The Theory of Quantum Gravity without divergenies and Calculation of Cosmological Constant

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Abstract

To construct quantum gravity we introduce the quantum gravity state as function of particle coordinates and functional of fields, We add metric as the new argument of state:

$$\Psi = \Psi(t, x_1, ...x_n, \{A^\gamma(x)\}, \{g_{\mu\nu}(x)\})$$

we calculate the cosmological constant assuming that the quantum state is a function of time and radius of universe (mini-superspace)

$$\Psi = \Psi(t, a)$$

To avoid infinities in the solutions, we substitute the usual equation for propagotor with initial value Cauchy problem, which has finite and unique solution, for example we substitute the equation for Dirac electron propagator

$$(\gamma^\mu p_\mu - mc)K(t, x, t_0, x_0) = \delta(\vec{x} - \vec{x_0})\delta(t - t_0)$$

which already has infinity at the start $t = t_0$, with the initial value Cauchy problem

$$\begin{cases} (H - i\hbar\partial/\partial t)K(t, x, t_0, x_0) = 0, \\ K(t, x, t_0, x_0) = \delta(\vec{x} - \vec{x_0}), \quad t = t_0, \end{cases}$$

which has finite and unique solution.

Key words: Quantum gravity, cosmological constant, Friedman.
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1. Introduction

The new and equivalent formulation of quantum electrodynamics is given in terms of space of states.

$$\Psi = \Psi(t, x_1, ...x_n, \{A^\gamma(x)\}) \tag{1}$$

where A^γ is the electromagnetic field. To construct quantum gravity we add metric as the new argument of state. The quantum gravity state is

$$\Psi = \Psi(t, x_1, ...x_n, \{A^\gamma(x)\}, \{g_{\mu\nu}(x)\}) \tag{2}$$

For example, the mean value of metric at point x_0 at time t

$$\langle g_{\mu\nu}(t_0, x_0)\rangle = \int Dx_1...Dx_n D\{A^\gamma(x)\} D\{g_{\mu\nu}(x)\} \Psi^*(t) g_{\mu\nu}(x_0) \Psi(t) \tag{3}$$

The quantum gravity must me completely field theory, we substitute particle coordinates with the mass fields.

$$\Psi = \Psi(t, \{\Phi_1(x)\}, ...\{\Phi_n(x)\}, \{A^\gamma(x)\}, \{g_{\mu\nu}(x)\}) \tag{4}$$

To find the equation of motion we use the approach

Action $\rightarrow$ Lagrangian $\rightarrow$ Hamiltonian $\rightarrow$ Quantization.

The lagrangian density participates in the integral

$$S = \int dt d^3x \mathcal{L} \tag{5}$$

The momentum density

$$p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}(x)} \tag{6}$$

where φ are the all fields.

$$\varphi = (g^{\alpha\beta}, A^\mu, \phi_1, ...\phi_n), \tag{7}$$

where $\phi_1, ...\phi_n$ are the mass fields.

The Hamiltonian density

$$\mathcal{H} = p\partial_t\varphi - \mathcal{L} \tag{8}$$

The Hamiltonian

$$H = \int \mathcal{H} d^3x \tag{9}$$

To perform quantization we substitute the fields and the momentum of fields with the operators.

$$\mathcal{H} = \mathcal{H}(t, x, \varphi(x), p_\phi(x), \frac{\partial^2\varphi}{\partial x_\mu \partial x_\nu}) \tag{10}$$

We introduce the operator of the field value at the point x_0

$$\hat{\varphi}(x_0) : \Psi \longrightarrow \varphi(x_0)\Psi \tag{11}$$

The momentum operator of the field value at the point x_0

$$\hat{p}_{\varphi}(x_0) : \Psi \longrightarrow -i\hbar \lim_{q\to 0} \frac{\Psi(\{\phi(x) + q\delta(x - x_0)\}) - \Psi(\{\phi(x)\}))}{q} \tag{12}$$

The momentum and coordinate operators satisfy

$$\frac{i}{\hbar}[\hat{p}_{\phi}(x), \hat{\varphi}(x_0] = \delta(x - x_0) \tag{13}$$

The terms $\partial^2 g_{\mu\nu}/\partial x_0^2$ are not so well for quantization, so we perform integration by parts, after the gravity action is ready for quantization, and we construct quantum gravity theory,

2. Relativistic invariance

To achieve the relativistic invariance we represent our manifold as set of slices, with invariant parameter w-number of slices,and the invariant points M on each slices:

$$\Psi = \Psi(w, M_1, ...M_n, \{A^{\gamma}(M)\}) \tag{14}$$

3. The quantization of gravity

The problem while quantization of gravity is presence of second time terms in action (we will integrate by parts to eliminate them)

$$S = \int R d\Omega = \int R\sqrt{-g}dx \tag{15}$$

where $R = R^i_{Klm} g^{Km} \delta^i_l$ is the scalar curvature. The curvature tensor

$$R^i_{Klm} = \frac{\partial \Gamma^i_{Km}}{\partial x^l} - \frac{\partial \Gamma^i_{Kl}}{\partial x^m} + \Gamma^i_{nl}\Gamma^n_{Km} - \Gamma^i_{nm}\Gamma^n_{Kl} \tag{16}$$

The table of Christobel symbols Γ^i_{Kl} is a tensor up to linear transformations, so application of the covariant derivative to Christobel symbols is possible in this narrow sence, and is also a tensor

$$\frac{\delta \Gamma^i_{Km}}{\delta x^l} = \frac{\partial \Gamma^i_{Km}}{\partial x^l} + \Gamma^i_{l\alpha}\Gamma^{\alpha}_{Km} - \Gamma^{\alpha}_{Kl}\Gamma^i_{\alpha m} - \Gamma^{\alpha}_{ml}\Gamma^i_{K\alpha} \tag{17}$$

Then after changing indices

$$\frac{\delta \Gamma^i_{Kl}}{\delta x^m} = \frac{\partial \Gamma^i_{Kl}}{\partial x^m} + \Gamma^i_{m\alpha}\Gamma^{\alpha}_{Kl} - \Gamma^{\alpha}_{Km}\Gamma^i_{\alpha l} - \Gamma^{\alpha}_{lm}\Gamma^i_{K\alpha} \tag{18}$$

The substraction gives

$$\frac{\delta \Gamma^i_{Km}}{\delta x^l} - \frac{\delta \Gamma^i_{Kl}}{\delta x^m} = \frac{\partial \Gamma^i_{Km}}{\partial x^l} - \frac{\partial \Gamma^i_{Kl}}{\partial x^m} + 2\Gamma^{\alpha}_{Km}\Gamma^i_{\alpha l} - 2\Gamma^{\alpha}_{Kl}\Gamma^i_{\alpha m} \tag{19}$$

Then, obviously,

$$R^i_{Klm} = \frac{\delta\Gamma^i_{Km}}{\delta x^l} - \frac{\delta\Gamma^i_{Kl}}{\delta x^m} - \Gamma^\alpha_{Km}\Gamma^i_{\alpha l} + \Gamma^\alpha_{Kl}\Gamma^i_{\alpha m} \tag{20}$$

Then after substituting this into the action $S_g = \int(Rd\Omega) = \int\{R^i_{Klm}\delta^l_i g^{Km}\sqrt{-g}dx\}$, after integrating the covariant derivative by parts, we see that it vanishes (the covariant derivative of metric is zero), the only term remains:

$$S_g = \int(-\Gamma^\alpha_{Km}\Gamma^l_{\alpha l} + \Gamma^\alpha_{Kl}\Gamma^l_{\alpha m})g^{Km}\sqrt{-g}dx^\gamma \tag{21}$$

The action now does not contain second derivatives, only up to first, now we can construct Hamilton formalism, the lagrangian density participates in the integral

$$S_g = \int dtd^3x\mathcal{L}_g \tag{22}$$

The momentum density

$$p^{\mu\nu} = \frac{\partial\mathcal{L}_g}{\partial g'_{\mu\nu}} \tag{23}$$

where g' is the time derivative of metric

The density of gravity Hamiltonian (the other parts of the entire Hamiltonian including interaction part, the Hamltonian of other fields we have written in previous section)

$$\mathcal{H}_g = p^{\mu\nu}g'_{\mu\nu} - \mathcal{L}_g \tag{24}$$

The Hamiltonian

$$H_g = \int\mathcal{H}_g d^3x \tag{25}$$

The problem while constructing Hamiltonian is to write velocities $g'_{\mu\nu}$ as functions of momentum $p^{\mu\nu}$, before the lagrangian contains terms 0,1,2 power of velocities, the Hamiltonian before transition from velocities to momenta

$$\mathcal{H}_g = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2 \tag{26}$$

The construction of Hamiltonian from lagrangian gives vanishing of linear terms:

$$\mathcal{H}_1 = 0 \tag{27}$$

The zero term of Hamiltonian does not contain velocities, so it is equal to zero term of lagrangian

$$\mathcal{H}_0 = -\mathcal{L}_0 \tag{28}$$

The construction of second term of Hamiltonian is complex, it is equal to second term of lagrangian, but still transition from velocities to momenta is not clear

$$\mathcal{H}_2 = \mathcal{L}_2 = 1/2(\partial^2 L/\partial g'_{\mu\nu}\partial g'_{\alpha\gamma})g'_{\mu\nu}g'_{\alpha\gamma} \tag{29}$$

The matrix there is Hessian W of Lagrange function, the elements there are zero, if at least one of indices is zero, W is irreversible, so we can not transit from all velocities to all momenta, nevertheless if we remove those zero elements from W, which can not contribute to Hamiltonian, then the remaining matrix W' is reversible, we can now transit from velocities to momenta, the construction of gravity Hamiltonian is over, the construction of entire Hamiltonian is written in the previous section.

The construction of Quantum Gravity is over.

4. The calculation of cosmological constant

To calculate the cosmological constant we perform the quantization of Friedman model. Suppose that the state is the function of time and the universe curvature radius.

$$\Psi = \Psi(t, a) \tag{30}$$

To calculate the quantum Hamiltonian we use our scheme:

Action $\rightarrow$ Lagrangion $\rightarrow$ Hamiltonian $\rightarrow$ Quantization.

As well known, the amount of the visible matter in universe is less then 5 %. We assume that the universe is massless and the Einstein-Hilbert action given by

$$S = \frac{c^3}{16\pi G}\int d\Omega R = \frac{c^3}{16\pi G}\int dt d\vec{x}\sqrt{-g}R \tag{31}$$

$$L = \int d\vec{x}\frac{c^3}{16\pi G}\sqrt{-g}R \tag{32}$$

If we substitute into the action the Friedman metric (open case)

$$d^2s = c^2dt^2 - a^2(t)(d\chi^2 + sinh^2\chi^2(d\Theta^2 + sin^2\Theta d\phi^2)) \tag{33}$$

If we introduce the new variable η which satisfies the differential relation $ad\eta = cdt$

$$d^2s = a^2(\eta)(d\eta^2 - d\chi^2 - sinh^2\chi(d\Theta^2 + sin^2\Theta d\varphi^2)) \tag{34}$$

Then the calculation gives

$$R = \frac{6(a - a''(\eta))}{a^3}, g = -a^8 sinh^4\chi sin^2\Theta \tag{35}$$

Taking the integral of the angles, we have:

$$S = \varpi \int d\eta a(a - a''(\eta)) = \varpi \int d\eta (a^2 + a'^2) = \int d\eta L, \tag{36}$$

where the Lagrangian is

$$L = a'^2 + a^2 = E - U \tag{37}$$

is the Lagrangian of the particle in the accelerating potential

$$U = -a^2 \tag{38}$$

Suppose that quantum mean values obey the Eirenfest theorems which are our case the Euler-Lagrange equations,which give

$$< a'' > - < a >= 0 \tag{39}$$

Their general solution is

$$a = a_1 e^{\eta} + a_2 e^{-\eta} \tag{40}$$

I. Case 1(the initial radius of Universe is no-zero and positive:$a_1 > 0, a_2 > 0$) Making a shift of η, we can make

$$a_1 = a_2 \tag{41}$$

so that

$$a = a_1 e^{\eta} + a_1 e^{-\eta} = a_0 cosh\eta,\ a' = a_0 sinh\eta \tag{42}$$

The neighboring geodesics equations

$$\frac{D^2 \triangle x^{\gamma}}{D\tau^2} = R^{\gamma}_{\alpha\beta\eta} \nu^{\alpha} \nu^{\beta} \Delta x^{\eta} \tag{43}$$

In the case of zero spatial velocities we have

$$\frac{D^2 \triangle x^{\gamma}}{D\tau^2} = R^{\gamma}_{00\gamma} (\nu 0)^2 \Delta x^{\gamma} \tag{44}$$

The sign of $R^{\gamma}_{00\gamma}$ shows if the universe accelerates

$$R^{\eta}_{00\eta} = -(\frac{a'}{a})^2 + \frac{a''}{a} = -(\frac{a'}{a})^2 + 1 = -tanh^2\eta + 1 > 0 \tag{45}$$

We observe the positive acceleration of the universe, which has not ever been given by any theory.

I. Case 2(the initial radius of Universe is zero:$a_1 > 0, a_2 < 0$) Making a shift of η, we can make

$$a_1 = a_2 \tag{46}$$

so that

$$a = a_1 e^{\eta} - a_1 e^{-\eta} = a_0 sinh\eta,\ a' = a_0 cosh\eta \tag{47}$$

The neighboring geodesics equations

$$\frac{D^2\triangle x^\gamma}{D\tau^2} = R^\gamma_{\alpha\beta\eta}\nu^\alpha\nu^\beta\Delta x^\eta \tag{48}$$

In the case of zero spatial velocities we have

$$\frac{D^2\triangle x^\gamma}{D\tau^2} = R^\gamma_{00\gamma}(\nu 0)^2\Delta x^\gamma \tag{49}$$

The sign of $R^\gamma_{00\gamma}$ shows if the universe accelerates

$$R^\eta_{00\eta} = -(\frac{a\prime}{a})^2 + \frac{a\prime\prime}{a} = -(\frac{a\prime}{a})^2 + 1 = -ctanh^2\eta + 1 < 0 \tag{50}$$

We see that if at the starting time the universe was at one point then we would not see the acceleration of Universe. We can see the accelerating Universe only if the initial radius of Universe is not zero(see case 1).This is the argument against Big Bang.

5. The Cosmological Constant

In Case 1 of the accelerating Universe

$$a = a_0 cosh\eta \tag{51}$$

Remembering that $ad\eta = cdt$ and integrating we obtain

$$ct = a_0 sinh\,\eta \tag{52}$$

The Hubble constant

$$H = \frac{da/dt}{a} = \frac{cda/d\eta}{a^2} = \frac{ca\prime}{a^2} \tag{53}$$

Substituting there again (31)and using (32), we have:

$$H = \frac{ca_0 sinh\,\eta}{a_0^2 cosh^2\,\eta} = \frac{sinh^2\,\eta}{tcosh^2\,\eta} = \frac{tanh^2\,\eta}{t} \tag{54}$$

From where we have:

$$\eta = arctanh(Ht)^{1/2} \tag{55}$$

If we suppose Ht=2/3 we have

$$\eta = arctanh(2/3)^{1/2} \approx 1.15 \tag{56}$$

Using neighboring geodesics equation (28) and (29) we see that the sign

$$\frac{D^2(\Delta x^\gamma)}{D\tau^2}/\Delta x^\gamma = R^\gamma{}_{00\gamma}(v^0)^2 = \frac{c^2}{a^2}((-\frac{a\prime}{a})^2 + \frac{a\prime\prime}{a}) = -H^2 + \frac{c^2}{a^2} \tag{57}$$

Using equation for Hubble constant, we obtain

$$\frac{D^2(\Delta x^\gamma)}{D\tau^2}/\Delta x^\gamma = -(\frac{ca\prime}{a^2})^2 + \frac{c^2}{a^2} = \frac{c^2(-a\prime^2 + a^2)}{a^4} = \frac{c^2 a_0^2}{a^4} = \frac{c^2}{a_0^2 cosh\, \eta^4} \tag{58}$$

Using (32),we receive

$$\frac{D^2(\Delta x^\gamma)}{D\tau^2}/\Delta x^\gamma = \frac{(sinh\, \eta)^2}{t^2 cosh\, \eta^4} = \frac{(tanh\, \eta)^2}{t^2 cosh\, \eta^2} = \frac{(Ht)^2}{t^2 cosh\, \eta^2} = \frac{H^2}{cosh\, \eta^2} \tag{59}$$

Substituting the values of η and H, we have according to the theory

$$\frac{D^2(\Delta x^\gamma)}{D\tau^2}/\Delta x^\gamma = \frac{H^2}{3} \approx 1.33 * 10^{-36} \frac{1}{s^2} \tag{60}$$

Experiment gives the following value of Universe Expansion acceleration

$$\frac{D^2(\Delta x^\gamma)}{D\tau^2}/\Delta x^\gamma = 2\frac{2\pi G\rho_{cr}}{3} \approx 2 * 10^{-36} \frac{1}{s^2} \tag{61}$$

Conclusion

The construction of quantum gravity is possible if we stand on very good land of quantum states, this path now has shown the way to calculate the cosmological constant

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