contortions for stresses, which function as a reciprocal or conjugate duality transforming and transporting global events among sub-coordinates. Consequently, for any type of the events, the Y^-Y^+ functions are always connected, coupled, and conjugated between each other, a duality of which defines entanglements as the virtually inseparable and physically reciprocal pairs of all natural functions.

Artifact 1.1: ***Y^-Y^+ Manifolds***. For the conceptual simplicity, this manuscript refers the states, events, and operations of "physical" functions to the yin supremacy, and of "virtual" functions to the yang supremacy. For a Y^- manifold, it implies that *space and time is* parallel to its global domain with the spatial relativistic dynamics, symmetry characteristics, For a Y^+ Manifold, it implies that *time and space* transformational to its reciprocal domain for physical observations. Between them, they are operated by the general commutative dynamics with asymmetry characteristics, respectively. Therefore, a world plane of universe is a global duality of virtual and physical worlds or yin $\mathbf{r}+i\mathbf{k}$ and yang $\mathbf{r}-i\mathbf{k}$ manifolds. The world line interval between the two imaginary events are entangling as a pair of conjugation:

$$\Delta s^2 = \pm\left(\Delta\mathbf{r} - i\Delta\mathbf{k}\right)\left(\Delta\mathbf{r} + i\Delta\mathbf{k}\right) \tag{1.5}$$

Philosophically, the interval is at a life of the virtual entanglement, which is associated the physical property with virtual duality. For example, at a classical spacetime manifold, it designates a constant speed c such that $\mathbf{k}$ is simply ict, and collapsed without imaginary

$$\Delta s^2 = -\left(\Delta\mathbf{r}^2 + \Delta\mathbf{k}^2\right) = (c\Delta t)^2 - (\Delta r)^2 \quad : \mathbf{k} = ict \tag{1.6}$$

Therefore, traditionally, for $\Delta s^2 > 0$, the curves are degenerated to or miss-interpreted as time-like, $\Delta s^2 = 0$ as light-like, and $\Delta s^2 < 0$ as space-like.

Artifact 1.2: Four-dimensional Spacetime. Historically, in 1905–06 *Henri Poincaré* showed [22] that by taking time to be an imaginary fourth spacetime coordinate *ict*, a *Lorentz* transformation can formally be regarded as a rotation of coordinates in a four-dimensional space with three real coordinates representing space, and one imaginary coordinate representing time, as the fourth dimension.

Artifact 1.3: Dual Minkowski space. Equipped with a non-degenerate, the *Minkowski* inner product with metric signature is selected either $(-+++)$ as the space-like vectors or $(+---)$ as the time-like vectors [15]. Unfortunately, most of the mathematicians and general relativists sticks to one choice regardless of the other or not both, such that, apparently, any object with the two "relative states" is "collapsed" at its state with the same collapsed outcome. Therefore, a duality of the two manifolds has been hidden in contemporary physics.

II. FOUR POTENTIAL FIELDS

Governed by a global event λ under the universal topology, an operational environment is initiated by the virtual scalar fields $\phi(\lambda)$ of a rank-0 tensor, a differentiable function of a complex variable in its *Superphase* nature, where the scalar function is also accompanied with and characterized by a single magnitude $\phi(x)$ in *Superposition* nature with variable components of the respective coordinate sets $\hat{x}$ or $\check{x}$ of their own manifold. Corresponding to its maximal set of commutative and enclave states, a wave function defines the states of a quantum system virtually and represses the degrees of freedom physically. Uniquely on both of the two-dimensional world planes, a wave potential functions as a type of virtual generators, potential modulators, or dark energies that lies at the heart of all events, instances, or objects. A wave field can be classified as a scalar field, a vector field, or a tensor field according to whether the represented physical horizon is at a scope of scalar, vector, or tensor potentials, respectively.

Under the universal topology, a field $\psi(x,\lambda)$ is incepted or operated under either virtual $\psi^+(\lambda)$ and physical $\psi^-(x)$ primacies of an Y^+ or Y^- manifold respectively and simultaneously

$$\rho(x,\lambda) = \psi^-\big(x(\lambda)\big)\psi^+\big(\lambda(x)\big) \quad : x \in \{x^\mu, x_m\} \tag{2.1}$$

where $x(\lambda)$ represents the spatial supremacy with the implicit event λ as an indirect dependence; and likewise, $\lambda(x)$ represents the virtual supremacy with the redundant degrees of freedom in the implicit coordinates x as an indirect dependence. Besides, each point of the fields $\phi^\pm(x,\lambda)$ is entangled with and appears as a conjugate function of the scalar field $\varphi^\mp$ in its opponent manifold. Therefore, the effects are stationary projected to and communicated from their reciprocal opponent, shown as the following conjugate pairs:

$$\psi^+ = \phi^+(\hat{x},\lambda) + i\varphi^-(\check{x},\lambda) \tag{2.2}$$

$$\psi^- = \phi^-(\check{x},\lambda) + i\varphi^+(\hat{x},\lambda) \tag{2.3}$$

where $\phi^\pm$ implies the local supremacy of the $Y^\pm$ manifold, respectively. A conjugate field of the Y^+ scalar is mapped to a field $\varphi^-(x^\mu \mapsto x_m)$ in the Y^- manifold, and vice versa that a conjugate field of the Y^- scalar is mapped to a field $\varphi^+(x_m \mapsto x^\mu)$ in the Y^+ manifold. Apparently, two pairs of the potential fields give rise to the *Double Streams* $\{\psi^-, \psi^+\}$ of life entanglements.

In order to regulate the redundant degrees of freedom in particle interruptions, the double streaming entanglements of a wave function consists of the complex-valued probability of relative amplitude $\psi(x)$ and spiral phase $\vartheta(\lambda)$, its formalism of which has the degrees of event λ actions shown by the following:

$$\psi^+ = \psi^+(\hat{x})\ exp[i\hat{\vartheta}(\lambda)] \quad : x^\mu = x^\mu(\lambda), \lambda = \lambda(x^\mu) \tag{2.4}$$

$$\psi^- = \psi^-(\check{x})\ exp[i\check{\vartheta}(\lambda)] \quad : x_\nu = x_\nu(\lambda),\ \lambda = \lambda(x_\nu) \tag{2.5}$$

The amplitude function $\psi(x) : x = x(\lambda)$ represents the spatial position of the wave function complying with *superposition* or implicit to its λ event. The spiral function $\vartheta(\lambda) : \lambda = \lambda(x)$ features superphase of the λ event at the quantum states implicit to the physical dimensions.

Artifact 2.1: Double Streaming Entanglements. A conjugate pair of the wave functions (2.4-2.5) or (2.2-2.3) constitutes the density distributions for each of the manifolds at the steady state balancing:

$$\rho = \rho^- + i\rho^+ = \psi^-(\check{x})\ \psi^+(\hat{x})\ exp\left\{i\left[\check{\vartheta}(\lambda) + \hat{\vartheta}(\lambda)\right]\right\} \tag{2.6}$$

For a given system, the set of all possible normalizable wave functions forms an abstract mathematical scalar or vector space such that it is possible to add together different wave functions, multiply the wave functions, and extend further into the complex functions under a duality of entanglements. With normalization condition, wave functions form a projective magnitudes of space and phase states because a location cannot be determined from the wave function, but is described by a probability distribution. These two formulae of the fields and densities represent that the four-potentials are entangling in *Double Streaming* between the Y^-Y^+ manifolds, simultaneously, reciprocally, and systematically.

Artifact 2.2: Decoherence. In physics of the twentieth century, the superposed wave functions are hardly correlated to a duality of the two-dimensional world planes. Instead, the four-dimensional manifold is limited to the physical existence within one world plane such that the reality is isolated or decoherence to the superposition: homogeneity and additivity. For example, a pair of the conjugate fields $\varphi \neq \phi^*$ becomes purely imaginary $\varphi = \phi^*$, upon which the superphase is collapsed at the physical states such as the density $\rho = |\phi|^2$. Unfortunately, this density decoherence has lost its meaning to neither fluxions nor entanglements, which are critical to both symmetric and asymmetric dynamics. Therefore, the wave decoherence of the system no longer exhibits the superphase interference or wave–particle duality as in a double-slit experiment, performed by *Thomas Young* in 1801 [19]. Incredibly, this superphase interference not only demonstrates a duality of the complex fields but also is a parallel fashion to *Gauge Theory,* shown briefly in the section below.

Artifact 2.3: Gauge Fields. It represents a duality of virtual supremacy of time and physical supremacy of space. Mathematically, a partial derivative of a function of several variables is its derivative with respect to one of those variables, while the others held as constant, shown by the examples.

$$\frac{\partial\left[\psi(x)e^{i\vartheta(\lambda)}\right]}{\partial\lambda} = \psi(x)\frac{\partial\vartheta(\lambda)}{\partial\lambda}e^{i\vartheta(\lambda)} = \frac{\partial x}{\partial\lambda}\frac{\partial\vartheta(\lambda)}{\partial x}\left[\psi(x)e^{i\vartheta(\lambda)}\right] \tag{2.9}$$

Therefore, an event λ operates a full derivative D^λ or D_λ to include all indirect dependencies of magnitude and phase wave function with respect to an exogenous λ argument:

$$D^\lambda\psi(x^\mu,\lambda) = \left[\frac{\partial x^\mu}{\partial\lambda}\frac{\partial}{\partial x^\mu}\psi(x^\mu)\right]e^{-i\hat{\vartheta}(\lambda)} + \psi(x^\mu)\frac{\partial}{\partial\lambda}e^{-i\hat{\vartheta}(\lambda)}$$

$$= \dot{x}^\mu\left(\frac{\partial}{\partial x^\mu} - i\Theta^\mu\right)\psi(x^\mu,\lambda) \qquad : \Theta^\mu = \frac{\partial\hat{\vartheta}(\lambda)}{\partial x^\mu},\ \dot{x}^\mu = \frac{\partial x^\mu}{\partial\lambda} \tag{2.10a}$$

$$D_\lambda\psi(x_\nu,\lambda) = \left[\frac{\partial x_\nu}{\partial\lambda}\frac{\partial}{\partial x_\nu}\psi(x_\nu)\right]e^{i\check{\vartheta}(\lambda)} + \psi(x_\nu)\frac{\partial}{\partial\lambda}e^{i\check{\vartheta}(\lambda)}$$

$$= \dot{x}_\nu\left(\frac{\partial}{\partial x_\nu} + i\Theta_\nu\right)\psi(x_\nu,\lambda) \qquad : \Theta_\nu = \frac{\partial\check{\vartheta}(\lambda)}{\partial x_\nu},\ \dot{x}_\nu = \frac{\partial x_\nu}{\partial\lambda} \tag{2.10b}$$

where the $\hat{\vartheta}$ or $\check{\vartheta}$ is the Y^+ or Y^- superphase, respectively. Furthermore, when $\Theta = eA_\nu/\hbar$ and $D_\nu \mapsto \partial_\nu + ieA_\nu/\hbar$, this is known as *Gauge derivative* for an object with the electric charge *e* and the gauge field A_ν.

The *Gauge Field*, A_ν or A^ν in terms of the field strength tensor, is exactly the electrodynamic field, or an antisymmetric rank-2 tensor:

$$F^{+n}_{\nu\mu} = (\partial^\nu A^\mu - \partial^\mu A^\nu)_n \qquad : F^{+n}_{\mu\nu} = -F^{+n}_{\nu\mu} \tag{2.11}$$

$$D^\nu \mapsto \partial^\nu - ieA^\nu/\hbar \qquad : \Theta^\nu = \frac{e}{\hbar}A^\nu \tag{2.12}$$

$$F^{-n}_{\nu\mu} = (\partial_\nu A_\mu - \partial_\mu A_\nu)_n \qquad : F^{-n}_{\mu\nu} = -F^{-n}_{\nu\mu} \tag{2.13}$$

$$D_\nu \mapsto \partial_\nu + ieA_\nu/\hbar \qquad : \Theta_\nu = \frac{e}{\hbar}A_\nu \tag{2.14}$$

where *n* represent either a particle or a quantum state. A *Gauge Theory* was the first time widely recognized by *Pauli* in 1941 [17] and followed by the second generally popularized by *Yang-Mills* in 1954 [18] for the strong interaction holding together nucleons in atomic nuclei. Classically, the *Gauge Theory* was derived mathematically for a *Lagrangian* to be conserved or invariant under certain *Lie* groups of *local* transformations. Apparently, the superphase fields Θ^ν and Θ_ν are the event modulators operated naturally at the heart of all potential fields.

Artifact 2.4: Eigenvalues. In the first horizon or a quantum system, a result of the measurement lies in an observable set of the reciprocal states at a duality of relative amplitudes $\{\phi^\pm(\hat{x}\,|\,\check{x}),\varphi^\mp(\check{x}\,|\,\hat{x})\}$, cohesive phases $\{\hat{\vartheta}(\lambda),\check{\vartheta}(\lambda)\}$ and their density distributions $\{\rho^+,\rho^-\}$. Introduced by *Max Born* in 1926 [16], an observation yields a result given by the eigenvalues or identified by eigenvectors. Besides, within each of the respective manifold or between the cohesive Y^-Y^+ manifolds, the field entanglements are characterized by either local or relativity of the linear continuity density and commutation, cohesively. At observations, the foundation of a quantum system consists of the entangling fields of the eigenvectors, continuities and commutations.

As a summary, the workings of *Universal Topology* reveals that a duality of the potential fields are operated by both of the explicit *Magnitude* $\{\psi(x^\nu),\psi(x_\nu)\}$ dimensions and the implicit *Superphase* $\{\hat{\vartheta}(\lambda),\check{\vartheta}(\lambda)\}$ modulations. It naturally consists of two pairs of the wave functions and transforms into a variety of energy forms of quantum fields that lies at the heart of all life events, instances or objects, essential to the operations and processes of creations, annihilations, reproductions and interactions.

III. MATHEMATICAL FRAMEWORK

Both time and space are the functional spectra of the events λ, operated by and associated with their virtual and physical structure, and generated by supernatural Y^-Y^+ events associated with their virtual and physical framework. The event states on spatial-time planes are open sets and can either rise as subspaces transformed from the other worlds or confined as locally independent existence within their own domain. As in the settings of spatial and time geometry for physical or virtual world, a global parameter $G(\lambda)$ of event λ on a world plane is complex differentiable not only at $W^\pm(\lambda)$, but also everywhere within neighborhood of W in the complex plane or there exists a complex derivative in a neighborhood. By a major theorem in complex analysis, this implies that any holomorphic function is infinitely differentiable as an expansion of a function into an infinite sum of terms.

As a part of the natural architecture, the mathematical regulation of terminology not only includes symbol notation, operators, and indices of vectors and tensors, but also classifies the mathematical tools and their interpretations under the universal topology. In order to describe the nature precisely, we define essentially a duality of the contravariant $Y^+ = Y\{\mathbf{r} - i\mathbf{k}\}$ manifold and the covariant $Y^- = Y\{\mathbf{r} + i\mathbf{k}\}$ manifold, respectively by the following regulations.

1) Contravariance ($\hat{\partial}^\lambda$) - One set of the symbols with the upper indices $\{x^\mu, u^\nu, M^{\nu\sigma}\}$, as contravariant forms, are the numbers for the $Y\{\hat{x}\}$ basis of the Y^+ manifold labelled by its identity symbols $\{\hat{},{}^+\}$. "Contravariance" is a formalism in which the nature laws of dynamics operates the event actions $\hat{\partial}^\lambda$, maintains its virtual supremacy of the Y^+ dynamics, and dominates the virtual characteristics under the manifold $\hat{x}$ basis.

2) Covariance ($\check{\partial}_\lambda$) - Other set of the symbols with the lower indices (x_m, u_n, M_{ab}), as covariance forms, are the numbers for the $Y\{\check{x}\}$ basis of the Y^- manifold labelled by its identity symbols of $\{\check{},{}^-\}$. "Covariance" is a formalism in which the nature laws of dynamics performs the event actions $\check{\partial}_\lambda$, maintains its physical supremacy of the Y^- dynamics, and dominates the physical characteristics under the manifold $\check{x}$ basis.

Either contravariance or covariance has the same form under a specified set of transformations to the lateral observers within the same or boost basis as a common or parallel set of references for the operational event.

The communications between the manifolds are related through the tangent space of the world planes, regulated as the following operations:

3) Communications ($\hat{\partial}_\lambda$ and $\check{\partial}^\lambda$) - Lowering the operational indices $\hat{\partial}_\lambda$ is a formalism in which the quantitative effects of an event λ under the contravariant Y^+ manifold are projected into, transformed to, or acted on its conjugate Y^- manifold. Rising the operational indexes $\check{\partial}^\lambda$, in parallel fashion, is a formalism in which the quantitative effects of an event λ under the covariant Y^- manifold are projected into, transformed to, or reacted at its reciprocal Y^+ manifold.

The dual variances are isomorphic to each other regardless if they are isomorphic to the underlying manifold itself, and form the norm (inner product) of the manifolds or world lines. Because of the reciprocal and contingent nature, the dual manifolds conserve their invariant quantities under a change of transform commutations and transport continuities with the expressional freedom of its underlying basis.

As a part of the universal topology, these mathematical regulations of the dual variances architecturally defines further framework of the event characteristics, its operational interactions and their commutative infrastructures. In the $Y^{\mp}$ manifolds, a potential field can be characterized by a scalar function of $\psi \in \{\phi^{+}, \phi^{-}, \varphi^{+}, \varphi^{-}\}$ as *Ground Fields*, to serve as a state environment of entanglements. Among the fields, their localized entanglements form up, but are not limited to, the density fields, as *First Horizon Fields*. The derivatives to the density fields are event operations of their motion dynamics, which generates an interruptible tangent space, named as *Second Horizon Fields*.

Artifact 3.1: Residual Operations. In order to operate the local actions, an event λ exerts its effects of the virtual supremacy within its Y^{+} manifold or physical supremacy within its Y^{-} manifold. Because of the local relativity, the derivative ∂^{λ} to the vector $x^{\nu}\mathbf{b}^{\nu}$, where $\mathbf{b}^{\nu}$ is the basis, has the changes of both magnitude quantity $\dot{x}^{\mu}(\partial x^{\nu}/\partial x^{\mu})\mathbf{b}^{\nu}$ and basis direction $\dot{x}^{\mu}x^{\nu}\overset{+}{\Gamma}_{\mu\nu a}\mathbf{b}^{\mu}$, where $\dot{x}^{\mu}=\partial x^{\mu}/\partial\lambda$, transforming between the coordinates of x^{ν} and x^{μ}, giving rise to the second horizon in its *Local* or *Residual* derivatives with the boost and spiral relativities.

$$\hat{\partial}^{\lambda}\psi = \dot{x}^{\mu}X^{\nu\mu}\big(\partial^{\nu} - i\Theta^{\mu}(\lambda)\big)\psi \quad : S_2^{+} = \frac{\partial x^{\nu}}{\partial x^{\mu}},\ R_2^{+} = x^{\mu}\overset{+}{\Gamma}_{\nu\mu a} \tag{3.1a}$$

$$X^{\nu\mu} = S_2^{+} + R_2^{+} \quad : \overset{+}{\Gamma}_{\nu\mu a} = \frac{1}{2}\Big(\frac{\partial\hat{g}^{\nu\mu}}{\partial x^{a}} + \frac{\partial\hat{g}^{\nu a}}{\partial x^{\mu}} - \frac{\partial\hat{g}^{\mu a}}{\partial x^{\nu}}\Big) \tag{3.1b}$$

Because the exogenous event λ has indirect effects via the local arguments of the potential function, the non-local derivative to the local event λ is at zero. Likewise, the Y^{-} actions can be cloned straightforwardly, which gives rise from the Y^{-} tangent rotations of both magnitude quantity $\dot{x}_{n}(\partial x_{m}/\partial x_{n})\mathbf{b}_{m}$ and basis rotation $\dot{x}_{n}x_{m}\overset{-}{\Gamma}_{nm\alpha}\mathbf{b}_{n}$ into a vector Y^{-} potentials of the second horizon:

$$\check{\partial}_{\lambda}\psi = \dot{x}_{m}X_{nm}\big(\partial_{n} + i\Theta_{m}(\lambda)\big)\psi \quad : S_2^{-} = \frac{\partial x_{n}}{\partial x_{m}},\ R_2^{-} = x_{m}\overset{-}{\Gamma}_{nma} \tag{3.2a}$$

$$X_{nm} = S_2^{-} + R_2^{-} \quad : \overset{-}{\Gamma}_{nm\alpha} = \frac{1}{2}\Big(\frac{\partial\check{g}_{nm}}{\partial x_{\alpha}} + \frac{\partial\check{g}_{n\alpha}}{\partial x_{m}} - \frac{\partial\check{g}_{m\alpha}}{\partial x_{n}}\Big) \tag{3.2b}$$

where the $\overset{-}{\Gamma}_{nm\alpha}$ or $\overset{+}{\Gamma}_{\nu\mu a}$ is an Y^{-} or Y^{+} metric connection, similar but extend the meanings to the *Christoffel* symbols of the *First* kind, introduced in 1869 [7]. The first partial derivative ∂^{λ} or ∂_{λ} acts on the potential argument's value x^{μ} or x_{m} with the exogenous event λ as indirect effects.

Artifact 3.2: Relativistic Operations. By lowering the index, the virtual Y^{+} actions manifest the first tangent potential $\hat{\partial}_{\lambda}$ projecting into its opponent basis of the Y^{-} manifold. Because of the relativistic interactions, the derivative ∂_{λ} to the vector $x^{\nu}\mathbf{b}^{\nu}$ has the changes of both magnitude quantity $\dot{x}_{a}(\partial x^{\nu}/\partial x_{a})\mathbf{b}^{\nu}$ and basis direction $\dot{x}^{a}x_{\mu}\overset{+}{\Gamma}{}^{\nu}_{\mu a}\mathbf{b}^{\nu}$, transforming from one world plane $W^{+}\{\mathbf{r}-i\mathbf{k}\}$ to the other $W^{-}\{\mathbf{r}+i\mathbf{k}\}$. This action redefines the Y^{+} event quantities of relativity and creates the *relativistic Boost S_1^{+} Transformation* and the *interweave Spiral Torque R_1^{+} Transportation* around a central point, which gives rise from the Y^{+} tangent rotations into a vector Y^{-} potentials for the second horizon.

$$\hat{\partial}_{\lambda}\psi = \dot{x}_{a}X^{\nu}{}_{a}\big(\partial^{\nu} - i\Theta^{\nu}(\lambda)\big)\psi \quad : S_1^{+} = \frac{\partial x^{\nu}}{\partial x_{a}},\ R_1^{+} = x^{\mu}\overset{+}{\Gamma}{}^{\nu}_{\mu a} \tag{3.3a}$$

$$X^{\nu}{}_{a} = S_1^{+} + R_1^{+}, \qquad \overset{+}{\Gamma}{}^{\nu}_{\mu a} = \frac{1}{2}\hat{g}_{\nu\epsilon}\Big(\frac{\partial\hat{g}^{\epsilon\mu}}{\partial x^{a}} + \frac{\partial\hat{g}^{\epsilon a}}{\partial x^{\mu}} - \frac{\partial\hat{g}^{\mu a}}{\partial x^{\epsilon}}\Big) \tag{3.3b}$$

Similarly, one has the Y^{-} derivative relativistic to its Y^{+} opponent:

$$\check{\partial}^{\lambda}\psi = \dot{x}^{\alpha}X_{m}{}^{\alpha}\big(\partial_{m} + i\Theta_{m}(\lambda)\big)\psi \quad : S_1^{-} = \frac{\partial x_{m}}{\partial x^{\alpha}},\ R_1^{-} = x_{s}\overset{-}{\Gamma}{}^{m}_{s\alpha} \tag{3.4a}$$

$$X_{m}{}^{\alpha} = S_1^{-} + R_1^{-}, \qquad \overset{-}{\Gamma}{}^{m}_{s\alpha} = \frac{1}{2}\check{g}^{me}\Big(\frac{\partial\check{g}_{e\alpha}}{\partial x_{s}} + \frac{\partial\check{g}_{es}}{\partial x_{\alpha}} - \frac{\partial\check{g}_{\alpha s}}{\partial x_{e}}\Big) \tag{3.4b}$$

where the matrix $\check{g}_{\sigma\epsilon}$ or $\hat{g}^{se}$ is the Y^{-} or Y^{+} metric, and the matrix $\check{g}^{\sigma\epsilon}$ or $\hat{g}_{se}$ is the inverse metric, respectively. Besides, the $\overset{-}{\Gamma}{}^{m}_{s\alpha}$ or $\overset{+}{\Gamma}{}^{\nu}_{\mu a}$ is an Y^{-} or Y^{+} metric connection, similar but extend the meanings to the *Christoffel* symbols of the *Second* kind.

Associated with the horizon actions, the partial derivative ∂^{λ} or ∂_{λ} is embedded in the event operations $\Theta^{\mu}(\lambda)$ or $\Theta_{m}(\lambda)$, gives rise to the horizons, and acts on the potential argument's value λ as direct effects. Shown by the artifact 2.3, the events operate $\Theta^{\mu} = e\dot{x}^{\mu}A^{\mu}/\hbar$ and $\Theta_{m} = e\dot{x}_{m}A_{m}/\hbar$ which give rise to the second horizon potentials. The residual transformation $S_2^{\pm}$ and transportation $R_2^{\pm}$ are communications between two coordinate frames that move at velocity relative to each other under their local Y^{+} or Y^{-} manifold, respectively. Vice versa for the cross-boost $S_1^{\mp}$ transformation and torque-spiral $R_1^{\mp}$ transportation, relativistically. Normally, for event $\lambda = t$, the speeds of transform amplitude and transport angular are at constant rate, known as the momentum and speed conservations of photon or graviton propagations.

Artifact 3.3: Vector Residual Operations. Following the local tangent curvature of the potential vectors through the next tangent vector of the curvature, the λ event gives rise to the *Third Horizon Fields*, shown by the expressions:

$$\hat{\partial}^{\lambda}V^{\mu} = \dot{x}^{\nu}\big(\partial^{\nu}V^{\mu} - \overset{+}{\Gamma}{}^{\sigma}_{\nu\mu}V^{\sigma}\big) \tag{3.5}$$

$$\check{\partial}_{\lambda}V_{m} = \dot{x}_{n}\big(\partial_{n}V_{m} - \overset{-}{\Gamma}{}^{s}_{nm}V_{s}\big) \tag{3.6}$$

where the reference of an observation is at the Y^{-} manifold. The event operates the *local* actions in the tangent space relativistically, where the scalar fields are given rise to the vector fields and the vector fields are given rise to the matrix fields.

Artifact 3.4: Vector Interactions. Through the tangent vector of the third curvature, the events $\hat{\partial}^{\lambda}\hat{\partial}^{\lambda}$ and $\check{\partial}_{\lambda}\check{\partial}_{\lambda}$ continuously entangle the residual vector fields, shown by the formulae:

$$\begin{aligned}\hat{\partial}^{\lambda}\hat{\partial}^{\lambda}V^{\mu} = {} & \left(\dot{x}^{\iota}\partial^{\iota}\right)\left(\dot{x}^{\nu}\partial^{\nu}\right)V^{\mu} - \dot{x}^{\iota}\overset{+}{\Gamma}{}^{s}_{\iota\nu}\left(\dot{x}^{\nu}\partial^{\nu}V^{\mu}\right) + \dot{x}^{n}\overset{+}{\Gamma}{}^{n}_{ms}\dot{x}^{\nu}\overset{+}{\Gamma}{}^{\sigma}_{\mu\nu}V^{\sigma} \\ & - \big(\ddot{x}^{\nu}\overset{+}{\Gamma}{}^{\sigma}_{\mu\nu} + \dot{x}^{\nu}\dot{x}^{\iota}\partial^{\iota}\overset{+}{\Gamma}{}^{\sigma}_{\mu\nu} + \dot{x}^{\nu}\overset{+}{\Gamma}{}^{\sigma}_{\mu\nu}\dot{x}^{\iota}\partial^{\iota}\big)V^{\sigma}\end{aligned} \tag{3.7}$$

$$\begin{aligned}\check{\partial}_{\lambda}\check{\partial}_{\lambda}V_{m} = {} & \left(\dot{x}_{e}\partial_{e}\right)\left(\dot{x}_{n}\partial_{n}\right)V_{m} - \dot{x}_{e}\overset{-}{\Gamma}{}^{s}_{en}\left(\dot{x}_{n}\partial_{n}V_{m}\right) + \dot{x}_{\nu}\overset{-}{\Gamma}{}^{\mu}_{\sigma\nu}\dot{x}_{n}\overset{-}{\Gamma}{}^{o}_{mn}V_{o} \\ & - \big(\ddot{x}_{n}\overset{-}{\Gamma}{}^{o}_{mn} + \dot{x}_{n}\dot{x}_{e}\partial_{e}\overset{-}{\Gamma}{}^{o}_{mn} + \dot{x}_{n}\overset{-}{\Gamma}{}^{o}_{mn}\dot{x}_{e}\partial_{e}\big)V_{o}\end{aligned} \tag{3.8}$$

Besides, the cross-entanglements, $\hat{\partial}_{\lambda}$ and $\check{\partial}^{\lambda}$, elaborate relativistic transformations between the manifolds and give rise to the next horizon fields, simply by the conversion $L^{\pm}_{\nu\mu}$ matrices:

$$\dot{x}_{\nu} \mapsto \dot{x}^{\mu}L^{-}_{\nu\mu} \qquad \dot{x}^{\nu} \mapsto \dot{x}_{\mu}L^{-}_{\nu\mu} \tag{3.9}$$

As an integrity, they perform full operational commutations of vector boosts and torque rotations operated between the $Y^{-}Y^{+}$ world planes. The event processes continue to build up the further operable and iterative horizons of the associated rank-n tensor fields. Systematically, sequentially, and progressively, a chain of these reactions constitutes various domains, each of which gives rise to the distinct field entanglements.

Artifact 3.5: Classical Operators. In quantum physics, a mathematical operator is driven by the event λ, which, for example at $\lambda = t$, can further derive the classical momentum $\hat{p}$ and energy $\hat{E}$ operators at the second horizon:

$$\hat{\partial}^{t} : \dot{x}^{\mu}\partial^{\mu} = \left(-ic\partial^{\kappa}, \mathbf{u}^{+}\partial^{r}\right) = \frac{i}{\hbar}\left(\hat{E}, \mathbf{u}^{+}\hat{p}\right) \quad : \partial^{\kappa} = \frac{\partial}{\partial x^{0}},\ \mathbf{u}^{+} = \frac{\partial x^{r}}{\partial t} \tag{3.11}$$

$$\check{\partial}_{t} : \dot{x}_{m}\partial_{m} = \left(+ic\partial_{\kappa}, \mathbf{u}^{-}\partial_{r}\right) = \frac{i}{\hbar}\left(\hat{E}, \mathbf{u}^{-}\hat{p}\right) \quad : \partial_{\kappa} = \frac{\partial}{\partial x_{0}},\ \mathbf{u}^{-} = \frac{\partial x_{r}}{\partial t} \tag{3.12}$$

$$\hat{E} = -i\hbar\frac{\partial}{\partial t}, \qquad \hat{p} = -i\hbar\nabla \qquad : \partial^{r} = \partial_{r} = \nabla \tag{3.13}$$

For $\mathbf{u}^{\mp} = \pm c$, one has the classical operators at the third horizon:

$$\check{\partial}^{\lambda}\check{\partial}_{\lambda} = \hat{\partial}^{\lambda}\hat{\partial}_{\lambda} = \hat{\partial}_{\lambda}\check{\partial}^{\lambda} = \hat{\partial}^{\lambda}\check{\partial}_{\lambda} = \frac{\partial^{2}}{\partial t^{2}} - c^{2}\nabla^{2} \equiv c^{2}\square^{+} \qquad : \lambda = t \tag{3.14}$$

$$\hat{\partial}_{\lambda}\hat{\partial}_{\lambda} = \check{\partial}^{\lambda}\check{\partial}^{\lambda} = \check{\partial}_{\lambda}\hat{\partial}_{\lambda} = \hat{\partial}^{\lambda}\check{\partial}^{\lambda} = \frac{\partial^{2}}{\partial t^{2}} + c^{2}\nabla^{2} \equiv c^{2}\square^{-} \qquad : \lambda = t \tag{3.15}$$

where the operator $\square^{\mp}$ extends the *d'Alembert* operator into the Y^-Y^+ properties. These operators can normally be applied to the diagonal elements of a matrix, observable to the system explicitly or externally.

It is worthwhile to emphasize that *a*) the manifold operators of $\{\partial^\mu, \partial_m\}$, including traditional "operators" of $\{\partial/\partial t, \partial/\partial x, \nabla, \hat{E}, \hat{p}, \cdots\}$ are exclusively useable as mathematical tools only, and *b*) the tools do not operate or perform by themselves unless they are driven or operated by an event λ, implicitly or explicitly.

Artifact 3.6: Flux Continuity. For the entanglement streams between the manifolds, the ensemble of an event λ is in a mix of the Y^- or Y^+-supremacy states such that each pair of the reciprocal states $\{\phi_n^-, \varphi_n^+\}$ or $\{\phi_n^+, \varphi_n^-\}$ is performed in alignment with an integrity of their probability $p_n^\pm = p_n(h_n^\pm)$, where $h_n^\pm$ are the $Y^\pm$ distributive or horizon factors, respectively. The parameter p_n^- or p_n^+ is a statistical function of horizon factor $h_n^-(T)$ or $h_n^+(T)$ and fully characterizable by *Thermodynamics*. Under the event operations, the interoperation among four types of scalar fields of $\phi_n^\pm$ and $\varphi_n^\pm$ correlates and entangles an environment of dual densities $\rho_\phi^+ = \phi_n^+\varphi_n^-$ and $\rho_\phi^- = \phi_n^-\varphi_n^+$ by means of the natural derivatives $\dot{\lambda}$ to form a pair of fluxions $\langle\dot{\lambda}\rangle^\mp$:

$$\dot{\lambda}\rho_\phi^- = \langle\check{\lambda}, \hat{\lambda}\rangle^- = \langle\dot{\lambda}\rangle^- = \sum\nolimits_n p_n^-\left(\varphi_n^+\check{\lambda}\phi_n^- + \phi_n^-\hat{\lambda}\varphi_n^+\right) \tag{3.17}$$

$$\dot{\lambda}\rho_\phi^+ = \langle\hat{\lambda}, \check{\lambda}\rangle^+ = \langle\dot{\lambda}\rangle^+ = \sum\nolimits_n p_n^+\left(\varphi_n^-\hat{\lambda}\phi_n^+ + \phi_n^+\check{\lambda}\varphi_n^-\right) \tag{3.18}$$

where $\dot{\lambda} \in \{\check{\lambda}, \hat{\lambda}\}$, $\check{\lambda} \in \{\check{\partial}_\lambda, \check{\partial}^\lambda\}$, and $\hat{\lambda} \in \{\hat{\partial}^\lambda, \hat{\partial}_\lambda\}$. The symbols $\langle\ \rangle^\mp$ are called Y^- or Y^+ *Continuity Bracket*. They represent the dual continuities of the Y^-Y^+ scalar densities, each of which extends its meaning to the classic anti-commutator or commutator,

$$\langle a, b\rangle = ab + ba, \qquad [a, b] = ab - ba \tag{3.19}$$

known as commutators or *Lei Bracket*, introduced in 1930s [2].

Artifact 3.7: Flux Commutation. In a parallel fashion, as another pair of the operational symbols $[\dot{\lambda}]^\mp$ at respective Y^- or Y^+ supremacy, the reciprocal entanglements of fluxion fields define the *Commutator Bracket* $[\ ]^\mp$:

$$[\hat{\lambda}, \check{\lambda}]^+ = [\dot{\lambda}]^+ = \sum\nolimits_n p_n^+\left(\varphi_n^-\hat{\lambda}\phi_n^+ - \phi_n^+\check{\lambda}\varphi_n^-\right) \tag{3.20}$$

$$[\check{\lambda}, \hat{\lambda}]^- = [\dot{\lambda}]^- = \sum\nolimits_n p_n^-\left(\varphi_n^+\hat{\lambda}\phi_n^- - \phi_n^-\hat{\lambda}\varphi_n^+\right) \tag{3.21}$$

$$\langle\dot{\lambda})_s^\pm = \sum\nolimits_n p_n^\pm\varphi_n^\mp\dot{\lambda}\phi_n^\pm, \qquad (\dot{\lambda}\rangle_s^\pm = \sum\nolimits_n p_n^\mp\phi_n^\pm\dot{\lambda}\varphi_n^\mp \tag{3.22}$$

where, in addition, the bracket $\langle\ )^\mp$ or $(\ \rangle^\mp$ are called Y^- or Y^+ *Asymmetry Brackets*. They are essential to ontological and cosmological dynamics.

Artifact 3.8: Vector Fluxions. Similarly, a set of the reciprocal vector fields of $V_m^\pm = -\partial\phi_m^\pm$ and $\Lambda_\mu^\pm \equiv -\partial\varphi_\mu^\pm$, has the brackets of Y^- or Y^+ continuity and commutation:

$$\langle\hat{\lambda}, \check{\lambda}\rangle_v^\pm \equiv \sum\nolimits_n p_n^\pm\left(\varphi_n^\mp\hat{\lambda}V_n^\pm + \phi_n^\pm\check{\lambda}\Lambda_n^\mp\right) \qquad \langle\dot{\lambda})_v^\pm = \varphi_n^\mp\dot{\lambda}V_n^\pm \tag{3.23}$$

$$[\hat{\lambda}, \check{\lambda}]_v^\mp \equiv \sum\nolimits_n p_n^\mp\left(\varphi_n^\pm\hat{\lambda}V_n^\mp - \phi_n^\mp\check{\lambda}\Lambda_n^\pm\right) \qquad (\dot{\lambda}\rangle_v^\pm = \phi_n^\pm\dot{\lambda}\Lambda_n^\mp \tag{3.24}$$

where the index n is corresponds to each type of particle, and *v* indicates entanglements of vector potentials, which respectively give rise to or balance each other's horizon environment.

Artifact 3.9: Interpretation of Entropy. A measure of the specific operations of ways is called entropy in which states of a universe system could be arranged and balanced towards its equilibrium. The total entropy $\mathcal{S}^\pm$ represent law of conservation of area commutation and defined by the following commutations. For a triplet quark system, the blackhole entropy $\mathcal{S}_A$ is at $\sum 2\varphi_a^\pm(\phi_b^\mp + \phi_c^\mp) \approx 4\varphi_a\phi_{b/c}$, which is about four times of the area entropy for the wave emission

$$\mathcal{S}_a = \mathcal{S}^+ + \mathcal{S}^- = 4\mathcal{S}_A \qquad : \mathcal{S}^\pm = \kappa_s\left[\hat{\partial}_\lambda\hat{\partial}_\lambda, \check{\partial}^\lambda\check{\partial}^\lambda\right]^\pm \tag{3.25}$$

where κ_s is factored by normalization of the potential fields for a pair of the world planes. As an operational duality, the entropy tends towards both extrema alternately to maintain a continuity of energy conservations, operated by each of the opponent *World Plane*. When a total entropy decreases, the intrinsic order, or Y^- development, of virtual into physical regime $\hat{\partial}_\lambda\hat{\partial}_\lambda$ is more dominant than the reverse process. This philosophy states that for the central quantity of *Motion Dynamics,* conversely, when a total entropy increases, the extrinsic disorder, or Y^+ annihilation $\check{\partial}^\lambda\check{\partial}^\lambda$, becomes dominant and conceals physical resources into virtual regime. For an observation at long range, the commutation becomes a conservation of the Y^-Y^+ thermodynamics, or is known as blackhole radiations, which yields law of the *Area Entropy* of the dual manifolds on the world planes.

Artifact 3.10: Interpretation of Lagrangians. To seamlessly integrate with the classical dynamic equations, it is critical to interpret or promote the natural meanings of *Lagrangian* mechanics $\mathcal{L}$ in forms of the dual manifolds. As a function of generalized information and formulation, *Lagrangians* $\mathcal{L}$ can be redefined as a set of densities, continuities, or commutators, entanglements of the Y^-Y^+ manifolds respectively. A few of the examples are:

a) The density *Lagrangians* for the *Artifact* 2.1 can be defined by the formulae:

$$\tilde{\mathcal{L}}_\rho = \check{\mathcal{L}}_\rho + i\hat{\mathcal{L}}_\rho = \psi^-(\check{x})\ \psi^+(\hat{x})\ exp\left(i\vartheta(\lambda)\right) \tag{3.26}$$

b) For a scalar or vector entanglement, the commutator *Lagrangians* can be expressed by their local- or inter-communications:

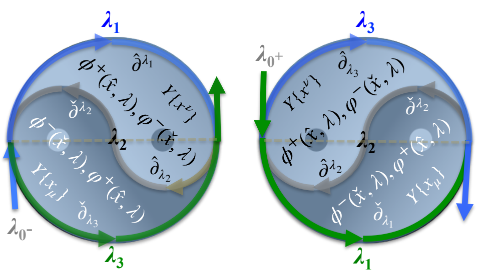
$$\tilde{\mathcal{L}}_L^\pm = -\frac{1}{c^2}\left[\hat{\partial}^\lambda\hat{\partial}^\lambda, \check{\partial}_\lambda\check{\partial}_\lambda\right]_{s/v}^\pm \qquad : \textit{Local-Commutators} \tag{3.27}$$

$$\tilde{\mathcal{L}}_I^\pm = -\frac{1}{c^2}\left[\hat{\partial}_\lambda\hat{\partial}_\lambda, \check{\partial}^\lambda\check{\partial}^\lambda\right]_{s/v}^\pm \qquad : \textit{Inter-Commutators} \tag{3.28}$$

Those formulae generalize the *Lagrangian* and state that the central quantity of *Lagrangian,* introduced in 1788*,* represents the bi-directional fluxions that sustain, stream, harmonize and balance the dual continuities of entanglements of the Y^-Y^+ dynamic fields. Apparently, there are a variety of ways to comprehend or empathize on a *Lagrangian* function under a scope of isolations.

IV. LAW OF EVENT PROCESSES

Following *Universal Topology*, world events, illustrated in the Y^-Y^+ flow diagram of Figure 4a, operate the potential entanglements that consist of the Y^+ supremacy (white background) at a top-half of the cycle and the Y^- supremacy (black background) at a bottom-half of the cycle. Each part is dissolving into the other to form an alternating stream of dynamic flows. Their transformations in between are bi-directional antisymmetric transportations crossing the dark tunnel through a pair of the end-to-end circlets on the center line. Both of the top-half and bottom-half share the common global environment of the state density ρ_n that mathematically represents the $\rho_n^+ = \phi^+\varphi^-$ for the Y^+ manifold and its equivalent $\rho_n^- = \phi^-\varphi^+$ for the Y^- manifold, respectively.



$W^+ : (\hat{\partial}^{\lambda_1} \to \hat{\partial}_{\lambda_2}), (\check{\partial}^{\lambda_2} \to \check{\partial}_{\lambda_3})$ $\qquad W^- : (\check{\partial}_{\lambda_1} \to \check{\partial}^{\lambda_2}), (\hat{\partial}^{\lambda_2} \to \hat{\partial}_{\lambda_3})$

Figure 4a: Event Flows of Y^-Y^+ YinYang Processes: $\hat{\partial}^\lambda \circlearrowleft \hat{\partial}_\lambda \rightleftharpoons \check{\partial}^\lambda \circlearrowright \check{\partial}_\lambda$

Besides, the left-side diagram presents the event flow acted from the inception of λ_{0^-} through $\lambda_1\ \lambda_2\ \lambda_3$ to intact a cycle process for the Y^+ supremacy. In parallel, the right-side diagram depicts the event flow initiated from the event λ_{0^+} through $\lambda_1\ \lambda_2\ \lambda_3$ to complete a cycle process for the Y^- supremacy. The details are described by the loops of *Evolutionary Processes* as the following interrelations:

1) Visualized in the left-side of Figure 4a, the transitional event process between virtual and physical manifolds involves a cyclic sequence throughout the dual manifolds of the environment: incepted at λ_0-, the event actor produces the virtual operation $\hat{\partial}_{\lambda_1}$ in $Y\{x^\nu\}$ manifold (the left-hand blue curvature) projecting $\hat{\partial}^{\lambda_2}$ to and transforming into its physical opponent $\check{\partial}^{\lambda_2}$ (the tin curvature transforming from the left-hand into right-hand), traveling through $Y\{x_\mu\}$ manifold (the right-hand green curvature), and reacting the event $\check{\partial}_{\lambda_3}$ back to the actor.

2) As a duality in the parallel reaction, exhibited in the right-side of Figure 4a, initiated at λ_0+, the event actor generates the physical operation $\check{\partial}_{\lambda_1}$ in $Y\{x_\mu\}$ manifold (the right-hand green curvature) projecting $\check{\partial}^{\lambda_2}$ to and transforming into its virtual opponent $\hat{\partial}^{\lambda_2}$ (the tin curvature transforming from right-hand into left-hand), traveling through $Y\{x^\nu\}$ manifold (the left-hand blue curvature), and reacting the event $\hat{\partial}_{\lambda_3}$ back to the actor.

With respect to one another, the two sets of the *Universal Event* processes, cycling at the opposite direction simultaneously, formulate $W^+(\rho_n^+) : (\lambda_1 \to \lambda_2)^+, (\lambda_2 \to \lambda_3)^-$ and $W^-(\rho_n^-) : (\lambda_1 \to \lambda_2)^-, (\lambda_2 \to \lambda_3)^+$ the flow charts in the quadrant-state expressions:

$$W^+ : (\hat{\partial}^{\lambda_1} \to \hat{\partial}_{\lambda_2}), (\check{\partial}^{\lambda_2} \to \check{\partial}_{\lambda_3}) \quad \textbf{(4.1)}$$

$$W^- : (\check{\partial}_{\lambda_1} \to \check{\partial}^{\lambda_2}), (\hat{\partial}^{\lambda_2} \to \hat{\partial}_{\lambda_3}) \quad \textbf{(4.2)}$$

This pair of the interweaving system pictures an outline of the internal commutation of dark energy and continuum density of the entanglements. It demonstrates that the two-sidedness of any event flows, each dissolving into the other in alternating streams, operate a life of situations, movements, or actions through least continuous helix-circulations aligned with the universal topology, which lay behind the context of the main philosophical interpretation of *World Equations*.

Artifact** 4.1: **Motion Operations. As a natural principle of motion dynamics, one of the flow processes dominates the intrinsic order, or development, of virtual into physical regime, while, at the same time, its opponent dominates the intrinsic annihilation or physical resources into virtual domain. Applicable to world expressions of (4.1)-(4.2), the principle of least-actions derives a set of the *Motion Operations*:

$$\check{\partial}^-\left(\frac{\partial W}{\partial(\hat{\partial}^+\phi^+)}\right) - \frac{\partial W}{\partial\phi^+} = 0 \quad : \check{\partial}^- \in \{\check{\partial}_\lambda, \check{\partial}^\lambda\}, \phi^+ \in \{\phi_n^+, \varphi_n^+\} \quad (4.3)$$

$$\hat{\partial}^+\left(\frac{\partial W}{\partial(\check{\partial}^-\phi^-)}\right) - \frac{\partial W}{\partial\phi^-} = 0 \quad : \hat{\partial}^+ \in \{\hat{\partial}^\lambda, \hat{\partial}_\lambda\}, \phi^- \in \{\phi_n^-, \varphi_n^-\} \quad (4.4)$$

This set of dual formulae extends the philosophical meaning to the *Euler-Lagrange* [9] *Motion Equation* for the actions of any dynamic system, introduced in the 1750s. The new sets of the variables of $\phi_n^\mp$ and the event operators of $\check{\partial}^-$ and $\hat{\partial}^+$ signify that both manifolds maintain equilibria and formulations from each of the motion extrema, simultaneously driving a duality of physical and virtual dynamics.

Artifact** 4.2: **Geodesic Routing. Unlike a single manifold space, where the shortest curve connecting two points is described as a parallel line, the optimum route between two points of a curve is connected by the tangent transportations of the Y^- and Y^+ manifolds. As an extremum of event actions on a set of curves, the rate of divergence of nearby geodesics determines curvatures that is governed by the equivalent formulation of geodesic deviation for the shortest paths on each of the world planes, given in local coordinates by:

$$\ddot{x}^\mu + \overset{+}{\Gamma}{}^{\mu}_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = 0 \quad (4.5a)$$

$$\ddot{x}_m + \overset{-}{\Gamma}{}^{m}_{ab}\dot{x}_a\dot{x}_b = 0 \quad (4.5b)$$

This set extends a duality to and is known as *Geodesic Equation* [10], where the motion accelerations of $\ddot{x}^\mu$ and $\ddot{x}_m$ are aligned in parallel to each of the world lines. It states that, during the inception of the universe, the tangent vector of the virtual Y^-Y^+ energies to the geodesic entanglements is either unchanged or parallel transport as an object moving along the world planes that creates the inertial transform generators and twist transport torsions to emerge a reality of the world.

V. WORLD EQUATIONS

In mathematical analysis, a complex manifold yields a holomorphic operation and is complex differentiable in a neighborhood of every point in its domain, such that an operational process can be represented as an infinite sum of terms:

$$f(\lambda) = f(\lambda_0) + f'(\lambda_0)(\lambda - \lambda_0)\cdots + f^n(\lambda_0)(\lambda - \lambda_0)^n/n! \quad (5.1)$$

known as the *Taylor* and *Maclaurin* series [8], introduced in 1715. Normally, a global event generates a series of sequential actions, each of which is associated with its opponent reactions, respectively and reciprocally. For any event operation as the functional derivatives, the sum of terms are calculated at an initial state λ_0 and explicitly reflected by a series of the ***Event Operations*** $\lambda_i \in \{\dot{\partial}_{\lambda_1}, \dot{\partial}_{\lambda_2}\dot{\partial}_{\lambda_1}, \cdots, \dot{\partial}_{\lambda_n\lambda_{n-1}\cdots\lambda_1}\}$ in the dual variant forms:

$$f(\lambda) = f_0 + \kappa_1\dot{\partial}_{\lambda_1} + \kappa_2\dot{\partial}_{\lambda_2}\dot{\partial}_{\lambda_1} \cdots + \kappa_n\dot{\partial}_{\lambda_n}\dot{\partial}_{\lambda_{n-1}}\dots\dot{\partial}_{\lambda_1} \quad (5.2a)$$

$$\kappa_n = f^n(\lambda_0)/n!, \quad \lambda_i \in \{\dot{\partial}_{\lambda_i}\} = \{\check{\partial}_\lambda, \check{\partial}^\lambda, \hat{\partial}^\lambda, \hat{\partial}_\lambda\} \quad (5.2b)$$

where κ_n is the coefficient of each order n. The event states of world planes are open sets and can either rise as subspaces transformed from the other horizon or remain confined as independent existences within their own domain, as in the settings of $Y^\mp$ manifolds expending from the world planes. Because the events are operated through the potential fields, it essentially incepts on the world planes a set of the λ_i derivatives, giving rise to the horizon infrastructures, simply given by the above $\phi_n^\mp(\lambda, x) = f(\lambda)\ \phi_n^\mp(\lambda, x)|_{\lambda=\lambda_0}$:

$$\hat{W}_n = \phi_n^+(\lambda, \hat{x})\phi_n^-(\lambda, \check{x}) \quad : \textit{First Type of World Equation} \quad (5.3)$$

$$\phi_n^\mp(\lambda, x) = \left(1 \pm \tilde{\kappa}_1\dot{\partial}_{\lambda_1} \pm \tilde{\kappa}_2\dot{\partial}_{\lambda_2}\dot{\partial}_{\lambda_1} \cdots\right)\phi_n^\mp(\lambda, x)|_{\lambda=\lambda_0} \quad (5.4)$$

where $\phi_n^+(\lambda, \hat{x})$ or $\phi_n^-(\lambda, \check{x})$ is the virtual or physical potential of a particle n, and $\hat{\kappa}_n$ is defined as the world constants. An integrity of the two functions is, therefore, named as *First Type* of ***World Equations***, because the function $\hat{W}_n$ represents that

a) The first two terms $(1 \pm \kappa_1\dot{\partial}_{\lambda_1})$ - The event drives both virtual and physical system and incepts from the world planes systematically breakup and extend into each of the manifolds.

b) The higher terms $\pm(\kappa_2\dot{\partial}_{\lambda_2}\dot{\partial}_{\lambda_1} + \cdots\kappa_i\dot{\partial}_{\lambda_i}\dot{\partial}_{\lambda_{i-1}}\dots\dot{\partial}_{\lambda_1})$ - The event operations transcend further down to each of its sub-coordinate system with extra degrees of freedoms for either physical dimensions $\mathbf{r}(\lambda)$ or virtual dimensions $\mathbf{k}(\lambda)$, reciprocally.

This *World Equation* $\hat{W}_n$ features the virtual supremacy for the processes of creations and annihilations. Amazingly, the higher horizon reveals the principles of *Force Fields,* which include, but are not limited to, and are traditionally known as the *Spontaneous Breaking* and fundamental forces. For the physical observation, the amplitude $|\hat{W}_n|$ features the Y^- behaviors of the forces explicitly while the phase attributes the Y^+ comportment of the superphase actions implicitly.

Once the physical three-dimensions are evolving or developed, the operational function $f(\lambda)$ for the event λ actions involves the local state densities $\rho_n(x)$ and its relativistic spacetime exposition of a system with N objects or particles. Assuming each of the $\phi_n^\pm$ particles is in one of three possible states: |-⟩, |+⟩, and |o⟩, the system has N_n^+ and N_n^- particles at non-zero charges with their reciprocal state functions of $\varphi_n^\mp$ confineable to the respective manifold $Y^\pm$ locally. Therefore, the horizon functions of the system can be expressed by:

$$\check{W}_c = k_w\int\check{W}_b d\Gamma, \quad \check{W}_b = \sum\nolimits_n h_n\check{W}_a, \quad \check{W}_a = f(\lambda)\rho_n \quad (5.5)$$

$$\rho_n = \psi_n^+(\hat{x})\psi_n^-(\check{x}) \quad : \psi_n^\pm \in \{\phi_n^\pm, \varphi_n^\mp\}, h_n = N_n^\pm/N \quad (5.6)$$

where h_n is a horizon factor, $N_n^\pm/N$ are percentages of the Y^-Y^+ particles, and k_w is defined as a world constant. During space and time dynamics, the density $\psi_n^-\psi_n^+$ is incepted at $\lambda = \lambda_0$ and followed by a sequence of the evolutions $\lambda_i \mapsto \dot{\partial}_{\lambda_1}\cdots\dot{\partial}_{\lambda_i\lambda_{i-1}\cdots\lambda_1}$. As a horizon infrastructure, this process engages and applies a series of the event operations of equations (5.2) to the equations of (5.4) in the forms of the following expressions, expressions, named as *Second Type of World Equations*:

$$\check{W}^{\pm} = k_w \int d\Gamma \sum_n h_n \left[W_n^{\pm} + \kappa_1 \check{\partial}_{\lambda_1} + \kappa_2 \check{\partial}_{\lambda_2} \check{\partial}_{\lambda_1} \cdots \right] \psi_n^{+}(\hat{x}) \psi_n^{-}(\check{x}) \quad (5.7)$$

where $\check{W}_n^{\pm} \equiv \check{W}(\hat{x} \mid \check{x}, \lambda_0)$ is the Y^+ or Y^- ground environment or an initial potential density of a system, respectively. This type of *World Equations* features the physical supremacy of kinetic dynamics or field equations as a part of the horizon infrastructure. Although, two types of the *World Equations* might be mathematically equivalent, they represents the real situations further favorable to a variety of variations. Generally, the first type is in the affirmative to the superphase evolutions whereas the second type to the horizon evolutions.

A homogeneous system has a trace of diagonal elements where an observer is positioned external to or outside of the objects. The source of the fields appears as a point object and has the uniform *Conservations* virtually at every point without irregularities in field strength and direction, regardless of how the source itself is constituted with or without its internal or surface twisting torsions.

Whereas, a heterogeneous system has the off-diagonal elements of the symmetric tensors where an observer is positioned internal to or inside of the objects, and the duality of virtual annihilation and physical reproduction are balanced to form the local *Continuity* or *Invariance*.

As the topological framework, various horizons are defined as, but not limited to, timestate, microscopic and macroscopic regimes, each of which is in a separate zone, emerges with its own fields, and aggregates or dissolves into each other as the interoperable neighborhoods, systematically and simultaneously. Through the Y^-Y^+ communications, the expression of the tangent vectors defines and gives rise to each of the horizons.

Artifact 5.1: First Horizon. The field behaviors of individual objects or particles have their potentials of the timestate or superposing functions in forms of, but not limited to, the dual densities, given by the (2.4-2.5) of the density (2.7) ρ_n^+ at Y^+ supremacy and the ρ_n^- (2.8) at Y^- supremacy. This horizon is confined by its neighborhoods of the potential fields and second horizons, which is characterizable by the scalar objects of $\phi^{\pm}$ and $\varphi^{\pm}$ fields of the ground horizon, individually, and reciprocally.

Artifact 5.2: Second Horizon. The effects of aggregated objects have their continuity and commutative entanglements towards the microscopic functions in forms of flux continuity and commutations of Y^+ and Y^- fields, respectively:

$$\mathbf{f}_x^{\pm} = \kappa_x \partial \rho_n^{\pm} \qquad : \mathbf{f}_c^{\pm} = \kappa_c \langle \partial \rangle^{\pm}, \mathbf{f}_s^{\pm} = \kappa_s^{\pm} [\partial]^{\pm} \quad (5.8)$$

This horizon summarizes the timestate functions $\mathbf{f}^{\pm} = \sum_n p_n \mathbf{f}_n^{\pm}$ confined between the first and third horizons of the microscopic forces and statistic p_n distributions.

Artifact 5.3: Fluxion Density and Currents. *Dark Fluxion* is an important type of energy flow, derivative of which gives rise to continuity for electromagnetism while associated with charge distribution, the gravitational force when affiliated with inauguration of mass distribution, or blackholes in connected with dark matters. At the energy $\tilde{E}_n^{\mp}$, the characteristics of time evolution interprets the Y^-Y^+ fluxions $\mathbf{f}_s^{\pm}$ of the densities $\rho_s^{\mp}$ and currents $\mathbf{j}_s^{\mp}$, generated by the first order of energy densities at the second horizon (5.4) as the following:

$$\tilde{\rho}_s^- = \sum_n p_n \left(\phi_n^- + \tilde{\kappa}_1^- \check{\partial}_\lambda \phi_n^- \cdots \right) \left(\varphi_n^+ + \tilde{\kappa}_1^+ \hat{\partial}^\lambda \varphi_n^+ \cdots \right)$$

$$= \sum_n p_n \left\{ \phi_n^- \varphi_n^+ + \frac{\tilde{\kappa}_0^-}{ic} \mathbf{f}_n^- + \tilde{\kappa}_1^- \tilde{\kappa}_1^+ \left(\check{\partial}_\lambda \phi_n^- \right) \wedge \left(\hat{\partial}^\lambda \varphi_n^+ \right) + \cdots \right\} \quad (5.9a)$$

$$\tilde{\rho}_s^+ = \sum_n p_n \left(\phi_n^+ + \tilde{\kappa}_1^+ D_\lambda \phi_n^+ \cdots \right) \left(\varphi_n^- + \tilde{\kappa}_1^- D_\lambda \varphi_n^- \cdots \right)$$

$$= \sum_n p_n \left\{ \phi_n^+ \varphi_n^- + \frac{\tilde{\kappa}_0^+}{ic} \mathbf{f}_n^+ + \tilde{\kappa}_1^- \tilde{\kappa}_1^+ \left(\hat{\partial}^\lambda \phi_n^+ \right) \wedge \left(\check{\partial}_\lambda \varphi_n^- \right) + \cdots \right\} \quad (5.9b)$$

$$\mathbf{f}_s^{\mp} = ic\rho_s^{\mp} + \mathbf{j}_s^{\mp} \qquad : \rho_s^{\mp} = \frac{i\hbar}{2E^{\pm}} \langle \partial_t \rangle_s^{\mp}, \qquad \mathbf{j}_s^{\mp} = \frac{\hbar c}{2E^{\mp}} \langle \mathbf{u} \nabla \rangle_s^{\mp} \quad (5.10)$$

where the wedge circulations $\wedge$ is the nature of the entangling processes. The Y^-Y^+ fluxions $\mathbf{f}_s^{\mp}$ are also known as the classic *Variant Density and Current* of the tetrad-coordinates $\left(ic\rho_s^{\pm}, \mathbf{j}_s^{\pm} \right)$. Upon the internal superphase modulations from the first horizon, the Y^-Y^+ duality inheres and forms up the higher horizons as the micro symmetry of a group community in form of flux continuities, characterized by their entangle components of transformation and standard commutations of the dual-manifolds. As one of the Y^-Y^+ entanglement principles, it results a pair of the fluxion equations: one for Y^- primary and the other Y^+ primary.

Artifact 5.4: Third Horizon. The integrity of massive objects characterizes their global motion dynamics of the matrices and tensors through an integration of, but not limited to, the derivative to the second horizon fields of densities and fluxions, defined as macroscopic *Force Fields*:

$$\dot{\mathbf{F}}^{\pm} = \kappa_{\mathbf{F}}^{\pm} \int \rho_a \dot{\partial} \dot{\mathbf{f}}_s^{\pm} d\Gamma \qquad : \dot{\partial} \in \{ \check{\partial}_\lambda, \hat{\partial}^\lambda \} \quad (5.11)$$

where $\kappa_{\mathbf{F}}^{\pm}$ are coefficients. This horizon is confined by its neighborhoods of the second and fourth horizons and characterizable by the acceleration fields $\dot{\partial} \mathbf{f}_m^{\pm}$.

Artifact 5.5: Continuity Equations of Y^-Y^+ Fluxions. The derivative to the density and current (5.10) represent and extend as the classical continuity equations into a pair of the matrix equations

$$\dot{\partial}_\lambda \mathbf{f}_s^{\pm} = \frac{\partial \rho_s^{\pm}}{\partial t} + (\mathbf{u}^{\pm} \nabla) \cdot \mathbf{j}_s^{\pm} = K_s^{\mp} \qquad : K_s^+ \to 0^+ \quad (5.12)$$

The above equations are defined as *Continuity Equations of Y^-Y^+ Fluxions*, the streaming forms of conservation laws for flows balancing between the Y^- and Y^+ manifolds. The scalar K_s^+ balancing the Y^- continuity is the virtual source of energy, producing Y^- continuity $\dot{\partial}_\lambda \mathbf{f}^-$ of dark fluxions. For a virtual object of energy and momentum, its massless entity to the external observers is cyclic surrounding a point object. Therefore, the Y^+ field may appear as if its virtual source were not existent, or physically empty: $K_s^+ \to 0^+$. The symbol 0^+ means that, although the fluxion may be physically hidden, its Y^+ field rises whenever there is a physical flow as its opponent in tangible interactions or entanglements. As a byproduct, we might redefine the *Aether Theory* in order for its interpretation to be more accurate.

Artifact 5.6: Higher Horizons. The horizon ladder continuously accumulates and gives a rise to the next objects in forms of a ladder hierarchy:

$$\iiiint \cdots \rho_c \dot{\partial} \int \rho_b \dot{\partial} \mathbf{F}^{\pm} d\Gamma \mapsto \check{\mathbf{W}}_x^{\pm} \quad (5.13)$$

They are orchestrated into groups, organs, globes or galaxies.

VI. UNIVERSAL FIELD EQUATIONS

The potential interweavement is a fundamental principle of the real-life streaming such that one constituent cannot be fully described without considering the other. As a consequence, the state of a composite system is always expressible as a sum of products of states of each constituents. Under the law of event operations, they are fully describable by the mathematical framework of the dual manifolds. As a foundation of the potential entanglements, the kinetic dynamics of field equations can be derived by the *World Equations*.

Artifact 6.1: First Universal Field Equations. During the events of the virtual supremacy, a chain of the event actors in the loop flows of Figure 4a and equation (4.1) can be shown by and underlined in the sequence of the following processes:

$$W^+ : (\underrightarrow{\hat{\partial}^{\lambda_1}} \to \underbrace{\hat{\partial}_{\lambda_2}}), (\underrightarrow{\check{\partial}^{\lambda_2}} \to \underbrace{\check{\partial}_{\lambda_3}}); \; W^- : (\underline{\check{\partial}_{\lambda_1}} \to \underleftarrow{\check{\partial}^{\lambda_2}}), (\underline{\hat{\partial}^{\lambda_2}} \to \underleftarrow{\hat{\partial}_{\lambda_3}}) \quad (6.1)$$

From the event actors $\hat{\partial}_{\lambda_2}$ and $\check{\partial}_{\lambda_3}$, the *World Equations* (5.5) becomes:

$$W_a^+ = \left(W_n^+ + \kappa_1 \underbrace{\hat{\partial}_{\lambda_2}} \right) \phi_n^+ \varphi_n^- + \kappa_2 \underbrace{\check{\partial}_{\lambda_3}} \left(\phi_n^+ \hat{\partial}_{\lambda_2} \varphi_n^- + \varphi_n^- \hat{\partial}_{\lambda_2} \phi_n^+ \right) \ldots \quad (6.2)$$

Meanwhile the event actors $\hat{\partial}^{\lambda_1}$ and $\check{\partial}^{\lambda_2}$ turn World Equations into:

$$W_a^{+*} = \left(W_n^- + \kappa_1 \underrightarrow{\hat{\partial}^{\lambda_1}} \right) \varphi_n^+ \phi_n^- + \kappa_2 \underrightarrow{\check{\partial}^{\lambda_2}} \left(\varphi_n^+ \hat{\partial}^{\lambda_1} \phi_n^- + \phi_n^- \hat{\partial}^{\lambda_1} \varphi_n^+ \right) \cdots \quad (6.3)$$

where $W_n^{\pm} = W_n^{\pm}(\mathbf{r}, t_0)$ is the time invariant Y^+Y^--energy area fluxion. Rising from the opponent fields of ϕ_n^- or φ_n^-, the dynamic reactions under the Y^- manifold continuum give rise to the *Motion Operation* (4.4) of the Y^+ fields ϕ_n^+ or φ_n^+ approximated at the first and second orders of perturbations in terms of *Second Type* of *World Equations*:

$$\frac{\partial W_a^+}{\partial \varphi_n^-} = W_n^+\phi_n^+ + \kappa_1\hat{\partial}_{\lambda_2}\phi_n^+ + \kappa_2\check{\partial}_{\lambda_3}\hat{\partial}_{\lambda_2}\phi_n^+ \tag{6.4a}$$

$$\check{\partial}^{\lambda_2}\Big(\frac{\partial W_a^+}{\partial(\underbrace{\hat{\partial}_{\lambda_2}}\ \varphi_n^-)}\Big) = \underleftarrow{\check{\partial}^{\lambda_2}}\big(\kappa_1 + \kappa_2\check{\partial}_{\lambda_3}\big)\phi_n^+ \tag{6.4b}$$

$$\hat{\partial}_{\lambda_3}\Big(\frac{\partial W_a^+}{\partial(\underbrace{\check{\partial}_{\lambda_3}}\ \varphi_n^-)}\Big) = \underleftarrow{\hat{\partial}_{\lambda_3}}\big(\kappa_2\hat{\partial}_{\lambda_2}\phi_n^+\big) \tag{6.4c}$$

$$\frac{\partial W_a^{+*}}{\partial \phi_n^-} = W_n^-\varphi_n^+ + \kappa_1\hat{\partial}^{\lambda_1}\varphi_n^+ + \kappa_2\check{\partial}^{\lambda_2}\hat{\partial}^{\lambda_1}\varphi_n^+ \tag{6.5a}$$

$$\check{\partial}_{\lambda_1}\Big(\frac{\partial W_a^{+*}}{\partial(\underrightarrow{\hat{\partial}^{\lambda_1}}\phi_n^-)}\Big) = \underline{\check{\partial}_{\lambda_1}}\big(\kappa_1 + \kappa_2\check{\partial}^{\lambda_2}\big)\varphi_n^+ \tag{6.5b}$$

$$\hat{\partial}^{\lambda_2}\Big(\frac{\partial W_a^{+*}}{\partial(\underrightarrow{\check{\partial}^{\lambda_2}}\phi_n^-)}\Big) = \underline{\hat{\partial}^{\lambda_2}}\big(\kappa_2\hat{\partial}^{\lambda_1}\big)\varphi_n^+ \tag{6.5c}$$

where the primary potentials of $\hat{\partial}_{\lambda_2}\varphi_n^-$ and $\check{\partial}_{\lambda_3}\varphi_n^-$ give rise simultaneously to their opponent's reactors of the physical to virtual $\check{\partial}^{\lambda_2}$ and the virtual to physical $\hat{\partial}_{\lambda_3}$ transformations, respectively. From these interwoven relationships, the motion operations (4.4) determine a pair of partial differential equations of the Y^-Y^+ state fields ϕ_n^+ and φ_n^+ under the supremacy of virtual dynamics at the $Y\{x^\nu\}$ manifold:

$$\kappa_1\big(\check{\partial}^{\lambda_2} - \hat{\partial}_{\lambda_2}\big)\phi_n^+ + \kappa_2\big(\check{\partial}_{\lambda_3}\check{\partial}^{\lambda_2} + \hat{\partial}_{\lambda_3}\hat{\partial}_{\lambda_2} - \check{\partial}_{\lambda_3}\hat{\partial}_{\lambda_2}\big)\phi_n^+ = W_n^+\phi_n^+ \tag{6.7}$$

$$\kappa_1\big(\check{\partial}_{\lambda_1} - \hat{\partial}^{\lambda_1}\big)\varphi_n^+ + \kappa_2\big(\hat{\partial}^{\lambda_2}\check{\partial}_{\lambda_1} + \hat{\partial}^{\lambda_2}\hat{\partial}^{\lambda_1} - \check{\partial}^{\lambda_2}\hat{\partial}^{\lambda_1}\big)\varphi_n^+ = W_n^+\varphi_n^+ \tag{6.8}$$

giving rise to the Y^+ *General Fields* from each respective opponent during their physical interactions.

In the events of the physical supremacy in parallel fashion, a chain of the event actors in the flows of Figure 4a and equation (4.2) can be shown by the similar sequence of the following processes:

$$W^- : (\underline{\check{\partial}_{\lambda_1}} \to \underbrace{\check{\partial}^{\lambda_2}}),(\hat{\partial}^{\lambda_2} \to \underbrace{\hat{\partial}_{\lambda_3}});\ W^+ : (\hat{\partial}^{\lambda_1} \to \hat{\partial}_{\lambda_2}),(\check{\partial}^{\lambda_2} \to \check{\partial}_{\lambda_3}) \tag{6.9}$$

$$W_a^- = \big(W_n^- + \kappa_1\underline{\check{\partial}_{\lambda_1}}\big)\ \varphi_n^+\phi_n^- + \kappa_2\underline{\hat{\partial}^{\lambda_2}}\big(\varphi_n^+\check{\partial}_{\lambda_1}\phi_n^- + \phi_n^-\check{\partial}_{\lambda_1}\varphi_n^+\big)\cdots \tag{6.10}$$

$$W_a^{-*} = \big(W_n^- + \kappa_1\ \underbrace{\check{\partial}^{\lambda_2}}\big)\phi_n^+\varphi_n^- + \kappa_2\ \underbrace{\hat{\partial}_{\lambda_3}}\ \big(\phi_n^+\check{\partial}^{\lambda_2}\varphi_n^- + \varphi_n^-\check{\partial}^{\lambda_2}\phi_n^+\big)\cdots \tag{6.11}$$

where $W_n^- = W_n^-(\mathbf{r}, t_0)$ is the time invariant Y^--energy area fluxion. Rising from its opponent fields of ϕ_n^+ or φ_n^+ in parallel fashion, the dynamic reactions (6.1) under the Y^+ manifold continuum give rise to the *Motion Operations* (4.3) of the Y^- state fields ϕ_n^- or φ_n^-, which determine a pair of linear partial differential equations of the state function ϕ_n^- or φ_n^- under the supremacy of physical dynamics at the $Y\{x_m\}$ manifold:

$$\kappa_1\big(\hat{\partial}^{\lambda_1} - \check{\partial}_{\lambda_1}\big)\phi_n^- + \kappa_2\big(\hat{\partial}^{\lambda_2}\hat{\partial}^{\lambda_1} + \hat{\partial}^{\lambda_2}\check{\partial}_{\lambda_1} - \check{\partial}^{\lambda_2}\check{\partial}_{\lambda_1}\big)\phi_n^- = W_n^-\phi_n^- \tag{6.12}$$

$$\kappa_1\big(\hat{\partial}_{\lambda_2} - \check{\partial}^{\lambda_2}\big)\varphi_n^- + \kappa_2\big(\hat{\partial}_{\lambda_3}\hat{\partial}_{\lambda_2} + \check{\partial}_{\lambda_3}\check{\partial}^{\lambda_2} - \hat{\partial}_{\lambda_3}\check{\partial}^{\lambda_2}\big)\varphi_n^- = W_n^-\varphi_n^- \tag{6.13}$$

giving rise to the Y^- *General Fields* from each of the respective opponents during their virtual interactions. For the sake of a completeness, the motion operations (4.3) that derives the above equations similar to a set of (6.4, 6.5) are shown as below:

$$\frac{\partial W_a^-}{\partial \varphi_n^+} = W_n^-\left(\mathbf{x}, t_0\right)\phi_n^- + \kappa_1\check{\partial}_{\lambda_1}\phi_n^- + \kappa_2\hat{\partial}^{\lambda_2}\check{\partial}_{\lambda_1}\phi_n^- \tag{6.14a}$$

$$\hat{\partial}^{\lambda_1}\Big(\frac{\partial W_a^-}{\partial(\overline{\check{\partial}_{\lambda_1}}\varphi_n^+)}\Big) = \underbrace{\hat{\partial}^{\lambda_1}}\left(\kappa_1 + \kappa_2\hat{\partial}^{\lambda_2}\right)\phi_n^- \tag{6.14b}$$

$$\check{\partial}_{\lambda_2}\Big(\frac{\partial W_a^-}{\partial(\underline{\hat{\partial}^{\lambda_2}}\varphi_n^+)}\Big) = \underbrace{\check{\partial}^{\lambda_2}}\left(\kappa_2\check{\partial}_{\lambda_1}\phi_n^-\right) \tag{6.14c}$$

$$\frac{\partial W_a^{-*}}{\partial \phi_n^+} = W^+\varphi_n^- + \kappa_1\check{\partial}^{\lambda_2}\varphi_n^- + \kappa_2\hat{\partial}_{\lambda_3}\check{\partial}^{\lambda_2}\varphi_n^- \tag{6.15a}$$

$$\hat{\partial}_{\lambda_2}\Big(\frac{\partial W_a^{-*}}{\partial(\check{\partial}^{\lambda_2}\phi_n^+)}\Big) = \hat{\partial}_{\lambda_2}\big(\kappa_1 + \kappa_2\hat{\partial}_{\lambda_3}\big)\varphi_n^- \tag{6.15b}$$

$$\check{\partial}_{\lambda_3}\Big(\frac{\partial W_a^{-*}}{\partial(\hat{\partial}_{\lambda_3}\phi_n^+)}\Big) = \check{\partial}_{\lambda_3}\big(\kappa_2\check{\partial}^{\lambda_2}\varphi_n^-\big) \tag{6.15c}$$

where the primary potentials of the local dynamics $\check{\partial}_{\lambda_1}\varphi_n^+$ and $\hat{\partial}^{\lambda_2}\varphi_n^+$ give rise simultaneously to their opponent's reactors of the virtual animation $\hat{\partial}^{\lambda_1}$ and the physical to virtual transformation $\check{\partial}^{\lambda_2}$, respectively.

Under the *Universal Topology*, the two pairs of the dynamic fields (6.7, 6.8) and (6.12, 6.13) are operated generically under horizon of the *World Events*. Together, the four formulae are named as ***First Universal Field Equations***, because they are fundamental and general to all fields of natural evolutions.

Artifact 6.2: Second Universal Field Equations. In the global environment, the Y^-Y^+ virtual energies have their commutations at operational uniformity to maintain a duality of their equal primacy. From the density equations, the physical events simultaneously operate another dual state $\{\phi_n^+, \varphi_n^-\}$ and their movements, $\dot{\partial}_\lambda\left(\varphi_n^+\phi_n^-\right)$, that give rise to the Y^+ fluxions of continuity. After the rearrangement given by the equations of (6.7) paired with (6.13), the successive operations entangle the scalar potentials in fluxions streaming a set of the Y^+ *Universal Fields* into another pair of the Y^-Y^+ motion dynamics:

$$\begin{aligned}\kappa_2\varphi_n^-\big(\hat{\partial}_{\lambda_3}\hat{\partial}_{\lambda_2}\big)\phi_n^+ &= \varphi_n^-W_n^+\phi_n^+ - \kappa_1\varphi_n^-\big(\check{\partial}^{\lambda_2} - \hat{\partial}_{\lambda_2}\big)^+\phi_n^+ \\ &\quad -\kappa_2\varphi_n^-\check{\partial}_{\lambda_3}\big(\check{\partial}^{\lambda_2} - \hat{\partial}_{\lambda_2}\big)\phi_n^+\end{aligned} \tag{6.16}$$

$$\begin{aligned}\kappa_2\phi_n^+\big(\check{\partial}^{\lambda_3}\check{\partial}^{\lambda_2}\big)\varphi_n^- &= \kappa_1\phi_n^+\big(\check{\partial}^{\lambda_2} - \hat{\partial}_{\lambda_2}\big)\varphi_n^- - \kappa_2\phi_n^+\big(\check{\partial}_{\lambda_3} - \hat{\partial}_{\lambda_2}\big)\check{\partial}^{\lambda_2}\varphi_n^- \\ &\quad +\kappa_2\phi_n^+\big(\check{\partial}^{\lambda_3}\check{\partial}^{\lambda_2}\big)\varphi_n^- - \kappa_2\phi_n^+\big(\hat{\partial}_{\lambda_3}\hat{\partial}_{\lambda_2}\big)\varphi_n^- + \phi_n^+W_n^-\varphi_n^-\end{aligned} \tag{6.17}$$

Add the above two equations together, we constitute a commutation of the Y^+ fluxion of density continuity $\dot{\partial}_\lambda\mathbf{f}_s^+ = \kappa_f\langle\hat{\partial}_\lambda\hat{\partial}_\lambda, \check{\partial}^\lambda\check{\partial}^\lambda\rangle_s^+$ in forms of the Y^+ symmetric formulation:

$$\dot{\partial}_\lambda\mathbf{f}_s^+ = \langle W_0\rangle^+ - \kappa_1\big[\check{\partial}^{\lambda_2} - \hat{\partial}_{\lambda_2}\big]_s^+ + \kappa_2\big\langle\check{\partial}_{\lambda_3}\big(\hat{\partial}_{\lambda_2} - \check{\partial}^{\lambda_2}\big)\big\rangle_s^+ + \mathbf{g}_a^-/\kappa_g^- \tag{6.18}$$

Representing a duality of the entangling environments as the dark flux continuity of the potential densities, a pair of potentials $\{\phi_n^+, \varphi_n^-\}$ is not only mapped the Y^+ fluxion to their symmetric commutation at second horizon and continuity at the third horizon, but also associated with an Y^- asymmetric accelerator $\mathbf{g}_a^-$.

In a parallel fashion, another dual state fields $\{\phi_n^-, \varphi_n^+\}$ in the dynamic equilibrium can be rewritten by φ_n^+ times (6.12) and ϕ_n^- times (6.8).

$$\begin{aligned}\kappa_2\varphi_n^+\big(\hat{\partial}^{\lambda_2}\hat{\partial}^{\lambda_1}\big)\phi_n^- &= \varphi_n^+W_n^-\phi_n^- + \varphi_n^+\kappa_1\big(\check{\partial}_{\lambda_1} - \hat{\partial}^{\lambda_1}\big)\phi_n^- \\ &\quad -\kappa_2\big(\check{\partial}^{\lambda_2} - \hat{\partial}^{\lambda_2}\big)\check{\partial}_{\lambda_1}\phi_n^-\end{aligned} \tag{6.19}$$

$$\begin{aligned}\kappa_2\phi_n^-\big(\check{\partial}_{\lambda_2}\check{\partial}_{\lambda_1}\big)\varphi_n^+ &= \phi_n^-W_n^+\varphi_n^+ - \phi_n^-\kappa_1\big(\check{\partial}_{\lambda_1} - \hat{\partial}^{\lambda_1}\big)\varphi_n^+ \\ &\quad +\kappa_2\check{\partial}^{\lambda_2}\big(\hat{\partial}^{\lambda_1} - \check{\partial}_{\lambda_1}\big)\varphi_n^+ + \kappa_2\phi_n^-\big(\check{\partial}_{\lambda_2}\check{\partial}_{\lambda_1}\big)\varphi_n^+ - \kappa_2\phi_n^-\big(\hat{\partial}^{\lambda_2}\hat{\partial}^{\lambda_1}\big)\varphi_n^+\end{aligned} \tag{6.20}$$

Adding the two formulae, we institute Y^- fluxion of density continuity $\dot{\partial}_\lambda\mathbf{f}_s^- = \langle\check{\partial}_\lambda\check{\partial}_\lambda, \hat{\partial}^\lambda\hat{\partial}^\lambda\rangle_s^-$ of the Y^- general formulation:

$$\dot{\partial}_\lambda\mathbf{f}_s^- = \langle W_0^-\rangle + \kappa_1\big[\check{\partial}_{\lambda_1} - \hat{\partial}^{\lambda_1}\big]_s^- + \kappa_2\big\langle\check{\partial}_{\lambda_1}\big(\hat{\partial}^{\lambda_2} - \check{\partial}^{\lambda_2}\big)\big\rangle_s^- + \mathbf{g}_a^+/\kappa_g^+ \tag{6.21}$$

where $\mathbf{g}_a^+$ is an Y^+ asymmetric accelerator. The entangle bracket $\dot{\partial}_\lambda\mathbf{f}_s^- = \langle\check{\partial}_\lambda\check{\partial}_\lambda, \hat{\partial}^\lambda\hat{\partial}^\lambda\rangle_s^-$ of the general dynamics features the Y^- continuity for their scalar potentials.

Consequently, driving the field dynamics at the second horizons, the system of N particles aggregates into fluxion domain associated with continuity at the second horizon developing the density commutation into the third horizon. These processes represent a set of the universal laws as the following,

1) Incepted in the virtual world, the events not only generate its opponent reactions but also create and conduct the real-life objects in the physical world, because the element embeds the bidirectional reactions $\hat{\partial}_\lambda$ and $\check{\partial}^\lambda$ entangling between the Y^-Y^+ manifolds, symmetrically and asymmetrically.

2) Initiated in the physical world, the events have to leave a life copy of its mirrored images in the virtual world without an intrusive effect into the virtual world, because the asymmetric element doesn't have the reaction $\hat{\partial}_\lambda$ to the Y^- manifold. In other words, the virtual world is aware of and immune to the physical world.

3) The fluxions are developed or operated by the symmetric commutative dynamics $[\ ]^\pm$ that give rise to the flux continuity $\langle\ \rangle^\pm$ of the next horizon and associate the accelerators $\mathbf{g}_a^\pm$ asymmetrically.

Because the virtual resources are massless, the accelerator might appear as if it were nothing or at zero resources $\mathbf{g}_a^+ \mapsto 0^+$, unlike the $\mathbf{g}_a^-$ as a physical accelerator. The Y^- or Y^+ supremacy of the flux continuity operates the symmetric commutation and continuity internally as well as the asymmetric acceleration of the system observable externally.

Artifact 6.3: Third Universal Field Equations. Under the Y^-Y^+ environment, it contains the energy continuity as the physical or virtual resources. The equations (6.18, 6.21) of the commutation fluxion $\dot{\partial}_\lambda \mathbf{f}^\pm$ give rise to both of the acceleration tensors $\mathbf{g}_a^\pm = \kappa_g^\pm \dot{\partial}_\lambda \mathbf{f}_a^\pm$ for motion dynamics and interactions balancing the virtual or physical forces, asymmetrically or resourcefully:

$$\mathbf{g}_a^- / \kappa_g^- = \left[\check{\partial}^\lambda \check{\partial}^\lambda, \hat{\partial}_\lambda \hat{\partial}_\lambda\right]_x^+ + \zeta^+ \quad : \zeta^+ = \left(\hat{\partial}_{\lambda_2} \check{\partial}^{\lambda_2} - \hat{\partial}_{\lambda_2} \check{\partial}_{\lambda_3}\right\rangle^+ \quad \mathbf{(6.22)}$$

$$\mathbf{g}_a^+ / \kappa_g^+ = \left[\check{\partial}_\lambda \check{\partial}_\lambda, \hat{\partial}^\lambda \hat{\partial}^\lambda\right]_x^- + \zeta^- \quad : \zeta^- = \left(\check{\partial}^{\lambda_2} \hat{\partial}^{\lambda_1} - \hat{\partial}^{\lambda_2} \check{\partial}_{\lambda_1}\right\rangle^- \quad \mathbf{(6.23)}$$

$$\left[\hat{\partial}_\lambda \hat{\partial}_\lambda, \check{\partial}^\lambda \check{\partial}^\lambda\right]_x^\pm \equiv \phi_n^\pm \left[\hat{\partial}_\lambda \hat{\partial}_\lambda, \check{\partial}^\lambda \check{\partial}^\lambda\right] \varphi_n^\mp \quad (6.24)$$

where the index x refers to either of the scalar or vector potential and the symbol [ ] is a commutator of *Lie* bracket. Apparently, a force is represented as and given by an asymmetric accelerator. Since the physical world is riding on the world planes where the virtual world is primary and dominant, the acceleration at a constant rate in universe has its special meaning different from the spacetime manifold. Mathematically when $\mathbf{g}_a^\pm = 0$, the above formulae can develop further applications without the $\phi_n^\pm$ potentials, because they are canceled out from the equations. Therefore, connected seamlessly to *Riemannian* geometry [15, 20], they are essential to our cosmology of the universe.

CONCLUSION

Universal Topology $W = P \pm iV$, has revealed a set of the following discoveries (in August 2016) or groundbreakings:

1) To align closely with life-streams of our natural world, the ***Dual complex manifolds*** are established that overcomes the limitations of a single spacetime manifold.

2) ***Two pairs of the potential fields*** lies at the heart of the field theory for the fundamental interactions among the virtual dark energies and physical motion dynamics. Naturally, one of its applications derives *Gauge* theory of quantum invariant.

3) ***Mathematical Framework*** is imperatively regulated on a new theoretical foundation by the dual variances to intimately mimic event actions of transform and transport processes.

4) Both of the ***Boost Generators*** and ***Twist Coordinators*** are manifested by the entangling alternators (3.1, 3.2, 3.5, 3.6), which lies at the heat of the light and gravitational fields: photon and graviton.

5) ***Law of Event Processes*** lies at the heart of the field entanglements reciprocally and consistently as the fundamental flows of loop interactions for the dark field entanglements.

6) ***Motion Operations*** are further regulated on and performed with a new theoretical foundation of the dual events intimately mimic operational actions (4.3, 4.4) on the geodesic covertures, extend the meanings to the *Euler-Lagrange Motion Equation.*

7) ***World Equations*** align a series of the infinite sequential actions (5.3, 5.7) concisely with potential-streams of the event operations that overcome the limitations of classical *Lagrangian* representations.

8) Four pairs of ***Universal Field Equations*** of (6.7, 6.8), (6.12, 6.13), (6.18, 6.21) and (6.22, 6.23) are discovered as three sets of general formulae, which are grounded for all horizon fields of *Universal and Unified Physics.*

As a result, it has laid out a ground foundation towards a unified physics that gives rise to the fields of quantum, photon, electromagnetism, graviton, gravitation, thermodynamics, ontology, cosmology, and beyond.

REFERENCES

1. Bryce Seligman DeWitt, R. Neill Graham, eds, The Many-Worlds Interpretation of Quantum Mechanics, Princeton Series in Physics, Princeton University Press (1973), ISBN 0-691-08131-X Contains Everett's thesis: The Theory of the Universal Wavefunction, pp 3–140.
2. Schrödinger, Erwin (November 1935). "Die gegenwärtige Situation in der Quantenmechanik (The present situation in quantum mechanics)". Naturwissenschaften. 23 (48): 807–812. doi:10.1007/BF01491891.
3. Hugh Everett (1956, 1973), Theory of the Universal Wavefunction, Thesis, Princeton University, pp 1–140
4. Everett, Hugh (1957). "Relative State Formulation of Quantum Mechanics". Reviews of Modern Physics. 29 (3): 454–462. Bibcode:1957RvMP... 29..454E. doi:10.1103/RevModPhys.29.454.
5. Exirifard, Q. (2010). "Phenomenological covariant approach to gravity". General Relativity and Gravitation. 43: 93–106. arXiv:0808.1962 . Bibcode: 2011GReGr..43...93E. doi:10.1007/s10714-010-1073-6.
6. Cecile M. DeWitt, John A. Wheeler eds, The Everett–Wheeler Interpretation of Quantum Mechanics, Battelle Rencontres: 1967 Lectures in Mathematics and Physics (1968)
7. Christoffel, E.B. (1869), "Ueber die Transformation der homogenen Differentialausdrücke zweiten Grades", Journal für die reine und angewandte Mathematik, B70: 46–70
8. Taylor, Brook (1715). Methodus Incrementorum Directa et Inversa [Direct and Reverse Methods of Incrementation] (in Latin). London. p. 21–23.
9. Courant, R; Hilbert, D (1953). Methods of Mathematical Physics. Vol. I. New York: Interscience Publishers, Inc. ISBN 978-0471504474.
10. Landau, L. D. & Lifshitz, E. M. (1975). Classical Theory of Fields (Fourth Revised English Edition). Oxford: Pergamon. ISBN 0-08-018176-7.
11. Bryce Seligman DeWitt, The Many-Universes Interpretation of Quantum Mechanics, Proceedings of the International School of Physics "Enrico Fermi" Course IL: Foundations of Quantum Mechanics, Academic Press (1972)
12. Aguirre, A & Tegmark, M (2004). Multiple universes, cosmic coincidences, and other dark matters. arXiv:hep-th/0409072
13. Xu, Wei; Unified Field Theory - Universal Topology and First Horizon of Quantum Fields, International Journal of Physics. 2017, 5(1), 16-20 doi: 10.12691/ijp-5-1-3
14. Jonathan G. Richens, John H. Selby, and Sabri W. Al-Safi. (Aug. 2017) "Entanglement is Necessary for Emergent Classicality in All Physical Theories." Physical Review Letters 119(8). DOI: 10.1103/PhysRevLett. 119.080503
15. Lee, J. M. (1997). Riemannian Manifolds – An Introduction to Curvature. Springer Graduate Texts in Mathematics. 176. New York Berlin Heidelberg: Springer Verlag. ISBN 978-0-387-98322-6.
16. Born, Max (1926). "I.2". In Wheeler, J. A.; Zurek, W. H. Zur Quantenmechanik der Stoßvorgänge [On the quantum mechanics of collisions]. Princeton University Press (published 1983). pp. 863–867. Bibcode:1926ZPhy(37)863B. doi:10.1007/BF01397477. ISBN 0-691-08316-9.
17. Pauli, Wolfgang (1941). "Relativistic Field Theories of Elementary Particles". Rev. Mod. Phys. 13: 203–32. Bibcode:1941RvMP...13..203P. doi:10.1103/revmodphys.13.203.
18. Yang C. N., Mills R. L. (1954). "Conservation of Isotopic Spin and Isotopic Gauge Invariance". Phys. Rev. 96: 191–195. Bibcode:1954PhRv...96..191Y. doi:10.1103/PhysRev.96.191.
19. Heavens, O. S.; Ditchburn, R. W. (1991). Insight into Optics. John Wiley & Sons. ISBN 978-0-471-92769-3.
20. Petersen, Peter (2006), Riemannian Geometry, Berlin: Springer-Verlag, ISBN 0-387-98212-4
21. Ricci, Gregorio; Levi-Civita, Tullio (March 1900), "Méthodes de calcul différentiel absolu et leurs applications" (PDF), Mathematische Annalen, Springer, 54 (1–2): 125–201, doi:10.1007/BF01454201
22. Poincaré, Henri (1905–1906), "Sur la dynamique de l'électron" [On the Dynamics of the Electron], Rendiconti del Circolo matematico di Palermo, 21: 129–176, doi:10.1007/BF03013466