CONVERGENCE OF THE RATIO OF PERIMETER OF A REGULAR POLYGON TO THE LENGTH OF ITS LONGEST DIAGONAL AS THE NUMBER OF SIDES OF POLYGON APPROACHES TO ∞

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ABSTRACT

Regular polygons are planar geometric structures that are used to a great extent in mathematics, engineering and physics. For all size of a regular polygon, the ratio of perimeter to the longest diagonal length is always constant and converges to the value of π as the number of sides of the polygon approaches to ∞. The purpose of this paper is to introduce Bishwakarma Ratio Formulas through mathematical explanations. The Bishwakarma Ratio Formulae calculate the ratio of perimeter of regular polygon to the longest diagonal length for all possible regular polygons. These ratios are called Bishwakarma Ratios- often denoted by short term BK ratios- as they have been obtained via Bishwakarma Ratio Formulae. The result has been shown to be valid by actually calculating the ratio for each polygon by using corresponding formula and geometrical reasoning. Computational calculations of the ratios have also been presented upto 30 and 50 significant figures to validate the convergence.

Keywords: Regular Polygon, limit, π*, convergence*

INTRODUCTION

A regular polygon is a planar geometrical structure with equal sides and equal angles. For a regular polygon of n sides there are $\frac{n(n-3)}{2}$ diagonals. Each angle of a regular polygon of n sides is given by $\left(\frac{n-2}{n}\right)180°$ while the sum of interior angles is $(n-2)180°$. The angle made at the center of any polygon by lines from any two consecutive vertices (center angle) of a polygon of sides n is given by $\frac{360°}{n}$. It is evident that each exterior angle of a regular polygon is always equal to its center angle.

The ratio of the perimeter to the longest diagonal or diameter is characteristic feature of any regular polygon. For a regular polygon of even number of sides, the ratio is given by $n\sin\left(\frac{180°}{n}\right)$, while for a regular polygon of odd number of sides the ratio is given by $2nsin\left(\frac{90°}{n}\right)$. As the number of sides of a regular polygon becomes infinitesimally large, i.e. $n \to \infty$, the resulting polygon is called ***regular apeirogon*** which resembles a circle. The regular polygon at this state has countably infinite number of equal edges. The ratio of the perimeter to the longest diagonal reduces to $C\frac{sin\theta}{p}$, where C is circumference of the resembling circle, θ is angle opposite to the perpendicular arm and p is perpendicular arm of the right angled triangle whose hypotenuse is diameter of the circle. The expression $C\frac{sin\theta}{p}$ provides exactly π. Angles are used both in degrees and in radians as required.

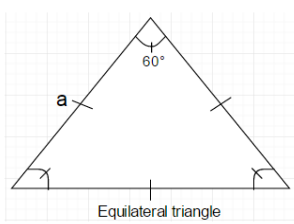
For inclusion of all regular polygons equilateral triangle poses a significant hurdle. It has no recognizable diagonal or diameter. The ratio of the perimeter of the equilateral triangle to the length of one of its sides is taken in this case which is equal to 3.

The Bishwakarma Ratio Formulae for the ratio of perimeter to the longest diagonal length can be summarized as follows:

I. f(n) = 3, for n = 3
II. $f(n) = n\sin\left(\frac{180°}{n}\right)$, for n is an even number, $n \geq 4$
III. $f(n) = 2n\sin\left(\frac{90°}{n}\right)$, for n is an odd number, $n \geq 5$
IV. $f(n) = C\frac{\sin\theta}{p}$, for $n \rightarrow \infty$

CALCULATION OF BK RATIOS FOR SOME REGULAR POLYGONS

1. For n=3, representing an equilateral triangle



$$\frac{total\ perimeter\ of\ equilateral\ triangle}{a\ side\ length} = \frac{3a}{a} = 3.$$

2. For a regular polygon with even number of sides

Square (n = 4), Hexagon (n = 6), Octagon (n = 8) are the basic examples of this types of polygons. The length of the longest diagonal (d) of these regular polygons is given by $\frac{a}{sin\left(\frac{180°}{n}\right)}$, where 'a' is the length of the side of the polygon. If a regular polygon has even number of edges, then length of the longest diagonal of that polygon will be hypotenuse of a right angled triangle for which one of the sides of thepolygon is the perpendicular side as illustrated in figure.

From figure 1, we get

$$sin\left(\frac{180°}{n}\right) = \frac{a}{d}\ i.e.\ d = \frac{a}{sin\left(\frac{180°}{n}\right)}$$

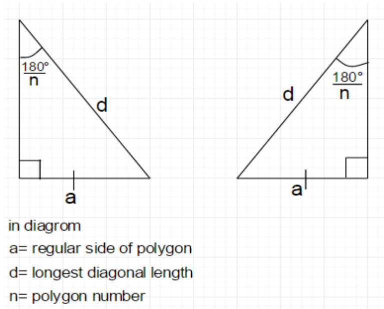


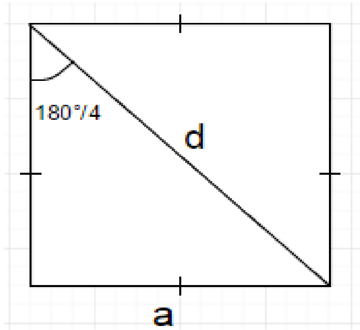
Figure 1: Illustration of the longest diagonal as the perpendicular of a right angled triangle

ILLUSTRATION I

The values of length of the longest diagonal for some regular polygons of have been geometrically presented below. The calculation of the ratio of the perimeter of the corresponding polygon to the length of the longest diagonal has been presented alongside.

I. Square (n=4)

$$sin\left(\frac{180°}{4}\right) = \frac{a}{d}\ i.e.\ d\ = \frac{a}{sin45°} = (\sqrt{2})a$$



From geometry, the ratio of the perimeter of the square to the length of its longest diagonal is

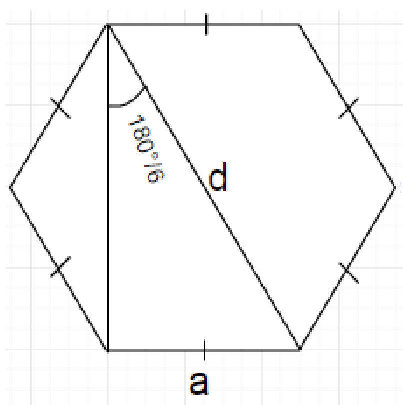
$$\frac{perimeter\ of\ the\ square}{length\ of\ the\ longest\ diagonal} = \frac{4a}{(\sqrt{2})a} = 2\sqrt{2}$$

Using formula, we get

$$f(n) = n\ sin\left(\frac{180°}{n}\right) = 4sin\left(\frac{180°}{4}\right) = 4\ sin\ 45° = 2\sqrt{2}$$

II. Regular Hexagon (n=6)

$$sin\left(\frac{180°}{6}\right) = \frac{a}{d}\ i.e.\ d = 2a$$



From geometry,

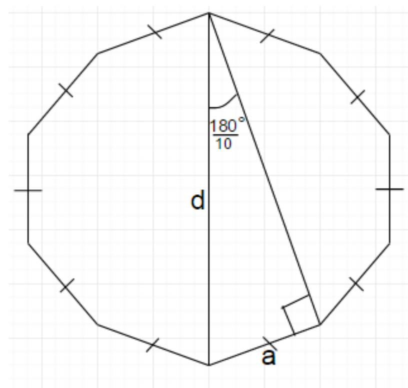
$$\frac{perimeter\ of\ the\ hexagon}{length\ of\ the\ longest\ diagonal} = \frac{6a}{2a} = 3$$

Using formula, we get

$$f(n) = n \sin\left(\frac{180°}{n}\right) = 6 \sin\left(\frac{180°}{6}\right) = 3$$

III. Regular Decagon (n=10)

$$sin\left(\frac{180°}{10}\right) = \frac{a}{d} \;\; i.e. \;\; d = \frac{a}{sin18°}$$



Regular decagon

From geometry,

$$\frac{perimeter\ of\ the\ decagon}{length\ of\ the\ longest\ diagonal} = \frac{10a}{\frac{a}{sin18°}} = 3.09016$$

Using formula, we get

$$f(n) = nsin\left(\frac{180°}{n}\right) = 10sin\left(\frac{180°}{10}\right) = \ 3.09016$$

Similarly, we can use $f(n) = nsin\left(\frac{180°}{n}\right)$ formula for all regular polygons having even number of edges. Therefore, $f(n) = nsin\left(\frac{180°}{n}\right)$ is a valid formula for given domain.

3. For a regular polygon with odd number of sides

Pentagon ($n = 5$), Heptagon ($n = 7$), Nonagon ($n = 9$) are the basic examples of this types of polygons.

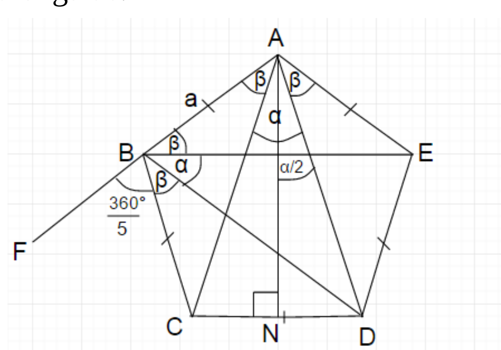
The length of the longest diagonal d of these regular polygons is given by $\frac{a}{2\sin\left(\frac{90°}{n}\right)}$, where a is the length of the side of the polygon.

ILLUSTRATION 2

The values of length of the longest diagonal for some regular polygons have been geometrically presented below. The calculation of the ratio of the perimeter of the corresponding polygon to the length of the longest diagonal has been presented alongside.

I. Regular Pentagon (n = 5)

Diagonals from D and E are drawn to B such that it subtends an angle α, and creates equal angles β on either side of the angle α.Similarly,diagonals AC and AD



Regular pentagon

subtend an angle α at A creating equal angles β on either side of the angle α.
Side AB is extended upto F and AN ⊥ CD such that

- ΔACD and ΔBDE are iscosceles and
- ΔCAN and ΔDAN are right angled triangles

From △ABD, we get

(α+ β) + (α+ β) + α = 180° (sum of interior angles of a triangle)

3α + 2β = 180°…………………….(i)

On straight line ABF, we get

∠ABE + ∠EBD + ∠ DBC + ∠CBF = 180° (angle add up on a straight line)

i.e. $\beta + \alpha + \beta + \frac{360°}{n} = 180°$

∴ $2\beta = 180° - (\alpha + \frac{360°}{n})$…………………(ii)

From equation (i) and (ii), we have
$180° - (\alpha + \frac{360°}{n}) = 180° - 3\alpha$
∴ $\alpha = \frac{360°}{2n}$……………………(iii)
In ΔACD,line AN divides ΔACD into equal two parts

∠CAN= ∠DAN= $\frac{a}{2}$ and CN = ND = $\frac{a}{2}$

From right angled triangles ΔCAN and ΔDAN, AC=AD, both AC and AD are longest diagonal (d) for pentagon and hypotenuse for right angled triangles ΔCAN and ΔDAN respectively.

$sin\left(\frac{\alpha}{2}\right) = \frac{a/2}{AC}$ i.e. AC = d = $\frac{a/2}{sin\left(\frac{\alpha}{2}\right)}$

∴ AC = $\frac{a}{2sin\left(\frac{\alpha}{2}\right)}$ ………………… (iv)

From equation (iii) and (iv), we get

$$AC = \frac{a}{2sin\left(\frac{360°}{2n} \times \frac{1}{2}\right)} = \frac{a}{2sin\left(\frac{90°}{n}\right)}$$

From Geometry,

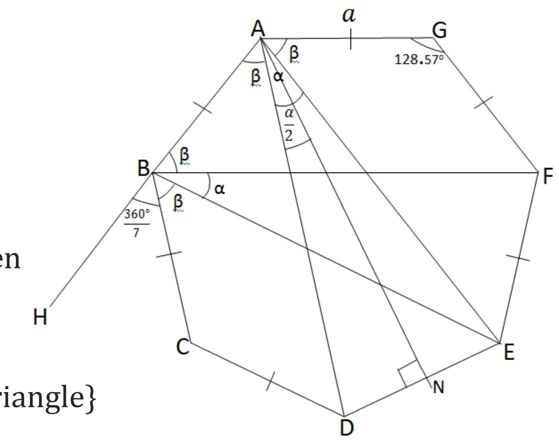
$$\frac{perimeter\ of\ the\ pentagon}{longestdiagonallength} = \frac{5a}{\frac{a}{2sin\left(\frac{90°}{5}\right)}} = 3.090169 \ldots$$

Using formula, we get

$$f(n) = 2nsin\left(\frac{90°}{n}\right) = 10\sin\left(\frac{90°}{5}\right) = 3.090169 \ldots$$

II. Regular Heptagon(n=7)

From both vertex angle A and B of Heptagon an angle ‘α’ taken from the middle so that by symmetry all angles marked ‘β’ are equal in diagram.



Regular Heptagon

Line AB is extended upto H,

ΔADE and ΔBEF are isosceles, and

ΔDAN and ΔEAN are right angled triangle

Let ∠ AEB= ∠DAE = ∠EBF = α(isosceles triangles), then

From ΔABE, we get

(α+ β) + (α+ β) + α = 180° {sum of interior angles of a triangle}

∴ 3α + 2β = 180°……………………(i)

On straight line ABH, we get

∠ABF + ∠FBE + ∠ EBC + ∠CBH = 180°

i.e. $\beta + \alpha + \beta + \frac{360°}{n} = 180°$

∴ $2\beta = 180° - \left(\alpha + \frac{360°}{n}\right)$…………………(ii)

From equation (i) and (ii), we get

$$180° - \left(\alpha + \frac{360°}{n}\right) = 180° - 3\alpha$$

∴ $\alpha = \frac{360°}{2\,n}$……………………(iii)

In ΔADE,line AN divides ΔADE into equal two parts

∠DAN= ∠EAN= $\frac{a}{2}$ andDN = NE =$\frac{a}{2}$

From right angled triangles ΔDAN and ΔEAN, AD=AE, both AD and AE are longest diagonal (d) for heptagon and hypotenuse for right angled triangles ΔDAN and ΔEAN respectively.

From right angled triangle ΔDAN, we get

$$sin\left(\frac{a}{2}\right) = \frac{a/2}{AD} \; i.e. AD = d = \frac{a/2}{sin\left(\frac{a}{2}\right)}$$

$$\therefore \; AD = \frac{a}{2sin\left(\frac{a}{2}\right)} \text{.................... (iv)}$$

From equation (iii) and (iv), we get

$$AD = \frac{a}{2sin\left(\frac{360°}{2n} \times \frac{1}{2}\right)} = \frac{a}{2sin\left(\frac{90°}{n}\right)}$$

Thus, this beautiful rule holds all the regular polygons having odd number of sides.

Since n=7 for heptagon, we get

$$\frac{\text{perimeter of the heptagon}}{longest\ diagonal\ length} = \frac{7a}{\frac{a}{2sin\left(\frac{90°}{n}\right)}} = (2 \times 7)sin\left(\frac{90°}{7}\right) = 3.115293 \ldots$$

Using formula, we get

$$f(n) = 2nsin\left(\frac{90°}{n}\right) = (2 \times 7)sin\left(\frac{90°}{7}\right) = 3.115293 \ldots$$

Similarly, this formula gives an accurate ratio of perimeter to the diagonal of all polygons having an odd number of sides.

4. For $n \to \infty$ representing a apeirogon

For a given perimeter, if n become very large (or countably infinite) then a regular polygon becomes regular apeirogon. Regular apeirogon is ultimate form of regular polygon and it has countably infinite number of equal edges.

Fact: *The ratio of the perimeter of this polygon to the length of its largest diagonal is calculated by the limiting value as $n \to \infty$. and it is calculated to be π.*

ANALYTICAL PROOF

We have two facts:

I. $180° = \pi\, radians$

II. $\lim_{x \to 0} f(x) = \lim_{n \to \infty} \frac{sinx}{x} = 1$

For n is an even number, we get

$$\lim_{n \to \infty} f(n) = \lim_{n \to \infty} nsin\left(\frac{180°}{n}\right)$$

$$\lim_{n \to \infty} f(n) = \lim_{n \to \infty} sin\left(\frac{\pi}{n}\right) n \times \frac{\pi}{\pi}$$

$$\lim_{n \to \infty} f(n) = \lim_{n \to \infty} \frac{sin\left(\frac{\pi}{n}\right) \times \pi}{\frac{\pi}{n}}$$

$$= 1 \times \pi$$

$$= \pi$$

For n is an odd number, we get

$$\lim_{n \to \infty} f(n) = \lim_{n \to \infty} = 2n\, sin\left(\frac{90°}{n}\right)$$

$$\lim_{n \to \infty} f(n) = \lim_{n \to \infty} sin\left(\frac{\pi}{2n}\right) \times 2n \times \frac{\pi}{\pi}$$

$$\lim_{n \to \infty} f(n) = \lim_{n \to \infty} \frac{sin\left(\frac{\pi}{2n}\right) \times \pi}{\frac{\pi}{2n}}$$

$$= 1 \times \pi$$

$$= \pi$$

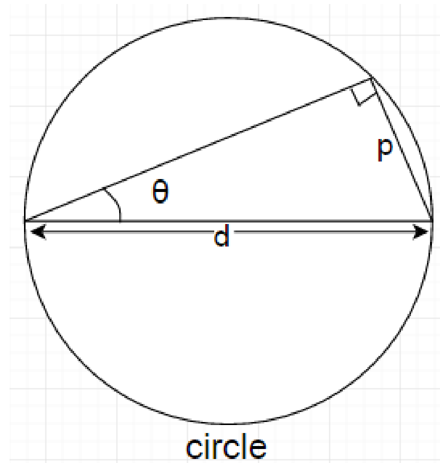
ANALOGY TO CIRCLE

By analogy to regular polygons with large number of edges, regular apeirogon resembles a circle. In the figure right, we can see that C is circumference of circle,θ is angle opposite to the perpendicular arm and p is length of the perpendicular arm of the right angled triangle whose hypotenuse is diameter of the circle. If two lines from diametrically opposite points of the circle intersect at any point on the circumference then angle formed on the - circumference by these two lines is always 90°.

ILLUSTRATION

From ΔABC, we get

$sin\theta = \frac{p}{d}$ i.e. $d = \frac{p}{\sin\theta}$ and the ratio is then given by



$$\lim_{n\to\infty} f(n) = \frac{c}{d} = \frac{C}{\frac{p}{\sin\theta}} = C\frac{\sin\theta}{p}$$

$$\therefore \lim_{n\to\infty} f(n) = C\frac{\sin\theta}{p} = \pi$$

COMPUTATIONAL ILLUSTRATION

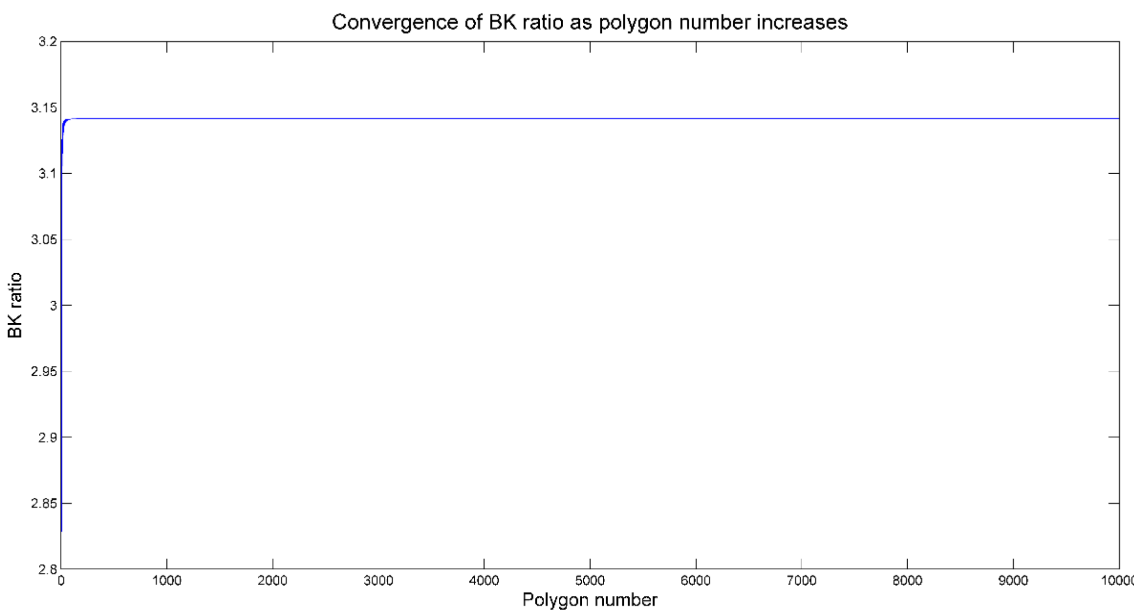


Figure2: convergence of the BK ratio as the number of sides of a regular polygon increases

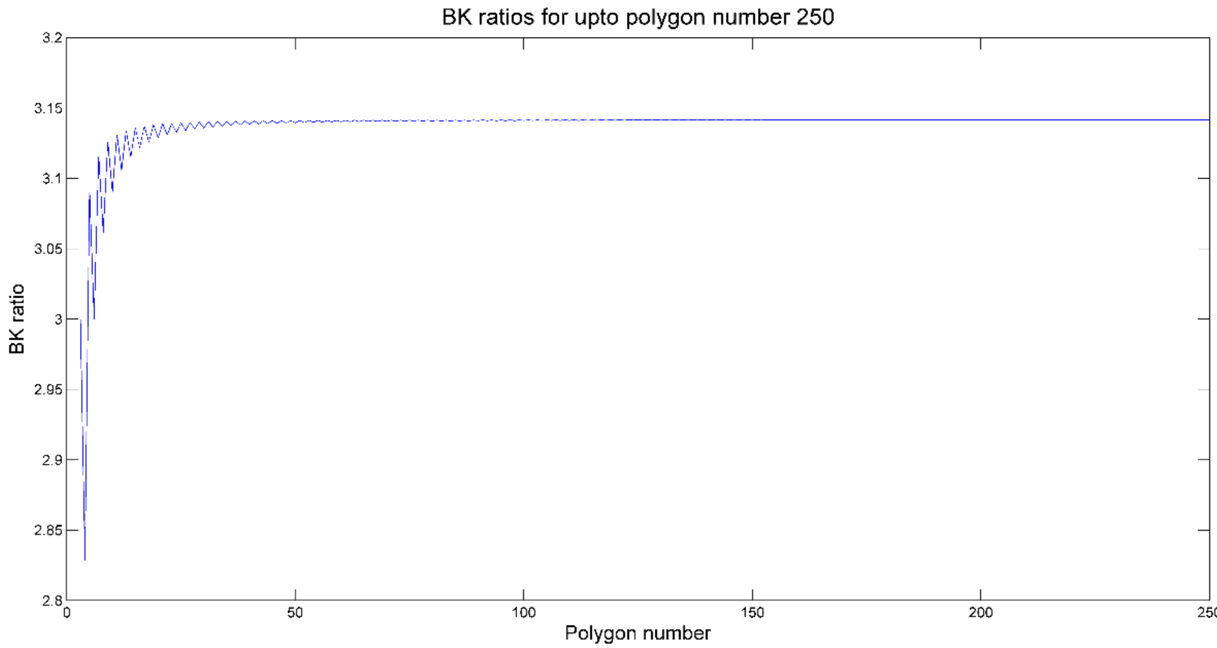


Figure 2: Fluctuations of the BK ratio for the number of sides of a regular polygon from 3 to 250

Figure 2 shows the convergence of the ratios as the polygon number increases. However, at lower polygon number there are significant fluctuations in the value of the ratio. As the polygon number exceeds 250 the fluctuation is approximately negligible and approaches the value of π.

APPENDIX

The numerical values of the ratios have been presented below:

Serial number	Name of regular polygon	Bishwakarma(BK) Ratio	BK ratio with precision upto 30 and 50 significant figures.
1	Equilateral triangle	3	3.00000000000000000000000000000
2	Square	$f(n) = 4\sin(\frac{180°}{4}) = 4\sin 45°$	2.82842712474619009760337744842
3	Pentagon	$f(n) = 2 \times 5\sin(\frac{90°}{5}) = 10\sin 18°$	3.09016994374947424102293417183
4	Hexagon	$f(n) = 6\sin 30°$	3.00000000000000000000000000000
5	Heptagon	$f(n) = 14\sin(90°/7)$	3.11529307538840166004463590295

6	Octagon	f(n)= $8\sin(180°/8)$	3.0614674589207181738276798 7224
7	Nonagon	f(n)= $18\sin 10°$	3.1256671980047462793308992 8185
8	Decagon	f(n)= $10\sin 18°$	3.0901699437494742410229341 7183
9	Undecagon	f(n)= $22\sin(90°/11)$	3.1309264420122730897634387 0956
10	Dodecagon	f(n)= $12\sin 15°$	3.1058285412302491481867860 5149
11	Tridecagon	f(n)= $26\sin(90°/13)$	3.1339536866383993870757598 7377
12	Tetradecagon	f(n)= $14\sin(180°/14)$	3.1152930753884016600446359 0295
13	Pentadecagon	f(n)= $30\sin 6°$	3.1358538980296041419950246 4407
14	Hexadecagon	f(n) = $16\sin(180°/16)$	3.1214451522580522855725578 9563
15	Heptadecagon	f(n) = $34\sin(90°/17)$	3.1371242217522678381481376 4325
16	Octadecagon	f(n) = $18\sin 10°$	3.1256671980047462793308992 8185
17	Enneadecagon	f(n) = $38\sin(90°/19)$	3.1380151279486283348130695 0102
18	Icosagon	f(n) = $20\sin(180°/20)$	3.1286893008046173802021063 8934
19	Icosihenagon	f(n)= $42\sin(90°/21)$	3.1386639306298186802194693 2086
20	Icosikaidigon	f(n)= $22\sin(180°/22)$	3.1309264420122730897634387 0956
21	Icosikaitrigon	f(n) = $46\sin(90°/23)$	3.1391510147748648923746700 3503
22	Icosikaitetragon	f(n)= $24\sin(180°/24)$	3.1326286132812381971617494 6949
23	Icosikaipentagon	f(n)= $50\sin(90°/25)$	3.1395259764656688038089112 2828
24	Icosikaihexagon	f(n)= $26\sin(180°/26)$	3.1339536866383993870757598 7377
25	Icosikaiheptagon	f(n)= $54\sin(90°/27)$	3.1398207611656947410923929 0974
26	Icosikaioctagon	f(n)= $28\sin(180°/28)$	3.1350053308926200371237661 8762
27	Icosikaienneagon	f(n)= $58 \sin(90°/29)$	3.1400566979542165166946829 0652
28	Tricontagon	f(n)= $30\sin 6°$	3.1358538980296041419950246 4407
29	Tricontakaihenagon	f(n)= $62\sin(90°/31)$	3.1402484680001881612826151 6408

30	Tricontakaidigon	f(n)= 32sin(180°/32)	3.136548490545939263814258044444
31	Tricontakaitrigon	f(n)= 66sin(90°/33)	3.140406444366991631685958106 08
32	Tricontakaitetragon	f(n)= sin(180°/34)	3.137124221752267838148137643 25
33	Tricontakaipentagon	f(n)= sin(90°/70)	3.140538124536044782066323576 13
34	Tricontakaihexagon	f(n)= 36sin 5°	3.137606738915694248090313750 15
35	Tricontakaiheptagon	f(n)= 74sin(90°/37)	3.140649036514974634977119877 55
36	Tricontakaioctagon	f(n)= 38sin(180°/38)	3.138015127948628334813069501 02
37	Tricontakaienneagon	f(n)= 78sin(90°/39)	3.140743328534381182232449116 86
38	Tetracontagon	f(n)= 40sin(180°/40)	3.138363829113797801318409839 74
39	Tetracontakaihenagon	f(n)= 82sin(90°/41)	3.140824162582898601850447584 80
40	Tetracontakaidigon	f(n)= 42sin(180°/42)	3.138663930629818680219469320 86
41	Tetracontakaitrigon	f(n)= 86sin(90°/43)	3.140893982958659807290564848 46
42	Tetracontakaitetragon	f(n)= 44sin(180°/44)	3.138924060766222974402861116 95
43	Tetracontakaipentagon	f(n)= 90sin 2°	3.140954703225087448139566346 28
44	Tetracontakaihexagon	f(n)= 46sin(180°/46)	3.139151014774864892374670035 03
45	Tetracontakaiheptagon	f(n)= 94sin(90°/47)	3.141007838721409662274849617 92
46	Tetracontakaioctagon	f(n)= 48sin(180°/48)	3.139350203046867207135146821 21
47	Tetracontakaienneagon	f(n)= 98sin(90°/49)	3.141054602022207074887951483 87
48	Pentacontagon	f(n)= 50sin(180°/50)	3.139525976465668803808911228 28
49	Pentacontakaihenagon	f(n)= 102sin(90°/51)	3.141095972729376095205619287 58
50	Pentacontakaidigon	f(n)= 52sin(180°/52)	3.139681865958874778777041642 20
.	.	.	.
99	Hectohenagan	f(n)= 202sin(90°/101)	3.141466007910876550051400241 23
100	Hectokaidigon	f(n)= 102sin(180°/102)	3.141095972729376095205619287 57
. .	. .	. .	. .

198	Dihectogon	f(n) = 200sin(180°/200)	3.14146346236413515065907066198
199	Dihectohenagon	f(n) = 402sin(90°/201)	3.14156067605829806847912624424
200	Dihectokaidigom	f(n) = 202sin(180°/202)	3.14146600791087655005140024123
. .	. .	. .	. .
498	Pentahectogon	f(n) = 500sin(180°/500)	3.14157198277947562486765507897
499	Pentahectohenagon	f(n) = 1002sin(90°/501)	3.14158750648854657344911867315
500	Pentahectokaidigon	f(n) = 502sin(180°/502)	3.14157214715870277377552989356
. .	. .	. .	. .
998	Chilliagon	f(n) = 1000sin(180°/1000)	3.14158748587956335193322703549
999	Chilliahenagon	f(n) = 2002sin(90°/1001)	3.14159136424174274196381045095
1000	Chilliaidigon	f(n) = 1002sin(180°/1002)	3.14158750648854657344911867315
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4998	Pentakischilliagon	f(n) = 5000sin(180°/5000)	3.14159244688128611672626766426
4999	Pentakischilliahenagon	f(n) = 10002sin(90°/5001)	3.14159260193333034429355391453
5000	Pentakischilliadigon	f(n) = 5002sin(180°/5002)	3.14159244704655375197152596898
. .	. .	. .	. .
9998	Myriagon	f(n) = 10000sin(180°/10000)	3.14159260191266569297934647928
9999	Myriahenagon	f(n) = 20002sin(90°/10001)	3.14159264067309477313310688367
10000	Myriakaidigon	f(n) = 10002sin(180°/10002)	3.14159260193333034429355391453
. .	. .	. .	. .
99998	Centachilliagon	f(n) = 100000sin(180°/100000)	3.14159265307302196048314802075
99999	Centachilliahenagon	f(n) = 200002sin(90°/100001)	3.14159265346060300278062061722
100000	Centachilliakaidigon	f(n) = 100002sin(180°/100002)	3.14159265307304263071415718326
.	.	.	.
999998	Megagon	f(n) = 1000000sin(180°/1000000)	3.14159265358462552568259596341

999999	Megahenagon	f(n) = 2000002sin(90°/1000001)	3.141592653588501312851483566440
1000000	Megakaidigon	f(n) = 1000002sin(180°/1000002)	3.141592653584625546353385071 20
. .	. .	. .	. .
10^{12}-2	Teragon(10^{12} sides)	f(n) = 10^{12}sin(180°/10^{12})	3.1415926535897932384626382 1556
10^{12} -1	(10^{12}+1)-gon	f(n) = 2(10^{12}+1)sin(90°/(10^{12}+1))	3.1415926535897932384626420 9135
10^{12}	(10^{12}+2)-gon	f(n) = (10^{12}+2)sin(180°/(10^{12}+2))	3.1415926535897932384626382 15566722854897991273463…
. .	. .	. .	. .
10^{15}-2	Petagon(10^{15} sides)	f(n) = 10^{15}sin(180°/10^{15})	3.1415926535897932384626433 832743351714171194293458
10^{15}-1	(10^{15}+1)-gon	f(n) = 2(10^{15}+1)sin(90°/(10^{15}+1))	3.1415926535897932384626433 832782109560021569094516
10^{15}	(10^{15}+2)-gon	f(n) = (10^{15}+2)sin(180°/(10^{15}+2))	3.1415926535897932384626433 832743351714171194500168
. . .	. . .	. . .	. . .
∞	Regular Apeirogon (Precise Circle) It is ultimate regular polygon.	$\lim_{n\to\infty} f(n) = \lim_{n\to\infty} nsin\left(\frac{180°}{n}\right) = \pi$ $\lim_{n\to\infty} f(n) = \lim_{n\to\infty} 2nsin\left(\frac{90°}{n}\right) = \pi$	3.1415926535897932384626433 832743351714171194500168

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