应用在数轴上逐步形成连续合数点的方法证明格林姆猜想

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摘要

让我们把具有一个共同素因子的正整数作为一种，那么，数轴的正方向射线就由无限多具有 χ 种整数点的相同排列的循环线段组成，这里 $\chi \geq 1$. 在本文中，我们将应用逐步改变原始数轴上每一种合数点符号的方法去证明格林姆猜想，以达到在已证明勒让德 —张猜想成真的有限范围内形成连续的合数点。

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关键词： 连续合数点，数轴，独特的素因子，勒让德 —张猜想

1. **介绍：** 格林姆猜想是以卡尔. 艾伯特. 格林姆命名的。这个猜想声明：如果 $n+1, n+2 \ldots n+k$ 全部是合数，那么，就有不同的素数 p^i_j 使得 $p^i_j \mid (n+j)$，对于 $1 \leq j \leq k$. 例如，对于在素数523 和 541之间的连续合数，从它们中取出各不相同的素因子如下：

合数：	**524,**	**525,**	**526,**	**527,**	**528,**	**529,**	**530,**
分解素因子：	$2^2\times131$,	$3\times5^2\times7$，	2×263，	17×31，	$2^4\times3\times11$，	23^2，	$2\times5\times53$，
不同素因子：	**131,**	**3,**	**263,**	**17,**	**11,**	**23,**	**53,**

531,	**532,**	**533,**	**534,**	**535,**	**536,**	**537,**	**538,**	**539,**	**540**
$3^2\times59$，	$2^2\times7\times19$,	13×41,	$2\times3\times89$，	5×107，	$2^3\times67$，	3×179，	2×269，	$7^2\times11$，	$2^2\times3^3\times5$
59,	**19,**	**41,**	**89,**	**107,**	**67,**	**179,**	**269,**	**7,**	**2**

这个猜想在1969年首先发表在美国数学月刊第76卷第1126-1128页.

然而，至今，它仍然是一个既没有被证明，又没有被否定的猜想。

2. 证明前的准备

首先，我们需要去引用已被证明成真的勒让德 —张猜想，用以限制连续合数的长度。

勒让德 —张猜想表明：在n^2 和 $n(n+1)$之间至少有一个奇素数，在$n(n+1)$ 和 $(n+1)^2$之间也至少有一个奇素数。对勒让德 —张猜想的一个证明来自于作者张天树，“*勒让德 —张猜想和吉尔布雷斯猜想以及对它们的证明*”，《综合数学》，ISSN 1221-5023，第 20 卷，2-3 期，2012 年，第 86-100 页.

由此可见，在n^2 和 $(n+1)^2$ 之间有至少两个奇素数，这里$n \geq 2$. 毫无疑问，连续合数的长度要比从n^2 到 $(n+1)^2$ 的距离要短。

另外，我们还需要去确定一个术语，就是让DPF (C_χ) 表示任意合数C_χ的一组及唯一一组不同的素因子。

因为在数轴正方向射线上的每一个整数点表示一个整数，而且无限多的两相邻整数点之间的距离彼此相等，于是，首先让我们使用符号 • 去表示一个整数点，无论它在简洁陈述，还是在数轴的正方向射线上。那么，这正方向射线被标上整数点符号后就成了原始的射线。请参见如下第一图。

第一图

除此之外，我们另使用符号 •S 去表示在简洁陈述中的至少两个整数点。在下文中，数轴的正方向射线简称为射线。

我们把最小的素数 2 作为第一个（№1）素数，又把素数 P_χ 作为第 χ 个（№χ）素数，这里$\chi \geq 1$．于是，素数 2 也被写成 P_1．然后，我们把共有素因子P_χ的正整数看作是 №χ 种整数。

显然，随着χ的值变得越来越大，P_χ 表示的素数会变得越来越大。

因为 №χ 种整数是以P_χ乘以每一个正整数的无限多个积，因此，在这射线上，每连续P_χ 个整数点，就有一个№χ 种整数点。

当一个整数含有许多不同的素因子时，这个正整数就应该同时属于这多种的每一种整数。

在№χ种整数中，有独一无二的素数 P_χ，因此，我们就把除了P_χ 外的另外的整数作为№χ 种合数。

因为有无限多素数，因此，也就有无限多种合数。相应地在这射线上就有无限多种合数点。

现在，我们需要的是从一串连续合数中一对一地找到并取出一组不同素因子的规律，于是，我们打算在这射线上从逐步形成合数点开始。

为此目的，在整数点的素、合性质被确定之后，有必要事先定义两个整数点符号。

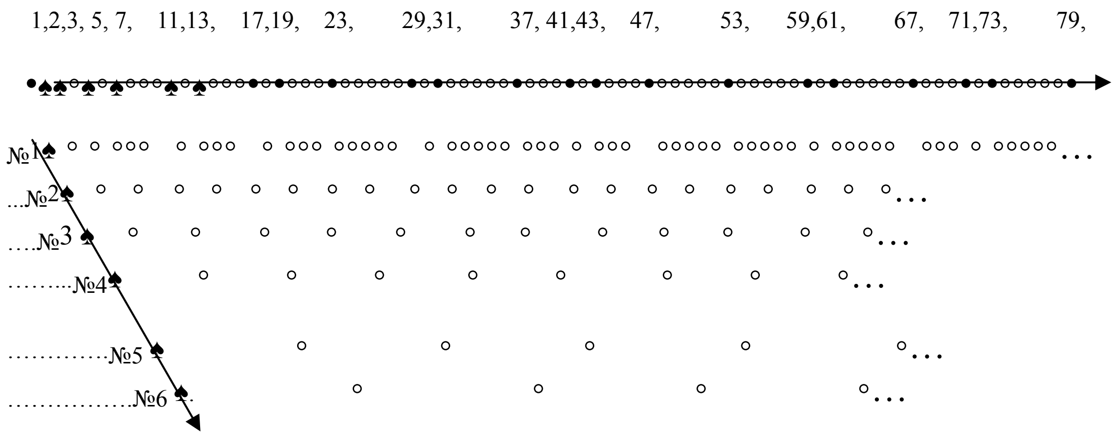
如果在这射线上一个整数点 • 被确定为一个合数点，那么就把它原先的符号 • 变成符号 ◦．加上，使用符号 ◦S 表示在简洁陈述中至少两个合数点。

如果在这射线上一个整数点 • 被确定为一个素数点，那么就把它原先的符号 • 变成符号♠．

接下来，我们将按照χ从小到大的次序，依次地改变 №χ 种合数点位

置上的 •s 为 ◦s，这里 $\chi\geq1$.

由此看来，在这射线上，χ 种整数点和他们之间的整数点的相互排列总是采取相同模式上的无限循环，但不考虑它们的素、合性质。我们按照χ从小到大的次序，逐一地分解在这射线上的 №χ 种整数点。请参见下面的一个示意图。



第二图

按照χ种整数点加上他们之间整数点的χ种整数点的相同排列，我们把在这射线上，彼此相等的最短的线段作为这χ种整数点排列的循环线段。而且，使用字符串 “$RLS_{№1\sim№\chi}$”表示一条$\sum$№χ 种整数点的循环线段，这儿$\chi\geq1$. 此外，至少两条这样的循环线段表为 $RLSS_{№1\sim№\chi}$. №1 $RLS_{№1\sim№\chi}$从 1 开始. 另外，每条$RLS_{№1\sim№\chi}$上有$\prod P_\chi$ 个整数点，这里$\chi\geq1$，并且 $\prod P_\chi=P_1P_2\ldots P_\chi$. 因此，一条$RLS_{№1\sim№(\chi+1)}$ 由连续的$P_{\chi+1}$条$RLSS_{№1\sim№\chi}$ 组成，而且它们一一连接。

显然，在每一条$RLSS_{№1\sim№\chi}$上，χ 种整数点加上它们之间的整数点之间的彼此排列，仅从χ 种整数点看来是完全相同的。

在按顺序排列的$RLSS_{№1\sim№\chi}$的每一条循环段上，用连续的自然数≥1从左到右的给每个整数点依次标上序号。

那样，在属于一条$RLS_{№1\sim№\chi+1}$的连续$P_{\chi+1}$条$RLSS_{№1\sim№\chi}$上，共有同一序号的$P_{\chi+1}$ 个整数点中就有一个№ $(\chi+1)$种整数点。而且，在№1 $RLS_{№1\sim№\chi+1}$右边的连续$P_{\chi+1}$ 条$RLSS_{№1\sim№\chi}$上，共有同一序号的$P_{\chi+1}$ 个整数点中就有一个№ $(\chi+1)$种合数点。

素数点$P_1, P_2 \ldots P_{\chi-1}$和 P_χ在 №1 $RLS_{№1\sim№\chi}$ 上。而，在№1 $RLS_{№1\sim№\chi}$右边，连续$RLSS_{№1\sim№\chi}$的每一条循环段的$P_1, P_2, P_3 \ldots$ 和 P_χ的序号位置上有χ 个合数点。因此，在与其它的每一条相比较之下，№1 $RLS_{№1\sim№\chi}$是一条特殊的$RLS_{№1\sim№\chi}$.

例如，在№1 $RLS_{№1\sim№4}$上多种整数点的排列是如下所示。

•♠♠○♠○♠○○○♠○♠○○○♠○♠○○○♠○○○○○♠○♠○○○○○♠○○○♠○♠○○○♠○○○○○♠○○○○○♠○♠○○○○○♠○○○♠

2,3, 5, 7, 11,13, 17,19, 23, 29,31, 37, 41,43, 47, 53, 59,61, 67, 71,

○♠○○○○○♠○○○♠○○○○○♠○○○○○○○♠○○○♠○♠○○○♠○♠○○♠○○○○○○○○○○○○○♠○○○♠○○○♠○♠○○○○○○○○○

73, 79, 83, 89, 97, 101,103,107,109,113, 127, 131, 137,139,

♠○♠○○○○○♠○○○○○♠○○○♠○○○○○♠○○○○○♠○♠○○○○○○○○○♠○○♠○○○♠○♠○○○○○○○○○○○→

149,151,157, 163, 167, 173, 179,181, 191,193, 197,199, 210

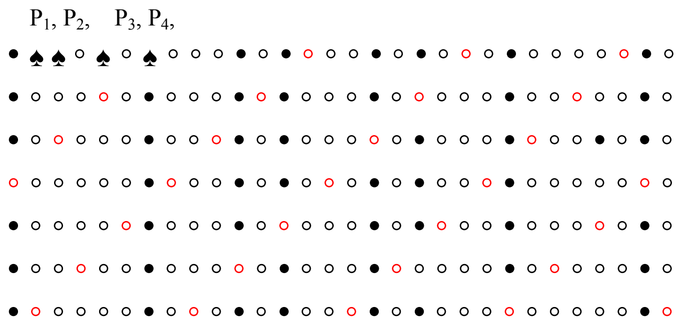
第三图

№1 $RLS_{№1}$ 结束在整数点 2; №1 $RLS_{№1\sim№2}$结束在整数点 6; №1 $RLS_{№1\sim№3}$结束在整数点 30; №1 $RLS_{№1\sim№4}$结束在整数点 210.

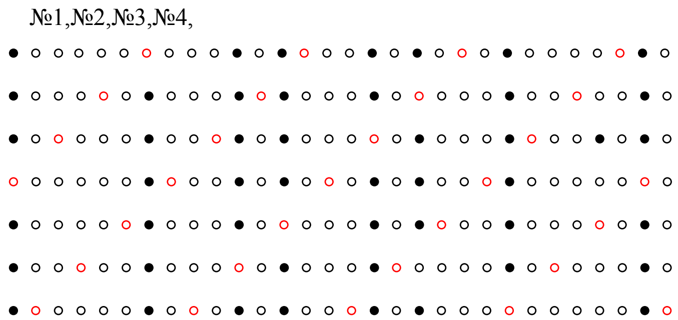
但是，对于这样的一种顺序排列，我们是难以去区别№1 $RLS_{№1\sim№\chi}$ 与排列在它右边的$RLSS_{№1\sim№\chi}$ 的每一条循环段间的差异，同样地也难以找出两条$RLSS_{№1\sim№\chi}$ 间的相同之处。

所以，属于一条$RLS_{№1\sim№(\chi+1)}$的$P_{\chi+1}$ 条$RLSS_{№1\sim№\chi}$ 可以被折叠在一个图中。

例如，№1 $RLS_{№1\sim№4}$ 的P_4 条$RLSS_{№1\sim№3}$ 和№n $RLS_{№1\sim№4}$ 的P_4 条$RLSS_{№1\sim№3}$ 被列在下面，这里$n \geq 2$.



在 №1 $RLS_{№1\sim№4}$ 上状况



在№n $RLS_{№1\sim№4}$上状况，这里 $n\geq2$

第四图

3. 证明格林姆猜想

依照在前一章的基本概念和有关论据的基础上，现在，我们将在原有射线上应用逐步形成合数点的方法，着手证明格林姆猜想，如下所述。在改变 №1 种合数点位置上的 •s 为◦s 后，在№1 $RLS_{№1}$ 右边，每一条$RLSS_{№1}$上，有一个 ◦ 点. 也就是说，在相邻两个•s 间有一个 ◦ 点. 因此，从№1种合数取出素因子作为不同素数的做法是如下:

形如 P_1^{α} 的合数被取出素因子P_1 作为一个不同的素数，这里 $\alpha\geq2$;

形如 $P_1^{\alpha}P_2^{\beta}$的合数被取出素因子 P_2作为一个不同的素数，这里 α 和 $\beta\geq1$;

形如$P_1^{\alpha}P_2^{\beta}P_3^{\eta}$的合数被取出素因子$P_3$作为一个不同的素数，这里$\alpha$, β 和 $\eta\geq1$. 其余的，可依此类推。

总的说来，对于每一个№1种的合数，比如C_1，它被取出 DPF (C_1) 中最大的一个素因子作为一个不同的素数。

因为在任意$RLS_{№1\sim№2}$的P_2条$RLSS_{№1}$上共有一个序号的P_2 个整数点中有一个№2 种合数点，所以，在相继地改变№2种合数点位置上的•s为◦s 后，在任意个№1种合数点的相邻的• 上或者它本身，就被重合一个№2 种合数点。因此，在相邻两个•s 之间，有一个 ◦ 或两个◦s. 那么，每一个新增加的№2 种合数点就包含了至少两个相同或不同的素因子$\geq P_2$. 然而，每一个含有素因子P_1 的 №2 种合数点，又仅仅是一个重复№1种合数点的合数点，于是，我们可完全忽略它。

随即，从№2种新增加的合数取出素因子作为不同素数的做法是如下：

形如P_2^{β}的合数被取出素因子P_2作为一个不同的素数，这里 $\beta\geq 2$;

形如$P_2^{\beta}P_3^{\eta}$的合数被取出素因子P_3作为一个不同的素数,这里β和$\eta\geq 1$;

形如 $P_2^{\beta}P_3^{\eta}P_4^{\mu}$的合数被取出素因子$P_4$ 作为一个不同的素数,这里β, η 和 $\mu\geq 1$. 其余的，可依此类推。

总的说来，每一个№2 种新增加的合数，比如C_2，它从DPF (C_2)中取出最大的一个素因子作为一个不同的素数。

对于从依次新增加的每一种合数中取出素因子作为不同素数的做法，全都可以用如上所作、依次类推去进行。

综上所述，在相继地改变№χ 种合数点位置上的•s为◦s后，形如P_{χ}^{α}的合数被取出素因子P_{χ} 作为一个不同的素数,这里$\alpha\geq 1$;

对于另外的每一个新增加的№χ 种合数，比如C_{χ}，就从DPF (C_{χ})中取出最大的一个素因子作为一个不同的素数。

对于从一串连续合数的任意一个取出一个素因子作为不同素数的上述做法，我们能够简单地概括成一句话，即 对于任意合数C_χ，总是从DPF (C_χ)中取出最大的一个素因子作为一个不同的素数，P_χ^α 亦不例外 。毫无疑问，如果一个合数属于至少两种，那么，不管从它取出的哪一种的一个素因子来看，都一律是与该种取出相同的一个。

前面已经提到过，连续合数的长度比从n^2 到 $(n+1)^2$ 的距离要短，那么，当 $n=P_\chi$ 时，就有 $n^2=P_\chi^2$ 和 $(n+1)^2=(P_\chi+1)^2<P_\chi^3$. 因此，连续合数的长度比从$P_\chi^2$ 到 P_χ^3 的距离短。

无疑地，连续合数的长度比从P_χ^α 到 $P_\chi^{\alpha+1}$的距离要短，因为 $P_\chi^{\alpha+1}- P_\chi^\alpha =P_\chi^\alpha(P_\chi-1)\geq P_\chi^3- P_\chi^2 =P_\chi^2(P_\chi-1)$，这里 $\alpha\geq 2$.

既然连续合数的长度是短于从P_χ^α 到 $P_\chi^{\alpha+1}$的距离，那么，对于属于同一种的合数而言， 无论χ 等于哪一个正整数，合数P_χ^α 和 $P_\chi^{\alpha+1}$ 都不能共存在相同的一串连续合数中，尽管它们两个被取出同一个素因子P_χ 作为各自不同的素数。

既然如此，在相继地改变 №χ 种合数点位置上的•s 为◦s 后，每一个既含有素因子P_χ 又含有至少一个小于P_χ的素因子的合数就是一个重复的合数，那么，每一个新增加的№χ种合数的全部素因子都不小于P_χ，这里$\chi\geq 2$.

加之，我们已经证明了：合数 P_χ^α 和 $P_\chi^{\alpha+1}$ 不能共存在同一串连续合数中，而且，对于任意合数C_χ，它都总是从DPF (C_χ) 中取出最大的一个素因子作为不同的素数。

由此可见，如果$P_\chi^\alpha *\ldots*P_{\chi+\varphi}^\eta < P_\chi^{\alpha+1}$，那么，不论 φ 等于哪一个自然

数，从含有合数$P_{\chi}^{\alpha}*\ldots*P_{\chi+\varphi}^{\eta}$的这串连续合数中取出的素因子$P_{\chi+\varphi}$，它都是独一无二的，这儿 $\eta\geq1$ 和 $\varphi\geq0$.

换一种说法，依照前面的做法，从连续的合数中一对一地取出素因子，那么，任意一个素因子与另外的每一个素因子都是不相同的。

当改变№χ 种合数点位置上的•s 为◦s 时，在素数点P_{χ} 左边的每一串连续合数点都是被固定下来了的。就是说，这样的每一串连续合数点都不能伸长到去增加一个合数点了。

对于前面说的从连续合数中取出素因子作为不同素数的做法，让我们举出一个例子来加以说明。例如，在素数1327 和素数 1361 之间的全部整数都是合数。我们就从这一串连续合数中，一对一地找出不同的素因子作为不同的素数，如下所示。

在1327 和 1361之间的№1种合数全部是偶数，对它们依次分解素因子是: $1328=2^4\times83$, $1330=2\times5\times7\times19$, $1332=2^2\times3^2\times37$, $1334=2\times23\times29$, $1336=2^3\times167$, $1338=2\times3\times223$, $1340=2^2\times5\times67$, $1342=2\times11\times61$, $1344=2^6\times3\times7$, $1346=2\times673$, $1348=2^2\times337$, $1350=2\times3^3\times5^2$, $1352=2^3\times13^2$, $1354=2\times677$, $1356=2^2\times3\times113$, $1358=2\times7\times97$ 和 $1360=2^4\times5\times17$. 按照这些偶数从小到大的次序，从№1种合数中分别取出的不同素因子是: 83, 19, 37, 29, 167, 223, 67, 61, 7, 673, 337, 5, 13, 677, 113, 97 和 17.

在1327 和 1361之间新增加的№2 种合数，以及对它们分解素因子是: $1329=3\times443$, $1335=3\times5\times89$, $1341=3^2\times149$, $1347=3\times449$, $1353=3\times11\times41$ and $1359=3^2\times151$.

按照新增加的№2 种合数从小到大的次序，从新增加的№2 种合数中

取出的不同素因子是: 443, 89, 149, 449, 41 和 151.

从新增加的№3, №4, №5, №6, №7, №8, №9 和 №11种合数中，按照种的次序分解素因子是: 1345=5×269, 1355=5×271, 1337=7×191, 1351=7×193, 1331=11^3, 1339=13×103, 1343=17×79, 1349=19×71, 1357= 23×59 和 1333=31×43.

按照上述新增加八种合数分解素因子的次序，从它们中分别取出的不同素因子是: 269, 271, 191, 193, 11, 103, 79, 71, 59 和 31.

为了比较方便的观察这样一串不同的素因子，于是按照从小到大的次序重新排列它们，是: 5, 7, 11, 13, 17, 19, 29, 31, 37, 41, 59, 61, 67, 71, 79, 83, 89, 97, 103, 113, 149, 151, 167, 191, 193, 223, 269, 271, 337, 443, 449, 673 和 677. 在这串素数中，显然每两个都是不相同的.

由此看来，使用这样的一种做法是完全能够从连续合数中取出彼此不同的素因子作为不同素数的.

也就是说，连续合数是能够分别被彼此不同的素数整除。

这个证明就到此结束了，作为一个结论：格林姆猜想是站得住脚的。

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附件一，原文：

Proving Grimm's Conjecture by Step-by-Step Forming Consecutive Composite Numbers' Points at the Number Axis

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Abstract

Let us regard positive integers which have a common prime factor as a kind, then the positive half line of the number axis consists of infinite many recurring line segments which have same permutations of χ kinds of integers' points, where χ≥1. In this article we shall prove Grimm's conjecture by the method which changes stepwise symbols of each kind

of composite numbers' points at the original number axis, so as to form consecutive composite numbers' points inside the limited field of proven Legendre- Zhang conjecture as the true.

AMS subject classification: 11A41, 11P82, 11B99

Keywords: consecutive composite numbers' points, number axis, distinct prime factors, Legendre-Zhang conjecture.

1. Introduction:

Grimm's Conjecture named after Carl Albert Grimm. The conjecture states that if n+1, n+2 … n+k are all composite numbers, then there are distinct primes $p^i{}_j$ such that $p^i{}_j \mid (n+j)$ for $1 \le j \le k$. For example, for consecutive composite numbers between primes 523 and 541, take out one another's distinct prime factors from them as follows.

Composite numbers:	**524,**	**525,**	**526,**	**527,**	**528,**	**529,**	**530,**
Decompose into prime factors:	$2^2\times131$,	$3\times5^2\times7$,	2×263,	17×31,	$2^4\times3\times11$,	23^2,	$2\times5\times53$,
Distinct prime factors:	**131,**	**3,**	**263,**	**17,**	**11,**	**23,**	**53,**

531,	**532,**	**533,**	**534,**	**535,**	**536,**	**537,**	**538,**	**539,**	**540**
$3^2\times59$,	$2^2\times7\times19$,	13×41,	$2\times3\times89$,	5×107,	$2^3\times67$,	3×179,	2×269,	$7^2\times11$,	$2^2\times3^3\times5$
59,	**19,**	**41,**	**89,**	**107,**	**67,**	**179,**	**269,**	**7,**	**2**

This conjecture was first published in American Mathematical Monthly, 76 (1969) 1126-1128. Yet, it is still both unproved and un-negated a conjecture hitherto.

2. Preparations before the Proof

First, we need to quote proven Legendre-Zhang's conjecture as the true, so as to use it to restrict the length of consecutive composite numbers.

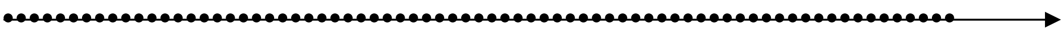
Legendre-Zhang's conjecture states that there is at least an odd prime between n^2 and n(n+1), and there is at least an odd prime between n(n+1) and $(n+1)^2$, where n≥2. A proof for Legendre-Zhang's conjecture comes from Zhang Tianshu, *"Legendre-Zhang's conjecture & Gilbreath's conjecture and proofs thereof"*, General Mathematics, ISSN 1221-5023, Volume 20, Issue 2-3, 2012, pp. 86-100. Or see also preprint Vixra.org: *http://vixra.org/abs/1401.0127.*

Thus it can be seen, there are at least two odd primes between n^2 and $(n+1)^2$ where n ≥2, undoubtedly the length of consecutive composite numbers is shorter than the distance from n^2 to $(n+1)^2$.

Moreover, we also need to define a terminology that let DPF (C_χ) express one and only set of all distinct prime factors of any composite number C_χ.

Since each and every integer's point at the positive half line of the number axis expresses a positive integer, and that infinite many a distance between two adjacent integers' points equals one another. So first let us use the symbol • to denote an integer's point, whether the symbol • is in a formulation or is at the positive half line of the number axis.

Then, the positive half line is marked with symbols of integers' points into the original half line. Please, see also first illustration as follows.



First Illustration

In addition to this, let us use symbol •s to denote at least two integers' points in formulations. Thereinafter, the positive half line of the number axis is called the half line for short.

We regard smallest prime 2 as №1 prime, and regard prime P_χ as №χ prime, where $\chi \geq 1$. Then, prime 2 is written as P_1 too. And then, we regard positive integers which share prime factor P_χ as №χ kind of integers.

Obviously, primes which P_χ expresses are getting greater and greater along with which values of χ are getting greater and greater.

Since №χ kind's integers are infinitely many products which multiply each and every positive integer by P_χ, thus there is one №χ kind's integer's point within consecutive P_χ integers' points at the half line.

When a positive integer contains many distinct prime factors, the positive integer must belong to each of the many kinds of integers concurrently.

There is unique prime P_χ within №χ kind's integers, so we regard others except for P_χ as №χ kind of composite numbers.

Since there are infinitely many primes, thus there are infinitely many kinds of composite numbers as well. Correspondingly there are infinitely many kinds of composite numbers' points at the half line.

By now, what we need is to find a law that take out a set of distinct prime factors from a string of consecutive composite numbers, one for one, so we are planning to start from step-by-step forming consecutive composite numbers' points at the half line.

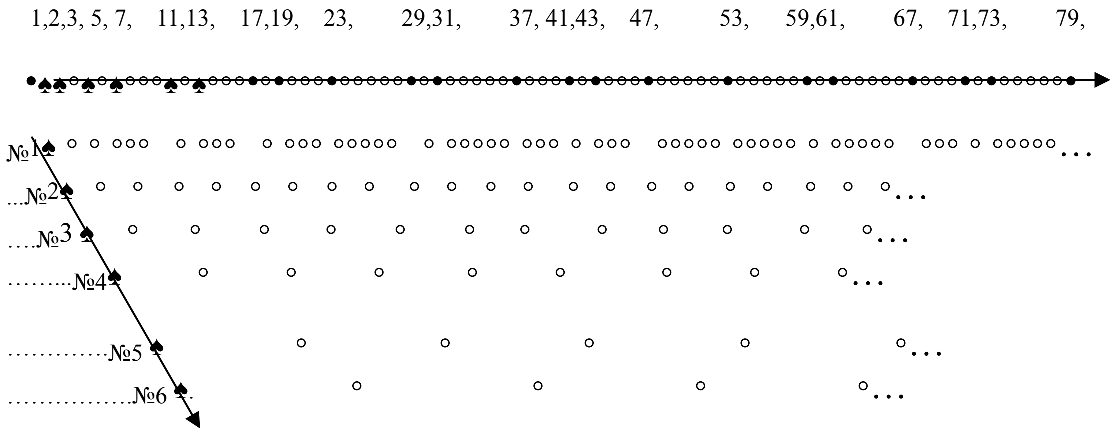
For this purpose, be necessary to define two symbols of integers' points beforehand after their prime/composite attribute is decided.

If an integer's point • at the half line is defined as a composite number's point, then change symbol ◦ for its original symbol •. Withal, use symbol ◦s to denote at least two composite numbers' points in formulations.

If an integer's point • at the half line is defined as a prime point, then change symbol ♠ for its original symbol •.

Next, we shall seriatim change ◦s for •s at places of №χ kind's composite numbers' points according as χ be from small to large, where χ≥1.

By this token, one another's permutations of χ kinds' integers' points plus integers' points amongst them assume always infinite many recurrences on same pattern at the half line, irrespective of their prime/composite attribute. We seriatim decompose №χ kind of integers' points at the half line according as χ be from small to large. Please, see also below a schematic illustration.



Second Illustration

Let us regard one another's equivalent shortest line segments at the half line according to same permutations of χ kinds' integers' points plus integers' points amongst them as recurring line segments of permutations of the χ kinds of integers' points. And that use character string "$RLS_{№1\sim№\chi}$" to express a recurring line segment of $\sum№\chi$ kind of integers' points, where $\chi\geq1$. Besides, at least two such recurring line segments are expressed by $RLSS_{№1\sim№\chi}$.

№1 $RLS_{№1\sim№\chi}$ begins with 1. Also, there are $\prod P_\chi$ integers' points per $RLS_{№1\sim№\chi}$, where $\chi\geq1$, and $\prod P_\chi=P_1P_2\ldots P_\chi$. Thus one $RLS_{№1\sim№(\chi+1)}$ consists of consecutive $P_{\chi+1}$ $RLSS_{№1\sim№\chi}$, and that they link one by one.

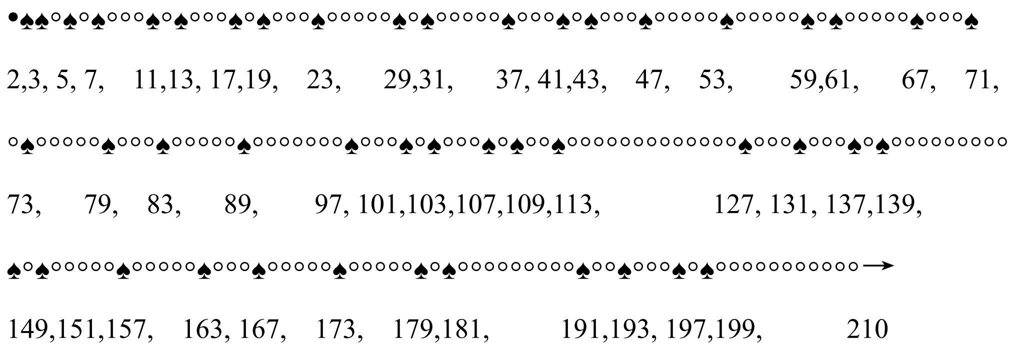
Evidently one another's permutations of χ kinds of integers' points plus integers' points amongst them at each of $RLSS_{№1\sim№\chi}$ are just the same.

Number the ordinals of integers' points at each of seriate $RLSS_{№1\sim№\chi}$ by consecutive natural numbers $\geq$ 1. Namely from left to right each and every integer's point at each of seriate $RLSS_{№1\sim№\chi}$ is marked with from small to great a natural number $\geq$1.

In that way, there is one № (χ+1) kind's integer's point within $P_{\chi+1}$ integers' points which share an ordinal at $P_{\chi+1}$ $RLSS_{№1\sim№\chi}$ of a $RLS_{№1\sim№\chi+1}$. Moreover, there is one № (χ+1) kind's composite number's point within $P_{\chi+y}$ integers' points which share an ordinal at $P_{\chi+1}$ $RLSS_{№1\sim№\chi}$ of each of seriate $RLSS_{№1\sim№\chi+1}$ on the right side of №1 $RLS_{№1\sim№\chi+1}$.

Prime points P_1, P_2 … $P_{\chi-1}$ and P_χ are at №1 $RLS_{№1\sim№\chi}$. Yet, there are χ composite numbers' points at places of ordinals of P_1, P_2, P_3... and P_χ at each of seriate RLSS$_{№1\sim№\chi}$ on the right side of №1 RLS$_{№1\sim№\chi}$. Thus №1 RLS$_{№1\sim№\chi}$ is a special RLS$_{№1\sim№\chi}$ as compared with each of others.

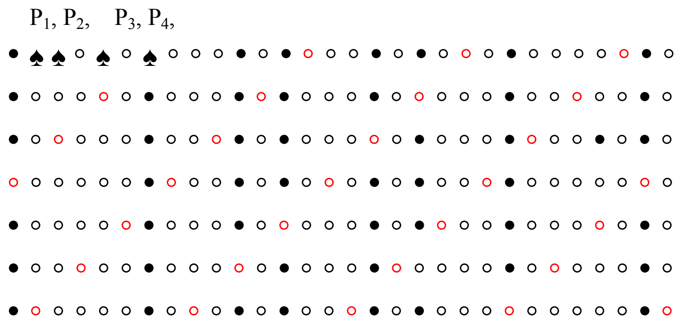
For example, the permutation of many kinds of integers' points at №1 RLS$_{№1\sim№4}$ is as follows.



Third Illustration

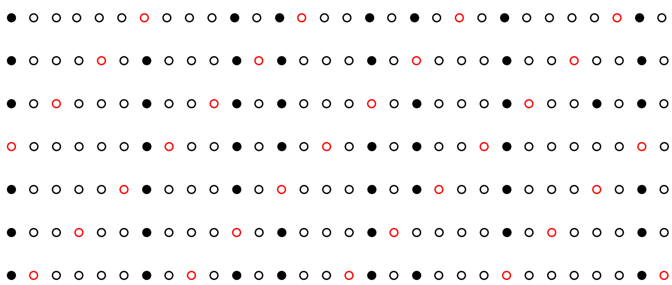
№1 RLS$_{№1}$ ends with integer's point 2; №1 RLS$_{№1\sim№2}$ ends with integer's point 6; №1 RLS$_{№1\sim№3}$ ends with integer's point 30; №1 RLS$_{№1\sim№4}$ ends with odd point 210.

But, for such a seriation, we are difficult to distinguish the differentia of №1 RLS$_{№1\sim№\chi}$ in comparison with each of RLSS$_{№1\sim№\chi}$ on the right side of №1 RLS$_{№1\sim№\chi}$, likewise difficult to find out the same of two RLSS$_{№1\sim№\chi}$. Therefore $P_{\chi+1}$ RLSS$_{№1\sim№\chi}$ of one RLS$_{№1\sim№(\chi+1)}$ may be folded at an illustration. For example, P_4 RLSS$_{№1\sim№3}$ of №1 RLS$_{№1\sim№4}$ and P_4 RLSS$_{№1\sim№3}$ of №n RLS$_{№1\sim№4}$ are listed as follows, where n≥2.



At №1 RLS$_{№1\sim№4}$

№1,№2,№3,№4,



At №n $RLS_{№1\sim№4}$, where n≥2

Fourth Illustration

3. Proving Grimm's Conjecture

Pursuant to basic conceptions and grounds of argument concerned within the preceding section, by now, we set about proving Grimm's conjecture by step -by- step forming consecutive composite numbers' points at the original half line, ut infra.

After change ◦s for •s at places of №1 kind of composite numbers' points, there is one ◦ at each of $RLSS_{№1}$ on the right side of №1 $RLS_{№1}$. That is to say, there is one ◦ between adjacent two •s.

Thus a way of doing which takes out prime factors from №1 kind of composite numbers as distinct primes is as follows.

Composite numbers like P_1^{α} is taken out prime factor P_1 as a distinct prime, where $\alpha \geq 2$;

Composite numbers like $P_1^{\alpha}P_2^{\beta}$ is taken out prime factor P_2 as a distinct prime, where α and $\beta \geq 1$;

Composite numbers like $P_1^{\alpha}P_2^{\beta}P_3^{\eta}$ is taken out prime factor P_3 as a distinct prime, where α, β and $\eta \geq 1$.

The rest may be deduced by analogy. Overall, for each and every №1 kind's composite number such as C_1, it is taken out greatest one within DPF (C_1) as a distinct prime.

Since there is one №2 kind's composite number's point within P_2 integers' points which share an ordinal at P_2 $RLSS_{№1}$ of any $RLS_{№1\sim№2}$. Therefore, after successively change ◦s for •s at places of №2 kind of composite numbers' points, it is coincided one №2 kind's composite number's point on adjacent • or itself of any of №1 kind's composite numbers' points. Thus there are one ◦ or two ◦s between adjacent two •s.

Well then, each and every new-increased №2 kind's composite number's point contains at least two identical or distinct prime factors $\geq P_2$. Yet each and every №2 kind's composite number's point which contains prime

factor P_1 is merely a composite number's point which repeats one №1 kind's composite number's point, hence we ignore it absolutely.

Thereupon, a way of doing which takes out prime factors from №2 kind's new-increased composite numbers as distinct primes is as follows.

Composite numbers like P_2^{β} is taken out prime factor P_2 as a distinct prime, where $\beta \geq 2$;

Composite numbers like $P_2^{\beta}P_3^{\eta}$ is taken out prime factor P_3 as a distinct prime, where β and $\eta \geq 1$;

Composite numbers like $P_2^{\beta}P_3^{\eta}P_4^{\mu}$ is taken out prime factor P_4 as a distinct prime, where β, η and $\mu \geq 1$.

The rest may be deduced by analogy. Overall, for each and every №2 kind's new-increased composite number such as C_2, it is taken out greatest one within DPF (C_2) as a distinct prime.

For a way of doing which takes out prime factors from successive new-increased each and every kind of composite numbers as a distinct prime, all may be deduced by analogy.

To sum up, after successively change ◦s for •s at places of №χ kind of composite numbers' points, composite numbers like P_χ^α is taken out prime factor P_χ as a distinct prime, where $\alpha \geq 1$; for other each and every new-increased №χ kind's composite number such as C_χ, it is taken out greatest one within DPF (C_χ) as a distinct prime.

For the aforesaid way of doing which takes out a prime factor from any of a string of consecutive composite number as distinct prime, we can generalize briefly a sentence that for any composite number C_χ, it is always taken out greatest one within DPF (C_χ) as a distinct prime, not excepting P_χ^α. Unquestionably, if a composite number belongs to at least two kinds, then a prime factor which taken out from it despite which kind is one and the same uniformly.

As previously mentioned that the length of consecutive composite numbers is shorter than the distance from n^2 to $(n+1)^2$, then when $n=P_\chi$, it has $n^2=P_\chi^2$ and $(n+1)^2=(P_\chi+1)^2<P_\chi^3$. Thus the length of consecutive composite numbers is shorter than the distance from P_χ^2 to P_χ^3.

Without doubt, the length of consecutive composite numbers is shorter than the distance from P_χ^α to $P_\chi^{\alpha+1}$, because $P_\chi^{\alpha+1}$- P_χ^α=P_χ^α(P_χ-1)≥P_χ^3- P_χ^2 =P_χ^2(P_χ-1), where α ≥2.

Now that the length of consecutive composite numbers is shorter than the distance from P_χ^α to $P_\chi^{\alpha+1}$, then for composite numbers which belong to identical a kind, no matter which positive integer which χ equals, composite numbers P_χ^α and $P_\chi^{\alpha+1}$ coexist not within identical a string of consecutive composite numbers though both are taken out an identical prime factor P_χ as respective distinct prime.

Such being the case, after successively change ◦s for •s at places of №χ kind of composite numbers' points where χ≥2, each and every composite number which contains both P_χ and at least a prime factor<P_χ is a repeating composite number, then all prime factors of each and every new-increased №χ kind's composite number are not less than P_χ.

Additionally, we have proven that composite numbers P_χ^α and $P_\chi^{\alpha+1}$ coexist not within an identical string of consecutive composite numbers, and that for any composite number C_χ, it is always taken out greatest one within DPF (C_χ) as a distinct prime.

Thus it can be seen, if P_χ^α *…*$P_{\chi+\varphi}^\eta$ < $P_\chi^{\alpha+1}$ where η≥1 and φ≥0, then no matter what natural number which φ equals, prime factor $P_{\chi+\varphi}$ which is taken out from the string of consecutive composite numbers which contain composite number P_χ^α*…*$P_{\chi+\varphi}^\eta$, it is unique.

In other words, pursuant to the aforesaid way of doing to take out prime factors from consecutive composite numbers, one for one, any of these prime factors and each of others are no alike.

When change ◦s for •s at places of №χ kind of composite numbers' points, every string of consecutive composite numbers' points on the left side of prime point P_χ is fixed, i.e. every such string of consecutive composite numbers' points is no longer to increase a composite number's point.

For the aforesaid way of doing which takes out prime factors from consecutive composite numbers to as distinct primes, let us illustrate with an example. For example, all integers between prime 1327 and prime 1361 are composite numbers. We find out distinct prime factors from this string of consecutive composite numbers, one for one, ut infra.

№1 kind of composite numbers between 1327 and 1361 is all even numbers, Decomposing prime factors for them orderly are: $1328=2^4\times83$, $1330=2\times5\times7\times19$, $1332=2^2\times3^2\times37$, $1334=2\times23\times29$, $1336=2^3\times167$, $1338=2\times3\times223$, $1340=2^2\times5\times67$, $1342=2\times11\times61$, $1344=2^6\times3\times7$, $1346=2\times673$, $1348=2^2\times337$, $1350=2\times3^3\times5^2$, $1352=2^3\times13^2$, $1354=2\times677$, $1356=2^2\times3\times113$, $1358=2\times7\times97$ and $1360=2^4\times5\times17$.

Distinct prime factors which №1 kind's composite numbers are taken out are: 83, 19, 37, 29, 167, 223, 67, 61, 7, 673, 337, 5, 13, 677, 113, 97 and 17 in the order of these even numbers from small to large.

New-increased №2 kind's composite numbers between 1327 and 1361 and decomposing prime factors for them are: 1329=3×443, 1335=3×5×89, 1341=3^2×149, 1347=3×449, 1353=3×11×41 and 1359=3^2×151.

Distinct prime factors which new-increased №2 kind's composite numbers are taken out are: 443, 89, 149, 449, 41 and 151 in the order of new-increased №2 kind's composite numbers from small to large.

New-increased №3, №4, №5, №6, №7, №8, №9 and №11 kinds' composite numbers between 1327 and 1361 and decomposing prime factors for each kind orderly are: 1345=5×269, 1355=5×271, 1337=7×191, 1351=7×193, 1331=11^3, 1339=13×103, 1343=17×79, 1349=19×71, 1357= 23×59 and 1333=31×43.

Distinct prime factors which the new-increased eight kinds' composite numbers are taken out are: 269, 271, 191, 193, 11, 103, 79, 71, 59 and 31 in the order of decomposing prime factors for these composite numbers.

In order to watch conveniently such a cluster of distinct prime factors, we renewedly arrange them according to the sequence from small to large: 5, 7, 11, 13, 17, 19, 29, 31, 37, 41, 59, 61, 67, 71, 79, 83, 89, 97, 103, 113, 149, 151, 167, 191, 193, 223, 269, 271, 337, 443, 449, 673 and 677. Evidently every two are not alike in the string of primes.

By this token, use such a way of doing be able to take out one another's-distinct prime factors from consecutive composite numbers.

That is to say, consecutive composite numbers are able to be divided exactly by one another's- distinct primes respectively.

The proof was thus brought to a close. As a consequence, Grimm’s conjecture is tenable.

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附件二，《数学和系统科学杂志》的录用通知

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Paper Acceptance Notice

July 25, 2017

Dear Zhang Tianshu,

We are pleased to inform you that your paper titled "**Proving Grimm's Conjecture by Step-by-Step Forming Consecutive Composite Numbers' Points at the Number Axis** (No. JMSS-E20170619-01)" submitted for consideration for **Journal of Mathematics and System Science**, has been processed utilizing a two person referee process and upon their recommendation your paper has been accepted for publication. Your paper will be arranged in nearest issues.

When your paper is published, one hard copy within your paper and receipt will be sent to you. Should you have any questions, feel free to contact us!

Best regards,

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