Dialogues on various relativistic paradoxes

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The author attempts to give a self-contained view of various paradoxes in the theory of relativity, and provides an extensive discussion of them in hopefully elementary terms.

I. FIRST DAY: THE USUAL TWIN PARADOX

SALVIATI: We are met today to discuss various paradoxes which our friend Simplicio has discovered in various publications he has recently read. Let us see whether indeed contradictions to the theory of relativity do arise, since this would no doubt be a major discovery.
SAGREDO: I am eager to know about such developments, and Simplicio is surely the right person to tell us all about it.
SIMPLICIO: Let me first quote the exact text [11] from the author, so there may be no doubt about what is meant:
"The French physicist Langevin in 1911, six years after the publication of Special Relativity, pointed out a contradiction that was present inside the dilation time by the famous twin paradox. He considered two twins with the same age, who at an initial time $t = 0$, referred to the reference frame S of the Earth, decided to go one's separate ways. One of the two twins departed with a spaceship that travelled with the constant and rectilinear speed v in order to reach a star placed at a distance d from the Earth, while the second twin stayed at Earth. The twins are in an inertial physical situation because of the inertial motion of the travelling twin with respect to the fixed twin. In Special Relativity a time dilation is theorized in this situation for which if the travelling twin spends a time T', measured with respect to his moving reference frame S', for completing his round trip, twin's clock who stays at Earth measures a dilated time $T > T'$ for the same trip. Naturally it seemed a contradiction to Langevin because if $t = t' = 0$ is the departure time of the travelling twin, on his return he has spent a time $t' = T'$ for completing the round trip in concordance with his clock, while the clock of the fixed twin would measure a time $t = T > T'$ with respect to his reference frame S. In the moment of reunion of twins, after that the trip is terminated, the twins are both again into the fixed reference frame of the Earth and consequently the spent time for the twins is the same, independently of prospective different times measured by the two clocks for which necessarily it is $T' = T$. It follows that the two reference frames proceed synchronous. Supporters of SR [special relativity] have criticized this paradox asserting that in actuality it needs to consider trip intervals in which the spaceship doesn't have constant speed but it would undergo acceleration periods and deceleration periods in the starting time, in the turn time of motion and in the arrival time, but these considerations have no sense because anyway, also in the presence of accelerations and decelerations, at last the twins however reunite and the spent time is the same for both."
SALVIATI: Merely as a matter of detail, I would like to point out that Langevin never viewed the twin effect as a paradox: he states, for example, immediately after having described the peculiar effect, and as a conclusion to his paper [7]: "Ceci montre par un exemple frappant à quelles conséquences éloignées des conceptions habituelles conduit la forme nouvelle des notions d'espace et de temps. Il faut se souvenir que c'est là le développement parfaitement correct de conclusions imposées par des faits expérimentaux indiscutables, dont nos ancêtres n'avaient pas connaissance lorsqu'ils ont constitué, d'après leur expérience que synthétisait le mécanisme, les catégories de l'espace et du temps dont nous avons hérité d'eux. A nous de prolonger leur oeuvre en poursuivant avec une minutie plus grande, en rapport avec les moyens dont nous disposons, l'adaptation de la pensée aux faits." (Rough translation: This shows, through a striking example, how distant from usual ideas are the consequences to which leads the new nature of the concepts of space and time. It must be borne in mind that this is the perfectly correct development of conclusions imposed by indisputable experimental facts, of which our ancestors had no knowledge when they built up, according to their experience which was summarised in the mechanistic worldview, these categories of space and time which we have inherited from them. It is up to us to carry their work further, pursuing the adaptation of thought to facts with a greater nicety, in relation to the resources we possess.)
SAGREDO: You are kind, Salviati, in viewing this as a matter of detail! For me, this attempt to use a great name in physics to give weight to the author's claims, when in fact no such support exists, is very close to dishonesty.
SIMPLICIO: I hope, indeed I am sure, that the author had no knowledge of this.

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SALVIATI: I gladly believe this and mention this only to avoid the impression that a general agreement exists as to the paradoxical nature of this effect. In fact, this has been discussed in a large number of earlier papers, of which one of the oldest, and in a sense among the simplest, is by Einstein himself [4].

But maybe it is easier to let you discover the problems in these arguments by yourselves. So let me remind you of the basics of the theory of relativity: we have essentially the two requirements, namely first that the speed of light should be the same in all reference frames, and second, that it should be always impossible to determine, between two reference frames in uniform rectilinear motion with respect to each other, which is in motion and which is at rest. From these assumptions, one finds that it is necessary to be quite explicit about what one means when one states that two distant clocks *show the same time*: there is no problem with this in the traditional, nonrelativistic view: indeed, in that case, velocities are always composed by addition: if a man walks at 2 meters per second on a train which goes at 20 meters per second with respect to the embankment, and if the man walks in the same direction as the train, then the man walks, with respect to the embankment, at a speed of 22 meters per second.

Under these circumstances, it is clearly possible to create arbitrarily rapid motions. This allows for an easy definition of simultaneity: an event A happens before another event B if there is a signal which starts from A and reaches the position where B will happen, before it does. Two events are then called simultaneous if neither A happens before B, nor B before A. In classical mechanics, simultaneity in that sense is enough to guarantee that both events happen at the same time, since signalling can occur arbitrarily fast.

In the theory of relativity, we cannot have signals faster than the speed of light, so we cannot specify when two clocks show the same time in this simple manner. We therefore *define* two distant clocks $C1$ and $C2$ to be synchronised if, when both emit a light signal, say, at $t = 0$ in the direction of the other clock, these signals cross at the midpoint between $C1$ and $C2$. Of course, this definition is only sensible if $C1$ and $C2$ are at rest with respect to each other.

Using this definition and the above hypotheses, it is easy to derive the Lorentz transformations, from which two interesting consequences [7] follow, which I formulate as a set of two *Rules*, one concerning Space, the other involving Time:

Rule S) Let two events $E1$ and $E2$ be simultaneous in a given reference frame S. Then the distance separating the positions at which these two events occur, is *shorter* than the length between these events' positions as viewed from any other uniformly moving reference frame S'.

Rule T) Let two events $E1$ and $E2$ occur at the same position in a given reference frame S. Then the time interval separating the instants at which these two events occur, is *shorter* than the time interval separating the instants at which these two events occur, as viewed from any other uniformly moving reference frame S'.

We can be more specific and say that the length or durations involved differ by a factor γ, where γ is a number larger than one defined by the relative velocity of the two reference frames involved. It is explicitly given by the formula:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{1}$$

An example we will often be using is the case in which v is $4c/5$ of the speed of light, which yields $\gamma = 5/3$, as can easily be calculated.

These consequences of the Lorentz transformations, viewed as general "rules", easily lead to a *correct* formulation of the concepts of "length contraction" and "time dilation". Let me therefore go through a few examples.

A rod, which measures 1 meter in its rest frame S', moves at speed v with respect to the frame S. The observer at S measures the distance between the positions of the moving rod's two ends simultaneously with respect to S. Is the result more or less than one meter?

SAGREDO: The answer should not be hard. The two events are simultaneous in S. By Rule S, their distance in S, which is what you ask for, is thus less than the distance viewed from any other reference frame, in particular, it must be less than the distance as viewed from S'. But in S', the two events happen at positions determined by the rod's endpoints, which, by assumption, are a meter apart as seen from S'. The distance as measured in S is therefore less than 1 meter, which must be the Lorentz contraction. Using the remark you made, it should in fact be $1/\gamma$ meter, in other words, 60 cm if $v = 4c/5$.

SALVIATI: Quite right! And what happens if, instead, the two ends of the rod emit, say, a light beam simultaneously in S', which is instantly observed in S? Are the positions of these two events as measured in S, closer or further apart than 1 meter?

SAGREDO: The two events are now simultaneous in S', so their distance is shortest in that reference frame. The distance in S' is, of course, still 1 meter, so the distance measured in S is *longer* than 1 meter. In fact, it should be γ meters, which amounts to about 1 meter and 33 cm if $v = 4c/5$.

This certainly shows the need for precision, indeed, when we speak of "length contraction"! Depending on issues which can easily be glossed over, it is quite as apt to be dilation as contraction.

SALVIATI: Exactly so! Now what happens if a clock, at rest in a system S', passes at velocity v two clocks at rest in the reference frame S? Assume that, when the moving clock passes the first clock of S, both show time equal to 12 o'clock. Assume further that, when the moving clock passes the second resting clock, the latter shows time equal to 1 o'clock. Will the moving clock show a time larger or less than 1 o'clock?
SAGREDO: The moving clock, in its own reference frame, is always at the same position. According to Rule T, the time interval it registers between these two events must therefore be less than the one registered by the clocks at rest. The "moving clock goes slow" mantra seems to be fully justified here. The interval should be $1/\gamma$ hours, that is, for $v = 4c/5$, the clock will show 12:36.
SALVIATI: Exactly! But now let us view the case of two moving clocks, again synchronised in their rest frame, which pass before one single clock. The clock at rest shows 12 o'clock as the first clock passes by, and 1 o'clock as the second one does. If the first clock, upon passing, also showed 12 o'clock, what does the second clock show?
SAGREDO: Again, since the clock at rest is always at the same position, the time intervals it registers must always be less than any other time intervals between the same two events. Therefore, the second clock will mark a time larger than 1 o'clock. Specifically, in the case we consider, the second moving clock should show 1:40 minutes for $v = 4c/5$. That is, if we measure things in this way, moving clocks go fast!

But how do we understand these two rules you have just shown us?
SALVIATI: They follow very easily from the Lorentz transformations, which themselves follow straightforwardly from the various assumptions of the theory of relativity. In Appendix A we derive the Lorentz transformations; (A17) together with the earlier two equations (A15, A16) then lead immediately to both rules.

These are the correct forms for "length contraction" and "time dilation". It is emphatically *not correct* to act as if any length whatsoever were contracted if it in any way moves. Similarly, we may not say that all durations between two events are immediately dilated. A large number of errors can be avoided by thinking such things over carefully. In doubt, one should simply go back to the Lorentz transformations.
SIMPLICIO: But in part it is the consistency of the theory of relativity which is in cause. Are we then not allowed to doubt the correctness of the Lorentz transformations, or the validity of the synchronisation procedures used?
SAGREDO: Actually, I do not think so. If you wish to show a contradiction in a theory, for example in the theory of relativity, you should first accept the theory's assumptions, and then, from these and these alone, obtain a contradiction. It is clear enough that, if you introduce additional assumptions in the theory, it is no more clear whether the contradictions arise from the theory of relativity itself or from your additional assumptions.
SALVIATI: Indeed, if we have time, I will gladly discuss the consequences of other synchronisation procedures. But under these circumstances we must, first of all, realise that the Lorentz transformations are in general no more valid [17], and that we must rethink everything, because we cannot use the same formulae as always. If we do attempt to reason using a different synchronisation procedure, and yet maintaining the usual formulae of the Lorentz transformation, we will most likely get into contradictions, which do not, however, in any way reflect on the theory of relativity, but on an inconsistent way of performing the computations.
SIMPLICIO: Can we now describe the twin problem in these terms?
SALVIATI: I think there is no difficulty here. We have three events: the first occurs when both twins are on Earth and take leave from each other. This event takes place at $x = 0$ and $t = 0$, say, which also corresponds to $x' = 0$ and $t' = 0$. We assume the travelling twin, call him Jim, to be always in inertial motion, except for negligible times. These times, in which Jim is accelerated, are important for some issues of principle, but do not affect the calculations, if we assume that the accelerations do not take up a large fraction of the time of the trip. Finally, to be specific, let us call Dan's reference frame D, Jim's reference frame in the first part of the trip J_1, and Jim's reference frame in the second part of the trip J_2
SIMPLICIO: I believe we can agree on that.
SALVIATI: Excellent! Now the second event arises when Paul reaches a distance L, by the reckoning of the twin who stayed at home, whom we shall call Dan. There Jim plans to turn around. But we maintain for the time being his velocity at v.
SAGREDO: Let me see if I can do the work myself: we are now at a distance L in the reference frame S of Dan. The time, as viewed by Dan, must be L/v, where v is Jim's velocity, by old fashioned definition of velocity.
SIMPLICIO: Indeed!
SAGREDO: Let us now go over to Jim's rest frame. There he, of course, remains at the same position throughout his trip. By Rule T, the time spent between his departure and his arrival at turnaround as viewed from J_1 must be less than the time between the same two events as viewed from D. But that last was L/v, so the time as observed by Jim must be less. By your earlier remark, it must be less by a factor γ, so that, by his own reckoning, Jim reaches turnaround at a time $L/(\gamma v)$.
SALVIATI: That is correct.
SIMPLICIO: And I believe we are nearing the contradiction.

SAGREDO: The rest appears to be simple enough: the way back is symmetric, so neglecting the time needed to accelerate, which we assume to be short with respect to the whole trip, Jim will be back after Dan has lived for $2L/v$ years, whereas Jim's clocks will only have marked $2L/(\gamma v)$ years, which can be quite different, if we assume, say, $\gamma = 5/3$, which follows if Jim travels at $v = 4c/5$.
SIMPLICIO: So, at this speed, if the trip lasted 10 years for Dan, 5 one way, 5 the other, it would have been a mere 6 years for Jim.
SALVIATI: I think this is indeed the simplest way to work out the calculation, which altogether avoids the problems linked to acceleration.
SAGREDO: So far, however, I see no contradiction. Of course, once Jim and Dan are happily reunited, there is no question that the rate at which they will both experience the time they spend together, is the same: if that is what the author means when he says "In the moment of reunion of twins, after that the trip is terminated, the twins are both again into the fixed reference frame of the Earth and consequently the spent time for the twins is the same..."

But he seems to argue that the time spent by both twins during the time in which they were separated cannot, as observed by the twins' clocks at reunion, have been different. I see no basis for this in the author's argument.
SIMPLICIO: You mean you do not accept the author's claim, that the fact that the time after the trip proceeds at the same rate for both twins, necessarily implies that they must have aged the same amount?
SAGREDO: Why should I? After all, after the trip, the twins are in the same position, yet it would surely be questionable to argue from this, that both twins had covered the same distance. Why then should they have "gone through" the same time?
SIMPLICIO: You argue then that time and space can be assimilated?
SAGREDO: Not necessarily. I do argue that I see no *logical* contradiction in Jim returning after having gone through less time than his brother Dan. Whether this is actually the case must surely be decided by experiment.
SALVIATI: As indeed it has: a large number of classic experiments, of which the older ones are reviewed in [14], show that time does go more slowly for particles moving more or less rapidly on a closed trajectory. Thus nuclei on a rotor emit gamma rays of lesser frequency than such as are at rest. An extraordinary recent experiment has even shown that an atom which performs an oscillatory motion at a mean speed of about 10 meters per second, emits light at a measurably slower frequency than when it is at rest [2]. That this can actually be seen is a masterpiece of experimental ingenuity.
SIMPLICIO: Do you then mean to suggest that Jim's velocity makes him grow less old? But that goes against the most basic tenet of relativity, as stated by yourself, Salviati, namely that it is impossible to distinguish uniform motion from rest. So what happened to Jim and Dan should be the same, since both were travelling uniformly nearly the whole time. Yet it is claimed that Jim is younger and Dan older. How is this asymmetry explained?
SALVIATI: That is a problem often stated, though in this case the author does not make this explicit. I believe, however, that the solution is clear enough: Jim goes through periods of acceleration, therefore we cannot view him as being in uniform motion. It is true that he is *most of the time* in uniform motion, but he is not, for all that, in *nearly uniform* motion. For that to be the case, he would have to move, to a good approximation, at constant velocity. But since Jim's velocity is assumed to lead to relativistic effects in both directions, Jim has a velocity $+v$ and a velocity $-v$, both of which are relativistic. The velocities vary therefore strongly [18], and it is not possible to view Jim as being "approximately not accelerated".
SAGREDO: The following analogy has occurred to me: let us look at a broken line, as drawn in Figure 1. This broken line is "almost everywhere" straight, by the same token as Jim's motion is supposed to be uniform for "almost all times". It is surely correct, in this case, to compute the broken line's length combining two formulae for the length of straight lines, but it is incorrect to argue that the broken line shares ABC with the straight line AC the property of being the shortest path between A and C!
SALVIATI: I think this is an excellent analogy indeed.
SIMPLICIO: But is there not an essential difference in the fact that, in the case of the straight lines, their combined length is greater than that of the direct path, whereas in the case of the twins, the "broken path" corresponding to Jim, is actually shorter than Dan's?
SALVIATI: This is truly an important difference: it is related to the fact that space and time, while inextricably intertwined in the theory of relativity, are nevertheless not equivalent. Were we to study geometric lengths, we would be transforming from one reference frame to another via rotations, not via Lorentz transformations.

It does not affect, however, the validity of Sagredo's criticism: the fact that a curve consists of pieces of straight lines does not allow one to view it as a straight line, though it does allow one to compute lengths by adding up all the straight line contributions to the total length. If there is a logical error in the simple geometric case, it is, at the very least, up to you, Simplicio, to show specifically *why* the logical problem disappears in the theory of relativity. In fact, it does not.
SIMPLICIO: I am not sure, however, that things are as simple as you make them out to be. Is your reasoning concerning acceleration not similar to one stated in Feynman [6]:

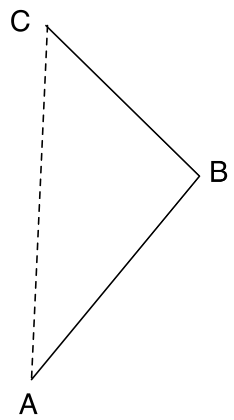

FIG. 1: A broken line connects A and C via B. It is clearly longer than the corresponding straight line, denoted by a dashed line, connecting A and C. Can we legitimately argue that this cannot be so, because the broken line is "nearly everywhere" straight, and thus cannot be distinguished from a straight line?

"This is called a "paradox only by people who believe that the principle of relativity means that all motion is relative; they say 'Heh, heh, heh, from the point of view of Paul cant we say that Peter was moving and should therefore appear to age more slowly? By symmetry, the only possible result is that both should be the same age when they meet. But in order for them to come back together and make the comparison, Paul must either stop at the end of the trip and make a comparison of clocks, or, more simply, he has to come back, and the one who comes back must be the man who was moving, and he knows this, because he had to turn around. When he turned around, all kinds of unusual things happened in his space-ship the rockets went off, things jammed up against one wall, and so on—while Peter felt nothing.

So the way to state the rule is to say that the man who has felt the accelerations, who has seen things fall against the walls, and so on, is the one who would be the younger; that is the difference between them in an absolute sense, and it is certainly correct."

SALVIATI: Yes, I can say I fully agree with these remarks.

SIMPLICIO: However, a noted philosopher, Tim Maudlin, has argued [9] that "everything in this 'explanation' is wrong".

SAGREDO: Could you tell us what he objects against it, please?

SIMPLICIO: His first and main remark is the following: "Notice, first, that we were able to predict the effect without calculating the acceleration of anything: all we computed was the ratio of the lengths of the two trajectories. The accelerations play no role in explaining the end result."

SAGREDO: That we could predict the *size* of the effect without referring to accelerations is clear enough. As we said above, the length of a broken line can be calculated by referring only to the properties of straight lines. But the issue is rather the following: how can we know which of the two twins follows an inertial, and which a non-inertial path. One might argue that all we can actually observe is the relative distance between Dan and Jim, and this does not allow us at all to say which is accelerating.

To this end, the points made by Feynman are quite relevant: we determine which of both twins has undergone non-inertial motion by asking which of both has felt accelerations.

More precisely, if both feel accelerations, we follow all the accelerations felt by either twin, and use them to work out the twins' velocities as a function of time. If, say for Dan, they remain approximately constant, then he will age much as if he were purely in inertial motion, whereas if the accelerations cause great variations in velocity over long times, as we assumed to happen with Jim, a significant difference in age will be found.

SIMPLICIO: The author does, in fact, mention that many short accelerations can lead to Dan being more accelerated than Jim, and yet still be older upon meeting Jim.

SALVIATI: There is no doubt that such is the case. Neither Feynman nor anyone else, to my knowledge, claims that accelerations *cause* the difference in ageing. It is emphatically not the case that differential ageing depends in any easy way on the "amount of acceleration", which in any way cannot easily be defined.

Rather, it is said that a complete *symmetry* between the two twins can never exist if the twins are to meet twice: one of the twins, at least, must have experienced different velocities along his trajectory, that is, accelerations.

Further, to determine for which twin such changes in velocity actually occurred, we must, in fact, as Feynman says, rely on the observable consequences of acceleration. A mere geometric description of the twin's relative motion will

not do.
SIMPLICIO: However, I am now curious about how things will appear as seen from Dan's viewpoint during the first half of Jim's trip, in which both twins are moving inertially.
SAGREDO: I think there should be no problem with this.
SALVIATI: Indeed there is none. But you should be careful to state what events you wish to look at, specifying both the time of occurrence as well as the position.
SAGREDO: All right. I suppose, Simplicio, you want to look at two events happening at Dan's position?
SIMPLICIO: In fact, I would like to define a bit more than two. I start by defining event T, which is the turnaround itself, happening, of course, where Jim then finds himself, that is, at a distance L from Dan as viewed from the reference frame D. We now further call E_1, Jim's departure, and as the second, which we call E_2, an event happening at Dan's position, but simultaneous to T.
SALVIATI: Simultaneous, in which reference frame?
SIMPLICIO: I am not used to ask this question every time, but of course I see that it is indeed necessary if one is to work according to the rules of the theory of relativity. So, for E_2 I want an event which occurs at a time $L/(\gamma v)$ in Jim's frame J_1 but at Dan's position. The way I see it, this event is then simultaneous with T in the reference frame J_1, since Jim does indeed turn around at the time $L/(\gamma v)$ as viewed from his own rest frame.
SALVIATI: All of this is quite as it should be.
SIMPLICIO: The time between these two events E_1 and E_2, as viewed in the reference frame D is thus shorter than in any other reference frame, by Rule T. In particular, it is shorter than in the J_1 frame, where that time is $L/(\gamma v)$. The time in the D frame is thus $L/(\gamma^2 v)$. Now turnaround is, for both twins, at half the trip, yet $L/(\gamma^2 v)$ is clearly quite a bit less than half the trip from Dan's viewpoint: take again the earlier example: for Dan, his brother's absence lasts 10 years, of which 5 are before turnaround. Now $\gamma = 5/3$, so that at $(9/25) * 5$ years, that is, after $9/5 = 1.8$ years, Dan will be simultaneous to turnaround in Jim's reference frame.
SALVIATI: That is quite correct as you say. And indeed, as we shall see, it could not be otherwise without violating relativity.
SAGREDO: I think this requires an explanation!
SALVIATI: Well, Sagredo, I think you will readily be able to work it out yourself, if you do not let prejudices influence your mind.
SAGREDO: I will try.

Yes, I see it now, and it is easy enough: E_1 and E_2 are separated by a time $L/(\gamma v)$ in Jim's frame, since E_2 is simultaneous to T, which happens at that time in $J1$. Jim thus observes Dan's clock to go slow, quite symmetrically to the way in which earlier we said that Dan observes Jim's clock to go slow.

Now let me define E_3 as the event which occurs at Dan's position but at time L/v. E_3 is therefore simultaneous to turnaround in Dan's frame, whereas E_2 is simultaneous to turnaround in Jim's frame. At first sight, there may not be any contradictions in this.
SIMPLICIO: I still remain puzzled: turnaround being at half the trip, what happens immediately after turnaround? Surely, there is no way in which the time $L/(\gamma^2 v)$ can be half of $2L/v$!
SALVIATI: This is presumably the only part that has some flavour of true paradox. Immediately after turnaround, the simultaneity relation between changes abruptly—as abruptly as the change of velocity was assumed to take place—and we have that after turnaround E_2 is no more simultaneous to T. Rather, in J_2, which is the reference frame after turnaround, T is simultaneous to an event E_4 occurring at Dan's position and at a time $2L/v - L/(\gamma^2 v)$, which is exactly the mirror image of E_2 with respect to E_3.
SIMPLICIO: But what physical process could possibly cause such a massive change?
SAGREDO: I believe the answer may well be, that the choice of simultaneity criteria, as you yourself suggested, Simplicio, is a matter of convention rather than physical necessity. A change of reference frame leads to a change in the concept of simultaneity, but nothing physical happens: just as nobody notices when an astronomer decides to change coordinates in his calculations.
SIMPLICIO: That may be, Sagredo, yet I would like to understand this issue of changes in synchronisation more clearly.
SAGREDO: Let me see if I can try. Remember, Salviati told us that when two clocks at rest with respect to each other are synchronised, then, if both emit a light beam towards the other when they both show the same time, these two light beams meet at the midpoint of these two clocks.
SIMPLICIO: Indeed I remember that quite well, and it seemed to me quite a reasonable way of defining synchronisation in the absence of infinitely rapid signals.
SAGREDO: All right, now look at Figure 2: we have two pairs of clocks, one moving, one not. Assume we here have an instant in which they all show the same time, and they are exactly in front of each other as shown. Let us further assume that the D clocks are correctly synchronised. What follows?

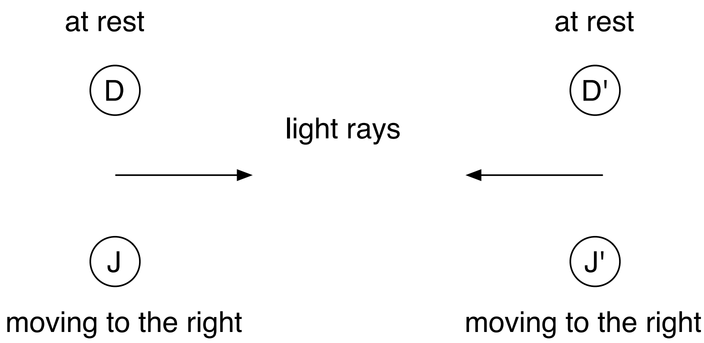

FIG. 2: Two pairs of clocks, one belonging to the D reference frame, the other to the J_1 reference frame. At a given instant, the two clocks are in front of each other as shown. Assume all show the same time, say noon. Can ith be that they are all synchronised? If the D and D' clocks are, then the light beams emitted will meet at the middle between the D and the D' clock. Since the J and J' clocks are moving to the right, the two light beams cannot meet at the middle between the J and the J' clocks, since by the time the light rays meet, these clocks, and hence their midpoint, will have moved somewhat to the right. The two light beams thus meet to the *left* of the midpoint of J and J', meaning that the right-going light beam left a bit late. In other words, the J clocks fail to be synchronised with the J' clock, and should be corrected by setting it forward by an appropriate amount. If, on the other hand, the J clocks move to the left, all the above considerations must be inverted.

SIMPLICIO: Let me see: the two light beams meet just at the midpoint between D and D'. While they move, the J and J' clocks also move to the right, so that the light beams cannot possibly meet at the midpoint of J and J'. Yes, I see now why you say that the J clocks cannot be synchronised if the D clocks are.

On the other hand, it is also clear that the motion of the J and J' clocks will, under ordinary circumstances, be extremely small indeed, so that it is easily understood why, in ordinary life, we fail to notice these phenomena.

SAGREDO: But, Salviati, I am now assailed by a new doubt: if the issue is indeed as simple as you make it out to be, surely this must have been stated before.

SALVIATI: Indeed, and many times. There are literally hundreds of papers on the issue, of which one of the first, and in my opinion among the best, is by Einstein himself [4]. But there are many, many others. Indeed, there is a whole book largely devoted to this question [8].

SAGREDO: Why then, Salviati, is the question the object of so much debate? It would seem as though the arguments you gave us should be enough to close the argument.

SALVIATI: That is not an easy question to answer. The papers published explaining the effect essentially always repeat the same arguments, each attempting to be somewhat clearer than the other. Several difficult points have been discussed, such as when, during Jim's trip does the age difference arise? It is not obvious, to me at least, that such a question has a meaningful answer, but that, I believe, is simply not required: since synchronisation of distant moving clocks is fraught with ambiguities, it is best to remain with the simple fact of an age difference when the twins meet again, which is unquestionable.

Why do these simple arguments not convince? I do not claim to know. In part, of course, it is due to the fact that those who claim the existence of a paradox usually only read the papers on "their" side of the debate, as was distressingly clear in the case of our author, who had no idea of what Langevin had actually said on the subject.

The role of acceleration has also played a great part in increasing the confusion. The fact, pointed out by Maudlin [9], that everything can be worked out without considering the values of the acceleration, has served to hide the more important truth of its necessary appearance and its role in destroying the symmetry between both twins.

There also has arisen an unaccountable superstition, even among capable physicists, that "acceleration cannot be treated in the framework of special relativity, but requires general relativity."

SIMPLICIO: I had indeed heard something to that effect. Is it not true, then?

SALVIATI: By no means: indeed, motion in those systems in which the largest accelerations have ever been created by Humanity, I mean particle accelerators [19], are fully analysed and understood in terms of special relativity alone. General relativity becomes essential only when gravity plays an important role.

There is, however, a rather distant connection with general relativity, which is the following: as Einstein points out in [4], one might be misled, by an incomplete understanding of this theory, to think that accelerations too can be transformed away, just as uniform motion can. There is some degree of truth in this, but the acceleration which one wishes to eliminate must be replaced by a gravitational field, and the twin effect can still be treated consistently [4].

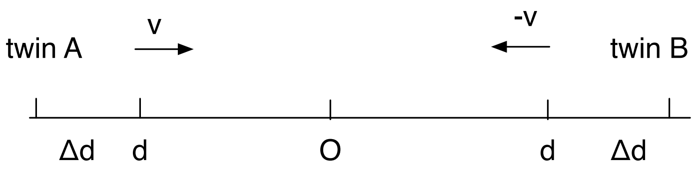

FIG. 3: Our favourite twins, now called A and B, accelerate over a distance Δd and cross two symmetric points, both at distance d from O simultaneously in the rest frame of O with opposite velocities v and $-v$. They then presumably arrive at O simultaneously, and having the same age. It is asked whether the treatment of the theory of relativity for this problem will not lead to contradictory answers.

II. SECOND DAY: AN ACCELERATION-FREE PARADOX

SAGREDO: I have been told by Simplicio that he has found, in the same tract [11] which we discussed extensively yesterday, an example which does not involve acceleration in any way.
SALVIATI: This is quite interesting indeed. How does this go?
SIMPLICIO: It is attributed by Sasso to one Suleiman [15], but the presentation in Sasso's tract [11] is quite clear, I believe:

"The two brothers after a symmetric transitory phase of acceleration (Δd) reach the same speed v into reverse at the same distance d from O and they maintain this constant speed (fig. 6). The two twins are now in an inertial situation of perfect symmetry because they move with a constant relative speed equal to $2v$. Every twin supposes that he is at rest while the other twin moves with approach speed equal to $2v$. As per SR [special relativity] every twin, supposed at rest, deduces to be older than the other who is in motion. It represents an evident contradiction that is a direct consequence of the time dilation theorized in the order of Lorentz's Transformations, because every brother deduces to be older than the other and it is impossible.

R. Suleiman concludes his paper with these manifest words: 'The TTP (Travelling Twin Paradox) poses an unsolvable problem within the framework of SR. We know that the twins approaching each other will meet sometime, somewhere, and compare clocks. The inability of SR to produce one prediction, instead of two contradictory predictions, should be highly disturbing to current physics'."

I have drawn a figure quite similar to the one given in [11].
SALVIATI: If such were the case, it *would* surely disturb physicists. However, we should first see whether Suleiman is in fact correct in assuming that the theory of relativity will yield contradictory answers.
SAGREDO: I do now notice that the way in which the problem is cast assumes the usual naive approach to time dilation, which you have shown us, Salviati, to be untenable. Surely, if the sole fact of motion were indeed to slow clocks down, then the author would be quite correct. But we have seen that this is a travesty of the theory of relativity.
SIMPLICIO: Perhaps. But I would like to see this solved in some way.
SALVIATI: Let us first dispose of a technical error. While it may indeed be claimed that the two twins have *relative velocity* $2v$ in the rest frame of O, since they do cover the distance $2d$ in a time d/v, yet it does not follow that, if one goes to the rest frame of A or B, the other twin's velocity will be $2v$. In fact, as follows from the formulae in Appendix A, it is given by

$$w = \frac{2v}{1+v^2/c^2} = \left[1 - \frac{(1-v/c)^2}{1+v^2/c^2}\right] c \tag{2}$$

which is always less than the speed of light. For $v = 4c/5$, to stick with the example, the resulting velocity is $40c/41$.

In principle this is a harmless detail and in no way affects the possibility of a paradox. It does indicate, however, a definite lack of familiarity with the basics of the theory of relativity.
SIMPLICIO: I agree that this is unfortunate. Still, I am struck by this paradox' absolute symmetry. If it can be solved at all, perhaps the solution will be instructive.
SAGREDO: Well, let us see what can be done with our two wonderful rules.
SIMPLICIO: Let me try it. We take as two events the crossing of the two points at distance d from the origin. These are simultaneous, so in any other frame, the distance will appear larger. Thus, say in twin A's frame, the distance between the two events will appear to be γ times larger than in the O frame, that is, $2\gamma d$.
SAGREDO: I agree with this.
SIMPLICIO: Now let us consider as a pair of events, the crossing of the point at distance d by twin A, and the meeting at point O of both twins. In the frame of twin A, both events are at the same position, so that the duration

separating them is less than in any other frame, in particular, it is less than the duration separating them in the O frame, where the duration is clearly d/v. The factor is γ, so the time between those two events as seen by A is in fact $d/(\gamma v)$. The same reasoning can also be made from B's viewpoint, so he will also reach O at a time $d/(\gamma v)$ from his viewpoint. That already seems to dispose of any paradox: both twins will arrive at O having spent the same time since they crossed the point at distance d.
SAGREDO: True enough, but let us pursue this some more: how long would the B twin take to meet his brother *as seen from twin A's reference frame*?
SIMPLICIO: Let us see: we know that initially twin B is at a distance $2\gamma d$ from twin A, in A's reference frame.
SALVIATI: Indeed we just saw that.
SIMPLICIO: Now, in twin A's frame, twin B is moving at a speed of $w = 2v/(1+v^2/c^2)$. He thus covers the distance $2\gamma d$ in a time

$$\tau = \frac{2\gamma d}{w} = \frac{2\gamma d}{2v}\left(1+v^2/c^2\right) = \frac{\gamma d}{v}\left(1+v^2/c^2\right). \tag{3}$$

So in twin A's frame, it seems to him that twin B takes a different, indeed a longer, time to reach the center than he did.
SAGREDO: You are right, indeed, but this need not be a contradiction. What we need is that both twins reach the center at the same time from their own viewpoints. We have already seen that they do so, since they both reach O at $d/(\gamma v)$, as you have shown. But now we have seen that A sees B take the time τ given by (3) to reach the center. What happens now if we transform the time τ from A' frame to B's? It should then give once more $d/(\gamma v)$, but I am not sure at all that it will.
SIMPLICIO: You are right, of course: it is not obvious that this will turn out to be consistent. The events consisting in B crossing the point at d and arriving at O occur at the same position in B's frame, the duration between them is shorter than as seen from any other reference frame. In particular τ must be γ' times larger than the time as viewed by B, where γ' is

$$\gamma' = \frac{1}{\sqrt{1-w^2/c^2}}, \tag{4}$$

where w is given by (2).This will get messy. Let me do the intermediate calculations separately. I use the first form of (2)

$$\begin{aligned} 1-\frac{w^2}{c^2} &= 1-\frac{4v^2/c^2}{(1+v^2/c^2)^2} \\ &= \left(\frac{1-v^2/c^2}{1+v^2/c^2}\right)^2 \end{aligned} \tag{5}$$

so that finally

$$\gamma' = \frac{1+v^2/c^2}{1-v^2/c^2} \tag{6}$$

To obtain the arrival time as viewed by B, we need to divide τ by γ'

$$\frac{\tau}{\gamma'} = \frac{\gamma d}{v}\left(1+v^2/c^2\right)\frac{1-v^2/c^2}{1+v^2/c^2} = \frac{\gamma d}{\gamma^2 v} = \frac{d}{\gamma v}. \tag{7}$$

SAGREDO: So even this fairly intricate consistency check comes out as it should! The theory is consistent whichever way you try to take it. I believe, dear Simplicio, you should agree that any arguments that claim to find contradictions in the theory of relativity are likely to be wrong, and that none of your stated arguments are built very soundly.
SIMPLICIO: I must admit that the construction of the theory of relativity is more subtle than I thought. I was deceived by an insufficient understanding of what is called "length contraction" and "time dilation", which I now understand are far richer concepts than these unfortunate expressions would indicate.
SALVIATI: Nevertheless, I will gladly admit that, in the main, your arguments, Simplicio, are quite correct: any theory which claims that motion alone causes a shortening of length or a lengthening of time, and *at the same time* claims that uniform motion is unobservable, is contradictory. All the paradoxes you have given rest upon such assumptions, and this last one is particularly convincing.

The issue, though, is that such contradictions do not argue against the theory of relativity, since it makes no such claims. Rather, it states that the effect of comparing two references frames S and S' in uniform motion will induce three effects acting together:

1. a contraction of lengths
2. a lengthening of time intervals
3. a desynchronisation of clocks

in just such a way that everything is perfectly symmetric, whether one views S' from S or S from S'. This symmetry should be perfectly clear if you look at the Lorentz transformations, say in the form given by (A15, A16), and do not attempt to dissect the formula in isolated effects concerning lengths and time intervals, but rather take all coordinates of any given event together and transform everything according to the Lorentz transformations.

Appendix A: The Lorentz transformations

For completeness' sake, let us here derive the Lorentz transformations from the assumption of relativity and the constancy of the speed of light. The approach follows closely that of Einstein in [5]. We simplify the problem from the start by assuming that the two reference frames coincide when $t = 0$ and that the moving frame S' moves along the x axis of S.

We now call x' and t' the coordinates of events in S', and x, t the corresponding coordinates in S. Since free motion goes along a straight line, the transformation must be linear, that is, of the form

$$x' = A(v)x + B(v)t \tag{A1}$$
$$t' = C(v)x + D(v)t \tag{A2}$$

Adding and subtracting the 2 equations, we obtain

$$x' - ct' = [A(v) - cC(v)]\,x + [(B(v)/c - D(v)]\,ct \tag{A3}$$
$$x' + ct' = [A(v) + cC(v)]\,x + [(B(v)/c + D(v)]\,ct \tag{A4}$$

From the invariance of the speed of light, we know that $x = ct$ implies $x' - ct' = 0$, and hence, putting x equal to ct in (A3), we obtain

$$A(v) - cC(v) = -\left(\frac{B(v)}{c} - D(v)\right) \tag{A5}$$

We perform the same calculation for $x = -ct$, which implies that $x' + ct' = 0$, and obtain

$$A(v) + cC(v) = \frac{B(v)}{c} + D(v) \tag{A6}$$

Substituting (A5, A6) into (A3, A4) one obtains

$$x' - ct' = f(v)(x - ct) \tag{A7}$$
$$x' + ct' = g(v)(x + ct), \tag{A8}$$

where $f(v) = A(v) - cC(v)$ and $g(v) = A(v) + cC(v)$. The inverse transform is, of course, given by

$$x - ct = f(v)^{-1}(x' - ct') \tag{A9}$$
$$x + ct = g(v)^{-1}(x' + ct') \tag{A10}$$

Using the principle of relativity, however, we obtain another expression for the inverse. Indeed, since all reference frames are equivalent, the transformation from S' to S must have the same form as that from S to S', that is

$$x - ct = f(-v)(x' - ct') \tag{A11}$$
$$x + ct = g(-v)(x' + ct'). \tag{A12}$$

Replacing v by $-v$ corresponds to replacing t by $-t$ and t' by $-t'$, so that we have eventually:

$$x + ct = f(v)(x' + ct') \tag{A13}$$
$$x - ct = g(v)(x' - ct'). \tag{A14}$$

Combining (A13, A14) with (A9, A10), we obtain $g(v) = 1/f(v)$ for all v and thus the final form for the Lorentz transformations:

$$
\begin{aligned}
x' - ct' &= f(v)(x - ct) && \text{(A15)}\\
x' + ct' &= \frac{1}{f(v)}(x + ct), && \text{(A16)}
\end{aligned}
$$

From these two equations, an immediate consequence obtained by multiplying both equations together, is the following

$$
(x')^2 - c^2 (t')^2 = x^2 - c^2 t^2 \tag{A17}
$$

Now we quickly derive Rules S and T. These both concern pairs of events. Let us choose, as we may, the first one to be at $x = t = 0$. Let the second one then be at $x = L$ and $t = T$. We may also always choose T to be positive, since we may always take the earlier event to be the first. First let $|L| < cT$. Then choosing v so that f has the following value

$$
f = \sqrt{\frac{cT + L}{cT - L}} \tag{A18}
$$

leads to $L' - cT' = -(L' + cT')$, which implies that $L' = 0$. Similarly, if $|L| > cT$, we may choose

$$
f = \sqrt{\frac{cT + L}{L - cT}}, \tag{A19}
$$

which leads to $L' - cT' = L' + cT'$ and hence to $T' = 0$. This shows that most pairs of events can be divided in two kinds: those which are separated by a time large enough for light to cross the distance separating their positions, and on the other hand, such as are separated by a time interval so short that light cannot cross the distance separating them. We have thus seen that the former can always be made to occur at the same position in an appropriate reference frame. On the other hand, the second type can always be made simultaneous in an appropriate frame. This means that Rules S and T cover all types of event pairs, except those that are exactly connected by a light beam.

Now let L be the position of an event simultaneous with the event occurring at the origin. Via (A17), we see that, if the event's coordinates in any other frame are L' and T', then

$$
L^2 = (L')^2 - c^2 (T')^2 \le (L')^2 . \tag{A20}
$$

This amounts to Rule S. Rule T is shown in an exactly similar manner, using an event at the origin occurring at time T:

$$
c^2 T^2 = c^2 (T')^2 - (L')^2 \le c^2 (T')^2 . \tag{A21}
$$

Rules S and T still contain an additional remark, namely that $L' = \gamma L$ and $T' = \gamma T$. This follows immediately from the Lorentz transformations as given, for example, in the formula (A29, A30) to be derived below. Thus, for the case of Rule S, we substitute the position L and the time $t = 0$ of the event in the right-hand side of the Lorentz transformations and obtain $L' = \gamma L$ from (A29). The case of Rule T is entirely similar.

Finally we need to connect the form given in (A15, A16) with the more traditional form of the Lorentz transformations involving the velocity v. Note that the latter is defined by saying that the equation $x' = 0$ corresponds to $x = vt$. Putting $x' = 0$ and $x = vt$ into (A15, A16) and adding, yields

$$
0 = -f(v)(c - v) + \frac{1}{f(v)}(c + v) = -\left(f(v) - \frac{1}{f(v)} \right) c + \left(f(v) + \frac{1}{f(v)} \right) v \tag{A22}
$$

or in other words

$$
f(v)^2 = \frac{c + v}{c - v}. \tag{A23}
$$

To ease the notation we now shall simply write f instead of $f(v)$. To obtain the usual form of the Lorentz transformations, we once more add and subtract equations (A15, A16) to obtain

$$
\begin{aligned}
x' &= \frac{1}{2}\left(f + \frac{1}{f} \right) x - \frac{1}{2}\left(f - \frac{1}{f} \right) ct && \text{(A24)}\\
t' &= -\frac{1}{2c}\left(f - \frac{1}{f} \right) x + \frac{1}{2}\left(f + \frac{1}{f} \right) t && \text{(A25)}
\end{aligned}
$$

Using (A22) one obtains

$$\frac{f-1/f}{f+1/f}=\frac{v}{c}. \tag{A26}$$

From this follows

$$\begin{aligned} 1-\frac{v^2}{c^2} &= \frac{(f+1/f)^2-(f-1/f)^2}{(f+1/f)^2} \\ &= \frac{4}{(f+1/f)^2}. \end{aligned} \tag{A27}$$

This leads immediately to

$$\gamma=\frac{1}{\sqrt{1-v^2/c^2}}=\frac{1}{2}\left(f+\frac{1}{f}\right). \tag{A28}$$

Using (A26) and (A28), (A24, A25) are now reduced to their usual form

$$x' = \gamma\,(x-vt) \tag{A29}$$

$$t' = \gamma\left(-\frac{v}{c^2}x+t\right). \tag{A30}$$

A last useful remark concerns the composition of velocities: if we transform from reference frame S to S' which moves at a velocity v_1 with respect to S, and if we now transform from S' to S'' which moves with velocity v_2 wth respect to S', and if we denote the corresponding factors arising in (A15, A16) by f_1 and f_2 respectively, then clearly the transformation from S to S'' wil be given again by (A15, A16) with $f_3 = f_1 f_2$, and hence, if we denote by v_3 the velocity of S'' with respect to S:

$$\frac{c-v_3}{c+v_3}=\frac{c-v_1}{c+v_1}\,\frac{c-v_2}{c+v_2} \tag{A31}$$

which leads, after a bit of tedious but straightforward algebra, to the well-known formula

$$v_3=\frac{v_1+v_2}{1+v_1 v_2/c^2} \tag{A32}$$

Appendix B: External synchronisation

Synchronisation is an intricate issue, but easy in its basics. A crucial issue is the following: there is no way to give meaning to the concept of velocity, in its simplest form, unless we are given a procedure for synchronising distant clocks: indeed, velocity is defined by reckoning the time an object takes to go from position A to position B, and then divide the distance separating A from B by the time. But the time is obtained by observing two clocks, one at A and the other at B, that is, two distant clocks. If we do not know whether the two clocks show the same time, we have no way to define the velocity. And further, any concept of velocity we might define will necessarily depend on the choice of synchronisation procedure.

From this follows that the usual statement "the speed of light is the same in all reference frames" lacks meaning until and unless we specify a given synchronisation procedure. At least, such is the case for so-called one-way speeds, which are defined as above.

Two-way speeds, on the other hand, are free from this problem: we send an object from A to B and back. The time taken is then measured at the *same* clock A, and synchronisation issues do not arise. It might be argued that we do not then know whether the object was going at the same speed on the way back as on the way out, but this question again leads us back to one-way speeds, and are unanswerable until we develop a synchronisation procedure.

The essential fact about light, then is the following: if two mirrors are separated by a distance L in their rest frame, then light will always take a time $2L/c$ to go back and forth, no matter in which uniformly moving frame this experiment is performed. This, and no other, is the "principle of the constancy of the speed of light", as confirmed, for example, by the Michelson–Morley experiment and several others [20].

Synchronisation, on the other hand, is a matter of *convention.* That is, there are many different ways to synchronise distant clocks. Once we accept *from experiment* the constancy of the (two-way) speed of light, we may, for example *arbitrarily decide* to synchronise distant clocks by assuming that the one-way speed of light is always the same and equal to the two-way speed. This is the so-called Einstein synchronisation, as introduced in [3], which is also sometimes attributed to Poincaré, since he described it in an earlier paper [10], in which he notices a connection with the so-called Lorentz "local time".

However, there are other ways to synchronise clocks. Perhaps the most obvious, apart from the Einstein approach, consists in taking an arbitrary reference frame, which we may call the *absolute* reference frame, synchronising all clocks in that frame by the Einstein method, and then deciding that any two clocks at rest with respect to each other are synchronised if they show the same time whenever the corresponding clocks of the absolute frame do so. This approach has first been argued to be unproblematic by Reichenbach, and later pioneered by Tangherlini [16] and Selleri [12, 13]. It has been called *external synchronization* in [16] and the corresponding transformations External Synchronisation Transformations (EST).

Let us first, however, dispel a prevalent myth: such a choice of synchronisation method *does not* constitute either a violation of the principle of relativity, nor yet a "refutation of relativity". Neither does it prove that "an absolute reference frame does exist, mainstream science notwithstanding", as some of my friends might be tempted to say. An analogy may help: if I choose to describe the stars' positions in a coordinate system in which the plane of the ecliptic plays a special role, this does not induce an anisotropy in the Universe. Rather, the plane of the ecliptic is used as part of a conventional construct which allows me to describe positions. Similarly, the absolute frame has no physical relevance, and all calculations performed with the help of one such frame will give results exactly equivalent to those obtained using another frame. Note that I say equivalent, not identical: clearly the numbers obtained for the different positions and times of various events will be different, but all the physical conclusions will be the same, just as, when changing coordinate systems, the coordinates of the points described will vary, but not the actual relationships between them.

To derive the resulting transformations, we may proceed as follows: let us call the absolute frame Σ and define the position and the time coordinates of any event in Σ to be X and T respectively. If we now consider a frame S moving at velocity v, the Lorentz transformations will yield the following expressions for x' and t' defined as the position and time of the event as obtained when the clocks of S are synchronised by the Einstein method

$$x' = \gamma (X - vT) \tag{B1}$$

$$t' = \gamma \left(-\frac{v}{c^2} X + T\right) \tag{B2}$$

This is just the old fashioned Lorentz transform.

Now let us see whether we can introduce new coordinates x and t such that the times t_1 and t_2 of two events are always equal when $T_1 = T_2$. To this end we ask: what changes will a change in synchronisation procedure lead to? Clearly it can only affect the time, so that we shall have $x = x'$, and since additionally we must assume that inertial motion is still described by a constant velocity, the connection between t, t' and x' must be linear:

$$t = t' + \alpha x', \tag{B3}$$

where α is some constant, possibly dependent on velocity, to be appropriately chosen. This leads to

$$t = \gamma \left(-\frac{v}{c^2} + \alpha\right) X + \gamma (1 - v\alpha) T. \tag{B4}$$

What we need for the synchronisation to have the desired property is that t should not depend on X, which is only the case if $\alpha = v/c^2$. We thus have, for the transformation from Σ to an arbitrary reference frame S:

$$x = \gamma (X - vT) \tag{B5}$$

$$t = \gamma \left(1 - \frac{v^2}{c^2}\right) T = \frac{T}{\gamma} \tag{B6}$$

These formulae may appear to be rather simpler than the Lorentz transformations. The impression dissipates somewhat, however, if one attempts to connect two arbitrary frames S_1 and S_2: one must first transform from S_1 to Σ and then from Σ back to S_2. If S_1 has velocity v_1 and S_2 has velocity v_2 with respect to Σ, the transformation is given by

$$x_2 = \frac{\gamma_2}{\gamma_1} \left[x_1 - (v_2 - v_1) t_1\right], \tag{B7}$$

$$t_2 = \frac{\gamma_2}{\gamma_1} t_1, \tag{B8}$$

$$\gamma_{1,2} = \frac{1}{\sqrt{1 - v_{1,2}^2/c^2}}. \tag{B9}$$

These transformations have several interesting properties. In particular, a signal propagating in Σ at the speed of light c will propagate with respect to S with speed $c - v$, as immediately follows from (B5, B6) by substituting X by cT. Since this only refers to the one-way speed of light, however, this does not in any way represent a contradiction to the physics of relativity: we have changed the convention for synchronisation and therefore the numerical values for the one-way speeds are affected.

This leads to a quick and natural explanation of things such as the Sagnac effect. Of course, as the EST and the Lorentz transformation are fully equivalent, there can be no question of the EST explaining something "for which there is a veritable explanatory inability of the two relativistic theories" as stated by Selleri [13]. But the Sagnac effect's explanation is easier when using EST, since these transformations naturally generate one-way speeds of light of the form $c \pm v$, which are what is observed in such experiments. An explanation using the Lorentz transformations must go into greater detail concerning which clocks do the measuring, and how they are synchronised. The same holds true for measurements of transit times between GPS satellites and the ground stations. Both descriptions, when correctly performed, are fully equivalent, but EST may be easier to grasp.

An analysis of the twin paradox is quite easy if we assume Dan to be in the Σ frame. It becomes messier in more general cases, but yields the same result. In the former case, we see that Jim's time is Dan's time divided by γ, so the result of differential ageing is trivially recovered.

On the other hand, there is no symmetry between the transformation from Σ to another frame S, but this is because the symmetry was broken from the start: since all synchronisation operations are referred to Σ, it is easy enough to see that the principle of relativity does not imply that the transformation should be symmetric. All observations that do not involve a synchronisation procedure, however, yield the same results as the Lorentz transformations. Thus, in the twin paradox, one gets the correct result when the twins meet again, no matter what the state of motion of Σ may have been, but the statements we would make concerning which events in Jim's life were simultaneous to which events in Dan's would depend on the choice of Σ, and would also markedly differ from the corresponding predictions of the Lorentz transformations.

Some other properties of the EST are disturbing. Thus the relative velocity of S_2 with respect to S_1, as follows from (B7) is given by

$$v_{12} = \frac{\gamma_2}{\gamma_1}(v_1 - v_2) \tag{B10}$$

This has two strange properties: one is that it is not symmetric in S_1 and S_2, that is, the velocity of S with respect to S_2 is not the negative of the the velocity of S_2 with respect to S_1. Further, it does not depend only on the difference of velocities, but rather on the absolute velocities with respect to the absolute reference frame Σ. All these features reflect the lack of symmetry introduced by the choice of synchronisation, but do not contradict the theory of relativity, since, as we have seen, the approach we describe here is fully equivalent to it.

An interesting example for what is similar and different in both theories, is provided by the following variant on the twin paradox, discussed in [13]: let John and Jane be in two different spaceships, initially at rest and separated by a distance L in their own rest frame. They synchronise clocks and start to accelerate according to a common schedule, that is, in the original reference frame, which we may take to be Σ. We are assuming that Jane is ahead of John. The two spaceships stop accelerating at a common time, as measured in Σ. The question is now: are the two clocks synchronised or not?

In this form, an easy calculation shows that the Lorentz transformations states John's clock to go behind Jane's, whereas according to the externally synchronised transformation, no such difference arises. On the one hand, there is clearly no contradiction there: we are synchronising clocks in inequivalent ways and obtain different answers. The question that remains is: which is more natural? According to the *buon senso* taken by Selleri to be a useful measure of a theory's adequacy, the latter might well appear to be the case.

However, let us now extend the story a bit: let John and Jane start out at the same position, at rest and with synchronised clocks. Then Jane moves very slowly ahead, so slowly that the effect of time dilation is negligible, until she reaches a distance L. Then both accelerate as before. When both have arrived at their final velocity, John rejoins Jane by moving, relative to her, just as slowly as she had relative to him in the first phase of the journey.

Again, a straightforward computation now yields the fact that John's clock, upon rejoining Jane, will show an earlier time than hers: the two clocks will be desynchronised, and this will now *not* depend in the way we perform the calculation, whether with Lorentz transformations or with externally synchronised transformations, since the final result involves only a direct comparison of clocks at the same position.

So how should we interpret this? That, at some point during the trip, John's clock became desynchronised from Jane's is, given this computation, an indisputable fact. The issue is: when, or perhaps more precisely, in which phase of the trip, did this happen? Following the Lorentz transformations, one sees that the two clocks remain synchronised during Jane's first forward sally, and become desynchronised after the acceleration step. Finally, the desynchronisation which obtains after the acceleration phase remains unaffected by John's sally to rejoin Jane. On the other hand, EST attributes the whole desynchronisation to John's final sally forward, at least if Σ is chosen to be the initial rest frame of John and Jane. If, on the other hand, Σ is taken as John and Jane's final rest frame, then the desynchronisation occurs entirely during Jane's sally. In more general situations, if Σ has an arbitrary velocity, the desynchronisation will happen to different extent during different stages of the trip. In other words, the answer to this question is not unique when we use EST, but rather depends on the motion of the two reference frames, the initial and the final one, with respect to Σ.

It is not clear to this author that *buon senso* will give preference to any one of the various scenarios in the complete example described above. From a mathematical viewpoint, on the other hand, the fact that the Lorentz transformations display the symmetry postulated in the relativity principle explicitly, in the sense that the inverse transformation to a Lorentz transformation again has the same form, as well as the fact that Lorentz transformations between two reference frames only depend on the relative velocity and not on 2 different velocities with respect to an unknown frame Σ, make it appear to me that the Lorentz transformations are simpler and more straightforward in their interpretation than the EST.

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[17] This fact, namely that the derivation of the Lorentz transformations requires the Einstein–Poincaré convention for synchronising clocks, is unfortunately not often emphasised with due clarity in textbooks.

[18] Of course, the *speed* does not vary. But we will not do our reader the injustice of doubting that she understands the difference between speed and velocity, and we assume her to be fully aware that v and $-v$ are different velocities.

[19] Even in fairly old experiments, such as that reported in [1], the accelerations arising are of the order of 10^{18} times larger than the acceleration of gravity.

[20] It is necessary to point out that several experiments exist that "measure" the one-way speed of light. The Rømer experiment, with Jupiter's moons, is one such. There also are several accurate such experiments using the GPS system. These experiments implicitly use the so-called "slow transport synchronisation". According to the theory of relativity, this must be equivalent to Einstein synchronisation, however generally this equivalence need not hold. Experiments which confirm that the one-way speed of light, as measured in this way, is in fact constant, are thus real tests of the theory. Note, on the other hand, that the constancy of the one-way speed of light is a trivial mathematical consequence of Einstein synchronisation, so that clocks thus synchronised cannot be used to shed light on the validity of the theory of relativity.