Complex Neutrosophic Graphs of Type 1

Said Broumi
Laboratory of Information Processing, Faculty of Science
Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco
broumisaid78@gmail.com

Assia Bakali
Ecole Royale Navale, Boulevard Sour Jdid, B.P 16303
Casablanca, Morocco.
assiabakali@yahoo.fr

Mohamed Talea
Laboratory of Information Processing, Faculty of Science
Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco
taleamohamed@yahoo.fr

Florentin Smarandache
Department of Mathematics, University of New Mexico,
705 Gurley Avenue, Gallup, NM 87301, USA
fsmarandache@gmail.com

Abstract—In this paper, we introduced a new neutrosophic graphs called complex neutrosophic graphs of type1 (CNG1) and presented a matrix representation for it and studied some properties of this new concept. The concept of CNG1 is an extension of generalized fuzzy graphs of type 1 (GFG1) and generalized single valued neutrosophic graphs of type 1 (GSVNG1).

Keywords complex neutrosophic set; Complex neutrosophic graph; Matrix representation.

I. Introduction

Smarandache [7] in 1998, introduced a new theory called Neutrosophic, which is basically a branch of philosophy that focus on the origin, nature, and scope of neutralities and their interactions with different ideational spectra. On the basis of neutrosophy, Smarandache defined the concept of neutrosophic set which is characterized by a degree of truth membership T, a degree of indeterminacy membership I and a degree falsehood membership F. The concept of neutrosophic set theory generalizes the concept of classical sets, fuzzy sets [14], intuitionistic fuzzy sets [13], interval-valued fuzzy sets [12]. In fact this mathematical tool is used to handle problems like imprecision, indeterminacy and inconsistency of data. Specially, the indeterminacy presented in the neutrosophic sets is independent on the truth and falsity values. To easily apply the neutrosophic sets to real scientific and engineering areas, Smarandache [7] proposed the single valued neutrosophic sets as subclass of neutrosophic sets. Later on, Wang et al. [11] provided the set-theoretic operators and various properties of single valued neutrosophic sets. The concept of neutrosophic sets and their particular types have been applied successfully in several fields [40]

Graphs are the most powerful and handful tool used in representing information involving relationship between objects and concepts. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes values from [0, 1]. Later on Atanassov [2] defined intuitionistic fuzzy graphs (IFGs) using five types of Cartesian products. The concept fuzzy graphs and their extensions have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects.

When description of the object or their relations or both is indeterminate and inconsistent, it cannot be handled by fuzzy intuitionistic fuzzy, bipolar fuzzy, vague and interval valued fuzzy graphs. So, for this reason, Smarandache [10] proposed the concept of neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. Then, Smarandache [4, 5] gave another definition for neutrosophic graph theory using the neutrosophic truth-values (T, I, F) and constructed three structures of neutrosophic graphs: neutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. Later on Smarandache [9] proposed new version of neutrosophic graphs such as neutrosophic offgraph, neutrosophic bipolar/tripola/ multipolar graph. Presently, works on neutrosophic vertex-edge graphs and neutrosophic edge graphs are progressing rapidly. Broumi et al.[24] combined the concept of single valued neutrosophic sets and graph theory, and introduced certain types of single valued neutrosophic graphs (SVNG) such as strong single valued neutrosophic graph, constant single valued neutrosophic graph, complete single valued neutrosophic graph and investigate some of their properties with proofs and examples. Also, Broumi et al. [25] also introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. In addition, Broumi et al.[26] proved a necessary and sufficient condition for a single valued neutrosophic graph to be an isolated single valued neutrosophic graph. After Broumi, the studies on the single valued neutrosophic graph theory have been studied increasingly [1, 16-20, 27-34, 36-38].

Recently, Smarandache [8]-initiated the idea of removal of the edge degree restriction of fuzzy graphs, intuitionistic fuzzy graphs and single valued neutrosophic graphs. Samanta et al [35] proposed a new concept named the generalized fuzzy

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graphs (GFG) and defined two types of GFG, also the authors studied some major properties such as completeness and regularity with proved results. In this paper, the authors claims that fuzzy graphs and their extension defined by many researches are limited to represent for some systems such as social network. Later on Broumi et al. [34] have discussed the removal of the edge degree restriction of single valued neutrosophic graphs and presented a new class of single valued neutrosophic graph called generalized single valued neutrosophic graph of type1, which is a is an extension of generalized fuzzy graph of type1 [35]. Since complex fuzzy sets was introduced by Ramot [3], few extension of complex fuzzy set have been widely discussed [22, 23]. Ali and Smarandache [15] proposed the concept of complex neutrosophic set which is a generalization of complex fuzzy set and complex intuitionstic fuzzy sets. The concept of complex neutrosophic set is defined by a complex-valued truth membership function, complex-valued indeterminate membership function, and a complex-valued falsehood membership function. Therefore, a complex-valued truth membership function is a combination of traditional truth membership function with the addition of an extra term.

Similar to the fuzzy graphs, which have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects. Also, complex fuzzy graphs presented in [21] have the same property. Until now, to our best knowledge, there is no research on complex neutrosophic graphs.

The main objective of this paper is to introduce the concept of complex neutrosophic graph of type 1 and introduced a matrix representation of CNG1.

The remainder of this paper is organized as follows. In Section 2, we review some basic concepts about neutrosophic sets, single valued neutrosophic sets, complex neutrosophic sets and generalized single valued neutrosophic graphs of type 1. In Section 3, the concept of complex neutrosophic graphs of type 1 is proposed with an illustrative example. In section 4 a representation matrix of complex neutrosophic graphs of type 1 is introduced. Finally, Section 5outlines the conclusion of this paper and suggests several directions for future research.

II. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, single valued neutrosophic graphs and generalized fuzzy graphs relevant to the present work. See especially [7,11, 15, 34] for further details and background.

Definition 2.1 [7]. Let X be a space of points and let x ∈ X. A neutrosophic set A in X is characterized by a truth membership function T, an indeterminacy membership function I, and a falsity membership function F. T, I, F are real standard or nonstandard subsets of $]^-0,1^+[$, and T, I, F: $X \to]^-0,1^+[$. The neutrosophic set can be represented as

$$A=\{(x, T_A(x), I_A(x), F_A(x)): x \in X\} \quad (1)$$

There is no restriction on the sum of T, I, F, So

$$^-0 \le T_A(x)+ I_A(x)+F_A(x) \le 3^+. \quad (2)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. Thus it is necessary to take the interval [0, 1] instead of $]^-0,1^+[$. For technical applications. It is difficult to apply $]^-0,1^+[$ in the real life applications such as engineering and scientific problems.

Definition 2.2 [11]. Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. A SVNS A can be written as

$$A=\{(x, T_A(x), I_A(x), F_A(x)): x \in X\} \ (3)$$

Definition 2.3 [15]

A complex neutrosophic set A defined on a universe of discourse X, which is characterized by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$ that assigns a complex-valued grade of $T_A(x)$, $I_A(x)$, and $F_A(x)$ in A for any x ∈ X. The values $T_A(x)$, $I_A(x)$, and $F_A(x)$and their sum may all within the unit circle in the complex plane and so is of the following form,

$T_A(x)= p_A(x).e^{j\mu_A(x)}$, $I_A(x)= q_A(x).e^{j\nu_A(x)}$ and $F_A(x)= r_A(x).e^{j\omega_A(x)}$

Where, $p_A(x)$, $q_A(x)$, $r_A(x)$ and $\mu_A(x)$, $\nu_A(x)$, $\omega_A(x)$ are respectively, real valued and $p_A(x)$, $q_A(x)$, $r_A(x) \in [0, 1]$ such that

$$0 \le p_A(x)+ q_A(x) + r_A(x) \le 3$$

The complex neutrosophic set A can be represented in set form as

$$A=\{(x, T_A(x)=a_T, I_A(x)=a_I, F_A(x)=a_F): x \in X\}$$

where $T_A: X \to \{a_T: a_T \in C, |a_T| \le 1\}$,
$I_A: X \to \{a_I: a_I \in C, |a_I| \le 1\}$,
$F_A: X \to \{a_F: a_F \in C, |a_F| \le 1\}$ and
$|T_A(x) + I_A(x) + F_A(x)| \le 3$.

Definition 2.4 [15] The union of two complex neutrosophic sets as follows:

Let A and B be two complex neutrosophic sets in X, where

$A=\{(x, T_A(x), I_A(x), F_A(x)): x \in X\}$ and
$B=\{(x, T_B(x), I_B(x), F_B(x)): x \in X\}$.

Then, the union of A and B is denoted as $A \cup_N B$ and is given as

$$A \cup_N B=\{(x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)): x \in X\}$$

Where the truth membership function $T_{A\cup B}(x)$, the indeterminacy membership function $I_{A\cup B}(x)$ and the falsehood membership function $F_{A\cup B}(x)$ is defined by

$T_{A\cup B}(x)= [(p_A(x) \vee p_B(x))].e^{j.\mu_{T_{A\cup B}}(x)}$,
$I_{A\cup B}(x)= [(q_A(x) \wedge q_B(x))].e^{j.\mu_{I_{A\cup B}}(x)}$,
$F_{A\cup B}(x)= [(r_A(x) \wedge r_B(x))].e^{j.\mu_{F_{A\cup B}}(x)}$
Where $\vee$ and $\wedge$ denotes the max and min operators respectively.
The phase term of complex truth membership function, complex indeterminacy membership function and complex falsity membership function belongs to $(0, 2\pi)$ and, they are defined as follows:

a) Sum:

$$\mu_{A\cup B}(x) = \mu_A(x) + \mu_B(x),$$
$$\nu_{A\cup B}(x) = \nu_A(x) + \nu_B(x),$$
$$\omega_{A\cup B}(x) = \omega_A(x) + \omega_B(x).$$

b) Max:

$$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)),$$
$$\nu_{A\cup B}(x) = \max(\nu_A(x), \nu_B(x)),$$
$$\omega_{A\cup B}(x) = \max(\omega_A(x), \omega_B(x)).$$

c) Min:

$$\mu_{A\cup B}(x) = \min(\mu_A(x), \mu_B(x)),$$
$$\nu_{A\cup B}(x) = \min(\nu_A(x), \nu_B(x)),$$
$$\omega_{A\cup B}(x) = \min(\omega_A(x), \omega_B(x)).$$

d) "The game of winner, neutral, and loser":

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & if \quad p_A > p_B \\ \mu_B(x) & if \quad p_B > p_A \end{cases},$$

$$\nu_{A\cup B}(x) = \begin{cases} \nu_A(x) & if \quad q_A < q_B \\ \nu_B(x) & if \quad q_B < q_A \end{cases},$$

$$\omega_{A\cup B}(x) = \begin{cases} \omega_A(x) & if \quad r_A < r_B \\ \omega_B(x) & if \quad r_B < r_A \end{cases}.$$

The game of winner, neutral, and loser is the generalization of the concept "winner take all" introduced by Ramot et al. in [3] for the union of phase terms.

Definition 2.5 [15] intersection of complex neutrosophic sets
Let A and B be two complex neutrosophic sets in X,
A=$\{(x, T_A(x), I_A(x), F_A(x)): x \in X\}$ and
B=$\{(x, T_B(x), I_B(x), F_B(x)): x \in X\}$.
Then the intersection of A and B is denoted as $A \cap_N B$ and is define as

$$A \cap_N B=\{(x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)): x \in X\}$$

Where the truth membership function $T_{A\cap B}(x)$, the indeterminacy membership function $I_{A\cap B}(x)$ and the falsehood membership function $F_{A\cap B}(x)$ is given as:

$T_{A\cap B}(x)= [(p_A(x) \wedge p_B(x))].e^{j.\mu_{T_{A\cap B}}(x)}$,
$I_{A\cap B}(x)= [(q_A(x) \vee q_B(x))].e^{j.\mu_{I_{A\cap B}}(x)}$,
$F_{A\cap B}(x)= [(r_A(x) \vee r_B(x))].e^{j.\mu_{F_{A\cap B}}(x)}$

Where $\vee$ and $\wedge$ denotes denotes the max and min operators respectively
The phase terms $e^{j.\mu_{T_{A\cap B}}(x)}$, $e^{j.\mu_{I_{A\cap B}}(x)}$ and $e^{j.\mu_{F_{A\cap B}}(x)}$ was calculated on the same lines by winner, neutral, and loser game.

Definition 2.6 [34]. Let V be a non-void set. Two function are considered as follows:
ρ=(ρ_T, ρ_I, ρ_F):V → [0, 1]^{3}and
ω= (ω_T, ω_I, ω_F):VxV → [0, 1]^3 . We suppose
A= {($\rho_T(x)$,$\rho_T(y)$) | ω_T(x, y) ≥ 0},
B= {($\rho_I(x)$,$\rho_I(y)$) |ω_I(x, y) ≥ 0},
C= {($\rho_F(x)$,$\rho_F(y)$) |ω_F(x, y) ≥ 0},
We have considered ω_T, ω_I and $\omega_F \geq 0$ for all set A,B, C , since its is possible to have edge degree = 0 (for T, or I, or F).
The triad (V, ρ, ω) is defined to be generalized single valued neutrosophic graph of type 1 (GSVNG1) if there are functions
α:A→ [0, 1] , β:B→ [0, 1] and δ:C→ [0, 1] such that
$\omega_T(x, y) = \alpha((\rho_T(x),\rho_T(y)))$
$\omega_I(x, y) = \beta((\rho_I(x),\rho_I(y)))$
$\omega_F(x, y) = \delta((\rho_F(x),\rho_F(y)))$ where x, y∈ V.
Here $\rho(x)$= ($\rho_T(x)$, $\rho_I(x)$, $\rho_F(x)$), x ∈ V are the truth-membership, indeterminate-membership and false-membership of the vertex x and $\omega(x, y)$=($\omega_T(x, y)$, $\omega_I(x, y)$, $\omega_F(x, y)$), x, y∈ V are the truth-membership, indeterminate-membership and false-membership values of the edge (x, y).

III. Complex Neutrosophic Graph of Type 1

By using the concept of complex neutrosophic sets [15] and the concept of generalized single valued neutrosophic graph of type 1 [34], we define the concept of complex neutrosophic graph of type 1 as follows:

Definition 3.1. Let V be a non-void set. Two functions are considered as follows:
ρ=(ρ_T, ρ_I, ρ_F):V → [0, 1]^{3}and
ω=(ω_T, ω_I, ω_F):VxV → [0, 1]^3 . We suppose
A= {($\rho_T(x)$, $\rho_T(y)$) | ω_T(x, y) ≥ 0},
B= {($\rho_I(x)$, $\rho_I(y)$) | ω_I(x, y) ≥ 0},
C= {($\rho_F(x)$, $\rho_F(y)$) | ω_F(x, y) ≥ 0},
We have considered ω_T, ω_I and $\omega_F \geq 0$ for all set A,B, C , since its is possible to have edge degree = 0 (for T, or I, or F).
The triad (V, ρ, ω) is defined to be complex neutrosophic graph of type 1 (CNG1) if there are functions
α:A→ [0, 1] , β:B→ [0, 1] and δ:C→ [0, 1] such that
$\omega_T(x, y) = \alpha((\rho_T(x), \rho_T(y)))$
$\omega_I(x, y) = \beta((\rho_I(x), \rho_I(y)))$
$\omega_F(x, y) = \delta((\rho_F(x), \rho_F(y)))$
Where x, y ∈ V.
Here $\rho(x)$=($\rho_T(x)$, $\rho_I(x)$, $\rho_F(x)$), x∈ V are the complex truth-membership, complex indeterminate-membership and complex false-membership of the vertex x and $\omega(x, y)$=($\omega_T(x, y)$, $\omega_I(x, y)$, $\omega_F(x, y)$), x, y ∈ V are the complex truth-membership, complex indeterminate-membership and complex false-membership values of the edge (x, y).

Example 3.2: Let the vertex set be V={x, y, z, t} and edge set be E={(x, y),(x, z),(x, t),(y, t)}

	x	y	z	t
ρ_T	$0.5\ e^{j.0.8}$	$0.9\ e^{j.0.9}$	$0.3\ e^{j.0.3}$	$0.8\ e^{j.0.1}$
ρ_I	$0.3\ e^{j.\frac{3\pi}{4}}$	$0.2\ e^{j.\frac{\pi}{4}}$	$0.1e^{j.2\pi}$	$0.5\ e^{j.\pi}$
ρ_F	$0.1\ e^{j.0.3}$	$0.6\ e^{j.0.5}$	$0.8\ e^{j.0.5}$	$0.4\ e^{j.0.7}$

Table 1: Complex truth-membership, complex indeterminate-membership and complex false-membership of the vertex set.

Let us consider the functions $\alpha(m,n)$= ($m_T \vee n_T$). $e^{j.\mu_{T_{m\cup n}}}$, $\beta(m,n)$=($m_I \wedge n_I$). $e^{j.\mu_{I_{m\cup n}}}$ and $\delta(m,n)$= ($m_F \wedge n_F$). $e^{j.\mu_{F_{m\cup n}}}$.

Here, A={(0.5 $e^{j.0.8}$, 0.9 $e^{j.0.9}$), (0.5 $e^{j.0.8}$, 0.3 $e^{j.0.3}$), (0.5 $e^{j.0.8}$, 0.8 $e^{j.0.1}$), (0.9 $e^{j.0.9}$, 0.8 $e^{j.0.1}$)}

B = {(0.3 $e^{j.\frac{3\pi}{4}}$, 0.2 $e^{j.\frac{\pi}{4}}$), (0.3 $e^{j.\frac{3\pi}{4}}$, 0.1 $e^{j.2\pi}$), (0.3 $e^{j.\frac{3\pi}{4}}$, 0.5 $e^{j.\pi}$), (0.2 $e^{j.\frac{\pi}{4}}$, 0.5 $e^{j.\pi}$)}

C = {(0.1 $e^{j.0.3}$, 0.6 $e^{j.0.5}$), (0.1 $e^{j.0.3}$, 0.8 $e^{j.0.5}$), (0.1 $e^{j.0.3}$, 0.4 $e^{j.0.7}$), (0.6 $e^{j.0.5}$, 0.4 $e^{j.0.7}$)}.Then

ω	(x,y)	(x,z)	(x,t)	(y,t)
ω_T	$0.9\ e^{j.0.9}$	$0.5\ e^{j.0.8}$	$0.8\ e^{j.0.8}$	$0.9\ e^{j.0.9}$
ω_I	$0.2\ e^{j.\frac{3\pi}{4}}$	$0.1\ e^{j.2\pi}$	$0.3e^{j.\pi}$	$0.2\ e^{j.\pi}$
ω_F	$0.1\ e^{j.0.5}$	$0.1\ e^{j.0.5}$	$0.1\ e^{j.0.7}$	$0.4\ e^{j.0.7}$

Table 2: Complex truth-membership, complex indeterminate-membership and complex false-membership of the edge set.

The corresponding complex neutrosophic graph is shown in Fig.2

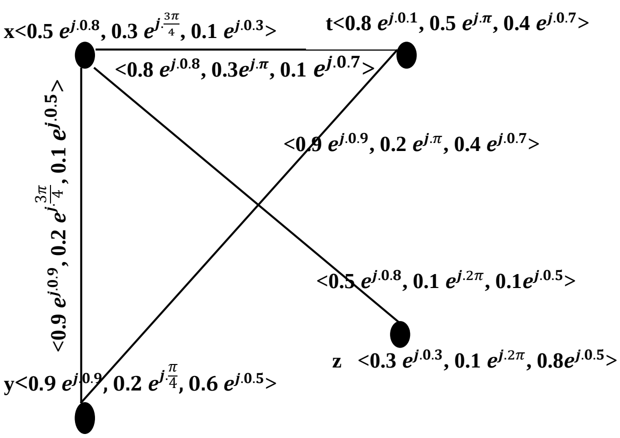

Fig 2.CNG of type 1.

The easier way to represent any graph is to use the matrix representation. The adjacency matrices, incident matrices are the widely matrices used. In the following section CNG1 is represented by adjacency matrix.

IV. Matrix Representation of Complex Neutrosophic Graph of Type 1

In this section, complex truth-membership, complex indeterminate-membership, and complex false-membership are considered independent. So, we adopted the representation matrix of generalized single valued neutrosophic graphs presented ~~in [34].~~

The complex neutrosophic graph (CNG1) has one property that edge membership values (T, I, F) depend on the membership values (T, I, F) of adjacent vertices. Suppose ξ = (V, ρ, ω) is a CNG1 where vertex set V={v_1, v_2,…, v_n}. The functions:

α :A→ (0, 1] is taken such that $\omega_T(x,y)$ = $\alpha((\rho_T(x), \rho_T(y)))$ where x, y∈ V and A= {$(\rho_T(x), \rho_T(y))$ | ω_T(x, y) ≥ 0},

β :B→ (0, 1] is taken such that $\omega_I(x,y)$ = $\beta((\rho_I(x), \rho_I(y)))$ where x, y∈ V and B= {$(\rho_I(x), \rho_I(y))$ | ω_I(x, y) ≥ 0}, and

δ :C→ (0, 1] is taken such that $\omega_F(x,y)$ = $\delta((\rho_F(x), \rho_F(y)))$ where x, y∈ V and C= {$(\rho_F(x), \rho_F(y))$ | ω_F(x, y) ≥ 0}.

The CNG1 can be represented by a (n+1)x(n+1) matrix $M_{G_1}^{T,I,F}$=[$a^{T,I,F}$(i, j)] as follows:

complex truth-membership (T), complex indeterminate-membership (I) and complex false-membership (F) values of the vertices are provided in the first row and first column.

The (i+1, j+1)-th entry are the complex truth-membership (T), complex indeterminate-membership (I), and complex false-membership (F) values of the edge (x_i, x_j), i, j=1,…,n if i≠j.

The (i, i)-th entry is $\rho(x_i)$=($\rho_T(x_i)$, $\rho_I(x_i)$, $\rho_F(x_i)$), where i=1,2,…,n. The Complex truth-membership (T), complex indeterminate-membership (I) and complex false-membership (F) values of the edge can be computed easily using the functions α, β and δ which are in (1,1)-position of the matrix. The matrix representation of CNG1, denoted by $M_{G_1}^{T,I,F}$, can be written as three matrix representation $M_{G_1}^{T}$, $M_{G_1}^{I}$ and $M_{G_1}^{F}$.

The $M_{G_1}^{T}$ can be represented as follows

α	$v_1(\rho_T(v_1))$	$v_2(\rho_T(v_2))$	$v_n(\rho_T(v_n))$
$v_1(\rho_T(v_1))$	$\rho_T(v_1)$	$\alpha(\rho_T(v_1), \rho_T(v_2))$	$\alpha(\rho_T(v_1), \rho_T(v_n))$
$v_2(\rho_T(v_2))$	$\alpha(\rho_T(v_2), \rho_T(v_1))$	$\rho_T(v_2)$	$\alpha(\rho_T(v_2), \rho_T(v_2))$
…	….	…	…
$v_n(\rho_T(v_n))$	$\alpha(\rho_T(v_n), \rho_T(v_1))$	$\alpha(\rho_T(v_n), \rho_T(v_2))$	$\rho_T(v_n)$

Table3. Matrix representation of T-CNG1

The $M_{G_1}^{I}$ can be represented as follows

β	$v_1(\rho_I(v_1))$	$v_2(\rho_I(v_2))$	$v_n(\rho_I(v_n))$
$v_1(\rho_I(v_1))$	$\rho_I(v_1)$	$\beta(\rho_I(v_1), \rho_I(v_2))$	$\beta(\rho_I(v_1), \rho_I(v_n))$
$v_2(\rho_I(v_2))$	$\beta(\rho_I(v_2), \rho_I(v_1))$	$\rho_I(v_2)$	$\beta(\rho_I(v_2), \rho_I(v_2))$
…	….	…	…
$v_n(\rho_I(v_n))$	$\beta(\rho_I(v_n), \rho_I(v_1))$	$\beta(\rho_T(v_n), \rho_I(v_2))$	$\rho_I(v_n)$

Table4. Matrix representation of I-CNG1

The $M_{G_1}^I$ can be represented as follows

δ	$v_1(\rho_F(v_1))$	$v_2(\rho_F(v_2))$	$v_n(\rho_F(v_n))$
$v_1(\rho_F(v_1))$	$\rho_F(v_1)$	$\delta(\rho_F(v_1), \rho_F(v_2))$	$\delta(\rho_F(v_1), \rho_F(v_n))$
$v_2(\rho_F(v_2))$	$\delta(\rho_F(v_2), \rho_F(v_1))$	$\rho_F(v_2)$	$\delta(\rho_F(v_2), \rho_F(v_2))$
…	….	…	…
$v_n(\rho_F(v_n))$	$\delta(\rho_F(v_n), \rho_F(v_1))$	$\delta(\rho_F(v_n), \rho_F(v_2))$	$\rho_F(v_n)$

Table5. Matrix representation of F-CNG1

Remark 1 : If the complex indeterminacy-membership and complex non-membership values of vertices equals zero, and phase term of complex truth membership of vertices equals 0, the complex neutrosophic graphs of type 1 is reduced to generalized fuzzy graphs type 1 (GFG1).

Remark 2: If the phase term of complex truth membership, complex indeterminacy membership and complex falsity membership values of vertices equals 0, the complex neutrosophic graphs of type 1 is reduced to generalized single valued neutrosophic graphs of type 1 (GSVNG1).

Here the complex neutrosophic graph of type 1 (CNG1) can be represented by the matrix representation depicted in table 9. The matrix representation can be written as three matrices one containing the entries as T, I, F (see table 6, 7 and 8).

α	x(0.5 $e^{j.0.8}$)	y(0.9 $e^{j.0.9}$)	z(0.3 $e^{j.0.3}$)	t(0.8 $e^{j.0.1}$)
x(0.5 $e^{j.0.8}$)	0.5 $e^{j.0.8}$	0.9 $e^{j.0.9}$	0.5 $e^{j.0.8}$	0.8 $e^{j.0.8}$
y(0.9 $e^{j.0.9}$)	0.9 $e^{j.0.9}$	0.9 $e^{j.0.9}$	0	0.9 $e^{j.0.9}$
z(0.3 $e^{j.0.3}$)	0.5 $e^{j.0.8}$	0	0.3 $e^{j.0.3}$	0
t(0.8 $e^{j.0.1}$)	0.8 $e^{j.0.8}$	0.9 $e^{j.0.9}$	0	0.8 $e^{j.0.1}$

Table 6: Complex truth-matrix representation of CNG1

β	x(0.3 $e^{j.\frac{3\pi}{4}}$)	y(0.2 $e^{j.\frac{\pi}{4}}$)	z(0.1 $e^{j.2\pi}$)	t(0.5 $e^{j.2\pi}$)
x(0.3 $e^{j.\frac{3\pi}{4}}$)	0.3 $e^{j.\frac{3\pi}{4}}$	0.2 $e^{j.\frac{3\pi}{4}}$	0.1 $e^{j.2\pi}$	0.3 $e^{j.2\pi}$
y(0.2 $e^{j.\frac{\pi}{4}}$)	0.2 $e^{j.\frac{3\pi}{4}}$	0.2 $e^{j.\frac{\pi}{4}}$	0	0.2 $e^{j.2\pi}$
z(0.1 $e^{j.2\pi}$)	0.1 $e^{j.2\pi}$	0	0.1 $e^{j.2\pi}$	0
t(0.5 $e^{j.2\pi}$)	0.3 $e^{j.2\pi}$	0.2 $e^{j.2\pi}$	0	0.5 $e^{j.2\pi}$

Table 7: Complex indeterminate- matrix representation of CNG1.

δ	x(0.1 $e^{j.0.3}$)	y(0.6 $e^{j.0.5}$)	z(0.8 $e^{j.0.5}$)	t(0.8 $e^{j.0.7}$)
x(0.1 $e^{j.0.3}$)	0.1 $e^{j.0.3}$	0.1 $e^{j.0.6}$	0.1 $e^{j.0.3}$	0.1 $e^{j.0.8}$
y(0.6 $e^{j.0.5}$)	0.1 $e^{j.0.5}$	0.6 $e^{j.0.5}$	0	0.6 $e^{j.0.7}$
z(0.8 $e^{j.0.5}$)	0.1 $e^{j.0.5}$	0	0.8 $e^{j.0.5}$	0
t(0.8 $e^{j.0.7}$)	0.1 $e^{j.0.7}$	0.6 $e^{j.0.7}$	0	0.8 $e^{j.0.7}$

Table 8: Complex falsity- matrix representation of CNG1

The matrix representation of CNG1 can be represented as follows:

(α, β, δ)	**x(0.5 $e^{j.0.8}$, 0.3 $e^{j.\frac{3\pi}{4}}$, 0.1 $e^{j.0.3}$)**	**y(0.9 $e^{j.0.9}$, 0.2 $e^{j.\frac{\pi}{4}}$, 0.6 $e^{j.0.5}$)**	**z(0.3 $e^{j.0.3}$, 0.1 $e^{j.2\pi}$, 0.8 $e^{j.0.5}$)**	**t(0.8 $e^{j.0.1}$, 0.5 $e^{j.1\pi}$, 0.4 $e^{j.0.7}$)**
x(0.5 $e^{j.0.8}$, 0.3 $e^{j.\frac{3\pi}{4}}$, 0.1 $e^{j.0.3}$)	**(0.5 $e^{j.0.8}$, 0.3 $e^{j.\frac{3\pi}{4}}$, 0.1 $e^{j.0.3}$)**	**(0.9 $e^{j.0.9}$, 0.2 $e^{j.\frac{3\pi}{4}}$, 0.1 $e^{j.0.5}$)**	**(0.5 $e^{j.0.8}$, 0.1 $e^{j.2\pi}$, 0.1 $e^{j.0.5}$)**	**(0.8 $e^{j.0.8}$, 0.3 $e^{j.1\pi}$, 0.1 $e^{j.0.7}$)**
y(0.9 $e^{j.0.9}$, 0.2 $e^{j.\frac{\pi}{4}}$, 0.6 $e^{j.0.5}$)	**(0.9 $e^{j.0.9}$, 0.2 $e^{j.\frac{3\pi}{4}}$, 0.1 $e^{j.0.5}$)**	**(0.9 $e^{j.0.9}$, 0.2 $e^{j.\frac{\pi}{4}}$, 0.6 $e^{j.0.5}$)**	(0,0,0)	**(0.9 $e^{j.0.9}$, 0.2 $e^{j.1\pi}$, 0.4 $e^{j.0.7}$)**
z(0.3 $e^{j.0.3}$, 0.1 $e^{j.2\pi}$, 0.8 $e^{j.0.5}$)	**(0.5 $e^{j.0.8}$, 0.1 $e^{j.2\pi}$, 0.1 $e^{j.0.5}$)**	(0,0,0)	**(0.3 $e^{j.0.3}$, 0.1 $e^{j.2\pi}$, 0.8 $e^{j.0.5}$)**	(0,0,0)
t(0.8 $e^{j.0.1}$, 0.5 $e^{j.1\pi}$, 0.4 $e^{j.0.7}$)	**(0.8 $e^{j.0.8}$, 0.3 $e^{j.\frac{3\pi}{4}}$, 0.1 $e^{j.0.7}$)**	**(0.9 $e^{j.0.8}$, 0.2 $e^{j.2\pi}$, 0.4 $e^{j.0.5}$)**	(0,0,0)	**(0.8 $e^{j.0.1}$, 0.5 $e^{j.1\pi}$, 0.4 $e^{j.0.7}$)**

Table 9: Matrix representation of CNG1.

Theorem 1. Let $M_{G_1}^T$ be a matrix representation of complexT-CNG1, then the degree of vertex $D_T(x_k) = \sum_{j=1, j\neq k}^{n} a^T(k+1, j+1)$, $x_k \in$ V or $D_T(x_p) = \sum_{i=1, i\neq p}^{n} a^T(i+1, p+1)$, $x_p \in$ V.

Proof: It is similar as in theorem 1 of [34].

Theorem 2. Let $M_{G_1}^I$ be a matrix representation of complex I-CNG1, then the degree of vertex $D_I(x_k) = \sum_{j=1, j\neq k}^{n} a^I(k+1, j+1)$, $x_k \in$ V or $D_I(x_p) = \sum_{i=1, i\neq p}^{n} a^I(i+1, p+1)$, $x_p \in$ V.

Proof: It is similar as in theorem 1 of [34].

Theorem 3. Let $M_{G_1}^F$ be a matrix representation of complex F-CNG1, then the degree of vertex $D_F(x_k) = \sum_{j=1, j\neq k}^{n} a^F(k+1, j+1)$, $x_k \in$ V or $D_F(x_p) = \sum_{i=1, i\neq p}^{n} a^F(i+1, p+1)$, $x_p \in$ V.

Proof: It is similar as in theorem 1 of [34].

Theorem4. Let $M_{G_1}^{T,I,F}$ be matrix representation of CNG1, then the degree of vertex $D(x_k) = (D_T(x_k), D_I(x_k), D_F(x_k))$ where

$D_T(x_k) = \sum_{j=1, j\neq k}^{n} a^T(k+1, j+1)$, $x_k \in$ V.

$D_I(x_k) = \sum_{j=1, j\neq k}^{n} a^I(k+1, j+1)$, $x_k \in$ V.

$D_F(x_k) = \sum_{j=1, j\neq k}^{n} a^F(k+1, j+1)$, $x_k \in$ V

Proof: the proof is obvious.

V. CONCLUSION

In this article, we presented a new concept of neutrosophic graph called complex neutrosophic graphs of type 1 and presented a matrix representation of it. The concept of complex neutrosophic graph of type 1 (CNG1) can be applied to the case of bipolar complex neutrosophic graphs (BCNG1). In the future works, we plan to study the concept of completeness, the concept of regularity and to define the concept of complex neutrosophic graphs of type 2.

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