

Question 337 : Integrals and Fractals

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abstract

This note presents fifteen integrals and ten fractals .

1. Integrals

$$\int_0^{\infty} \frac{e^{-2x}}{1 - e^{-2x} + e^{-4x}} dx = \frac{\pi}{3\sqrt{3}} \quad (1)$$

$$\int_0^{\infty} \frac{e^{-2x}}{1 - \sqrt{2}e^{-2x} + e^{-4x}} dx = \frac{3\pi}{8\sqrt{2}} \quad (2)$$

$$\int_0^{\infty} \frac{e^{-2x}}{1 - \sqrt{3}e^{-2x} + e^{-4x}} dx = \frac{5\pi}{12} \quad (3)$$

$$\int_0^1 \ln \sqrt{\frac{1+x + \sqrt{1+2x-3x^2}}{2x}} dx = \frac{\pi}{3\sqrt{3}} \quad (4)$$

$$\int_0^{(2-\sqrt{2})^{-1}} \ln \sqrt{\frac{1+\sqrt{2}x + \sqrt{1+2\sqrt{2}x-2x^2}}{2x}} dx = \frac{3\pi}{8\sqrt{2}} \quad (5)$$

$$\int_0^{(2-\sqrt{3})^{-1}} \ln \sqrt{\frac{1+\sqrt{3}x + \sqrt{1+2\sqrt{3}x-x^2}}{2x}} dx = \frac{5\pi}{12} \quad (6)$$

$$\int_1^{\infty} \frac{1}{x^2} \ln \sqrt{\frac{1+x+\sqrt{x^2+2x-3}}{2}} dx = \frac{\pi}{3\sqrt{3}} \quad (7)$$

$$\int_{2-\sqrt{2}}^{\infty} \frac{1}{x^2} \ln \sqrt{\frac{\sqrt{2}+x+\sqrt{x^2+2\sqrt{2}x-2}}{2}} dx = \frac{3\pi}{8\sqrt{2}} \quad (8)$$

$$\int_{2-\sqrt{3}}^{\infty} \frac{1}{x^2} \ln \sqrt{\frac{\sqrt{3}+x+\sqrt{x^2+2\sqrt{3}x-1}}{2}} dx = \frac{5\pi}{12} \quad (9)$$

$$\int_0^1 \cosh^{-1} \left(\frac{1}{2} + \frac{1}{2x} \right) dx = \frac{2\pi}{3\sqrt{3}} \quad (10)$$

$$\int_0^{(2-\sqrt{2})^{-1}} \cosh^{-1} \left(\frac{1}{\sqrt{2}} + \frac{1}{2x} \right) dx = \frac{3\pi}{4\sqrt{2}} \quad (11)$$

$$\int_0^{(2-\sqrt{3})^{-1}} \cosh^{-1} \left(\frac{\sqrt{3}}{2} + \frac{1}{2x} \right) dx = \frac{5\pi}{6} \quad (12)$$

$$\int_0^1 \sinh^{-1} \sqrt{\left(\frac{1}{2x} - \frac{1}{2} \right) \left(\frac{1}{2x} + \frac{3}{2} \right)} dx = \frac{2\pi}{3\sqrt{3}} \quad (13)$$

$$\int_0^{(2-\sqrt{2})^{-1}} \sinh^{-1} \sqrt{\frac{1}{4x^2} + \frac{1}{\sqrt{2}x} - \frac{1}{2}} dx = \frac{3\pi}{4\sqrt{2}} \quad (14)$$

$$\int_0^{(2-\sqrt{3})^{-1}} \sinh^{-1} \sqrt{\frac{1}{4x^2} + \frac{\sqrt{3}}{2x} - \frac{1}{4}} dx = \frac{5\pi}{6} \quad (15)$$

2. Fractals

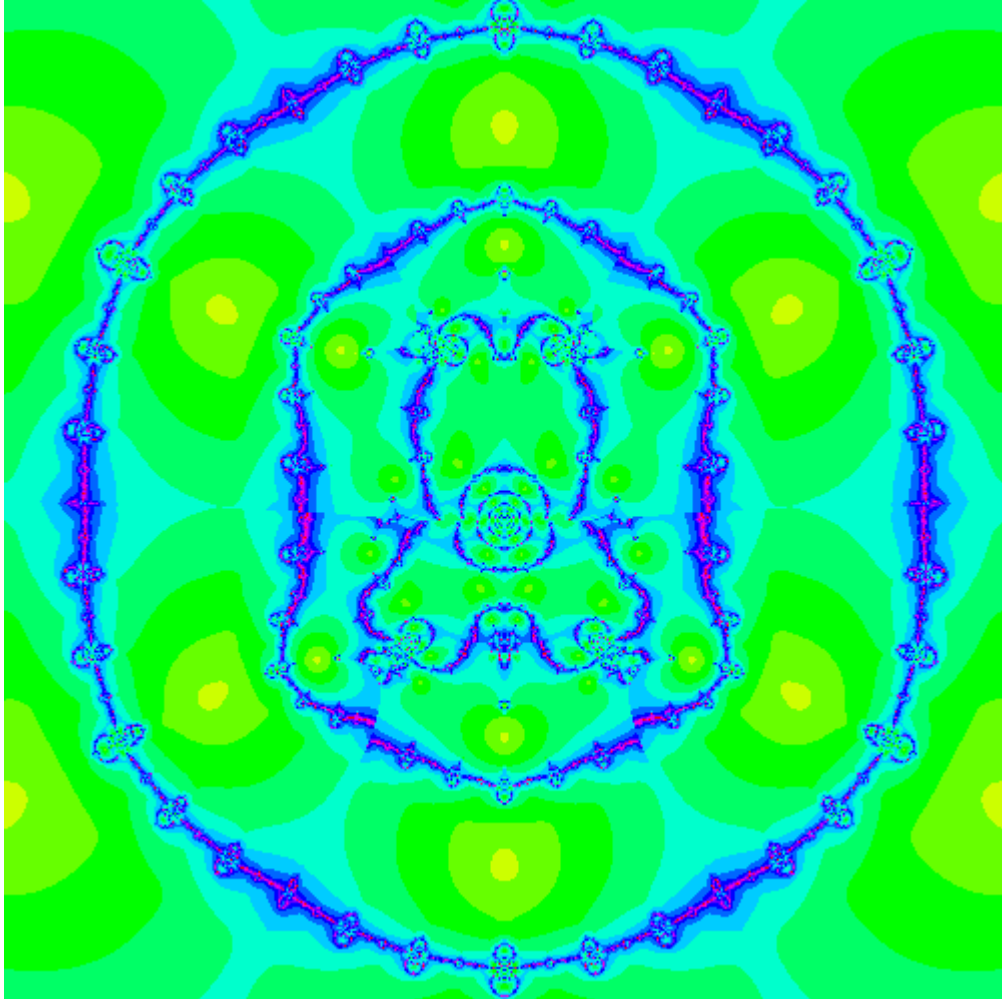


Fig. 1.

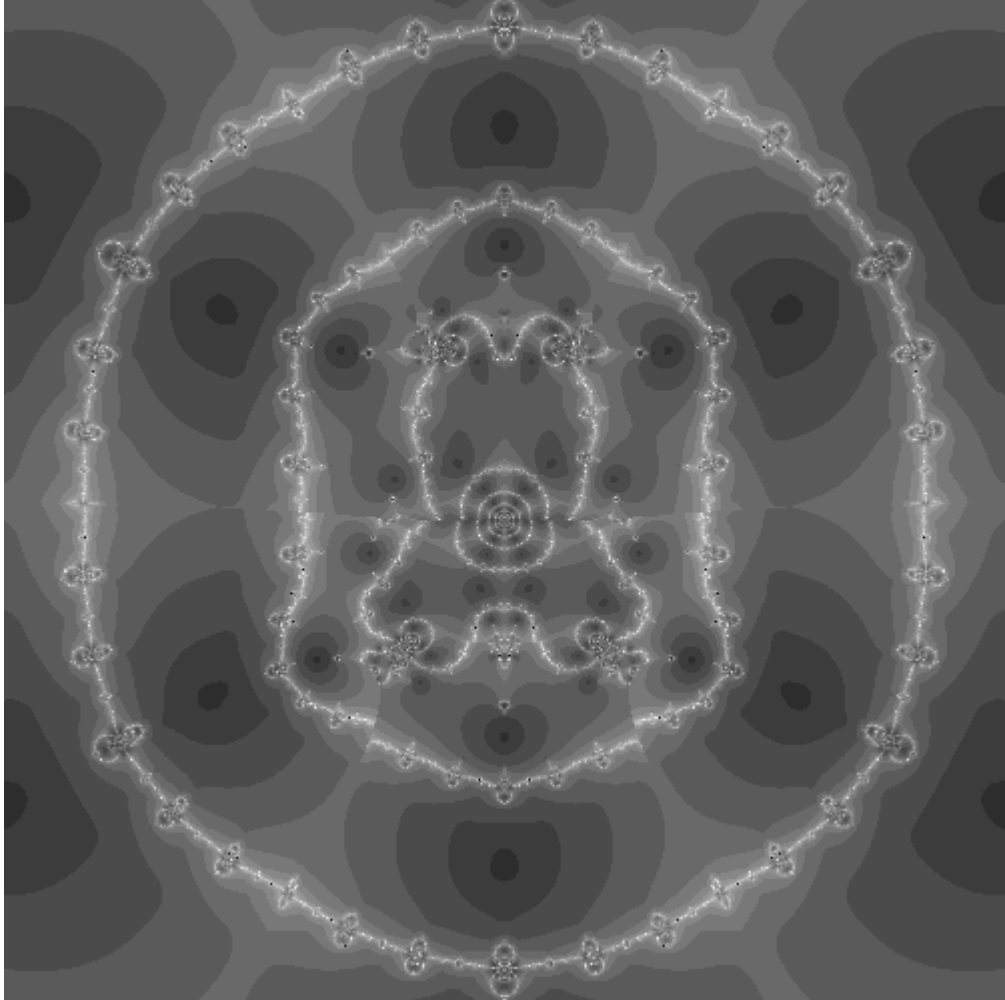


Fig. 2.

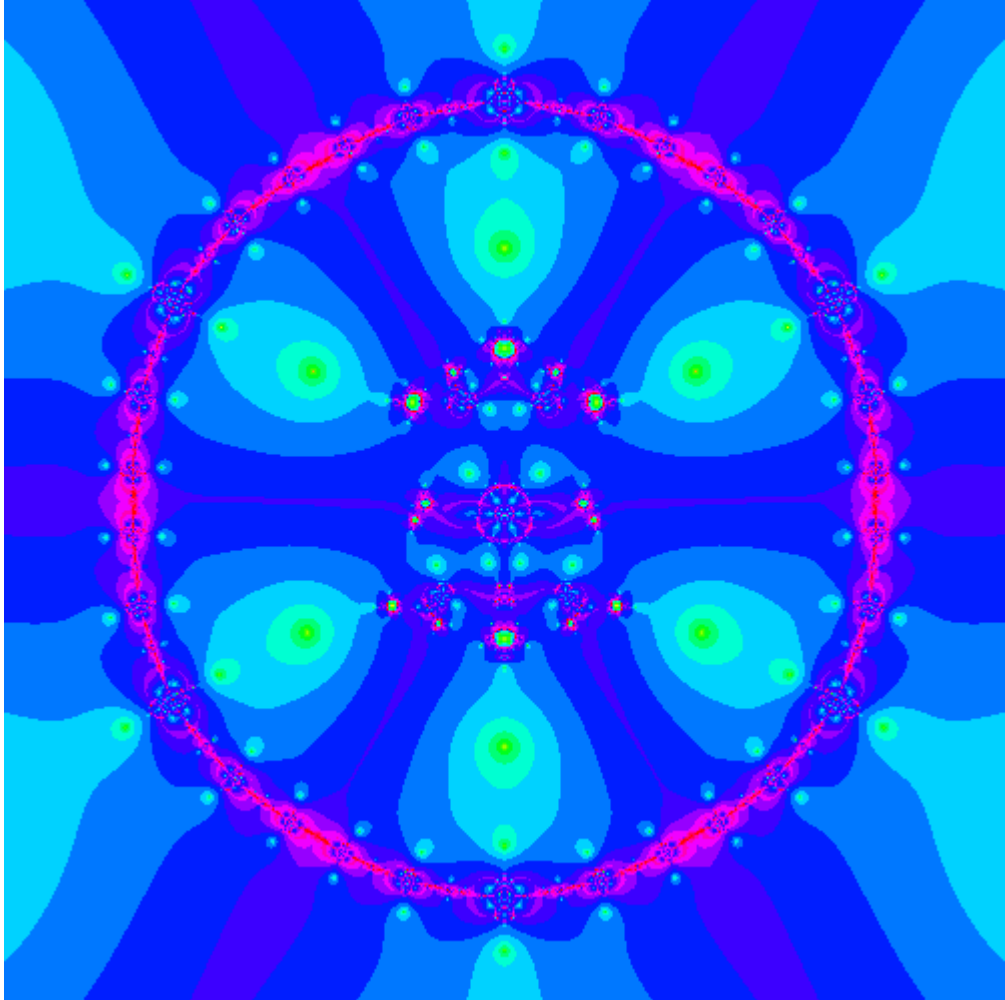


Fig. 3.

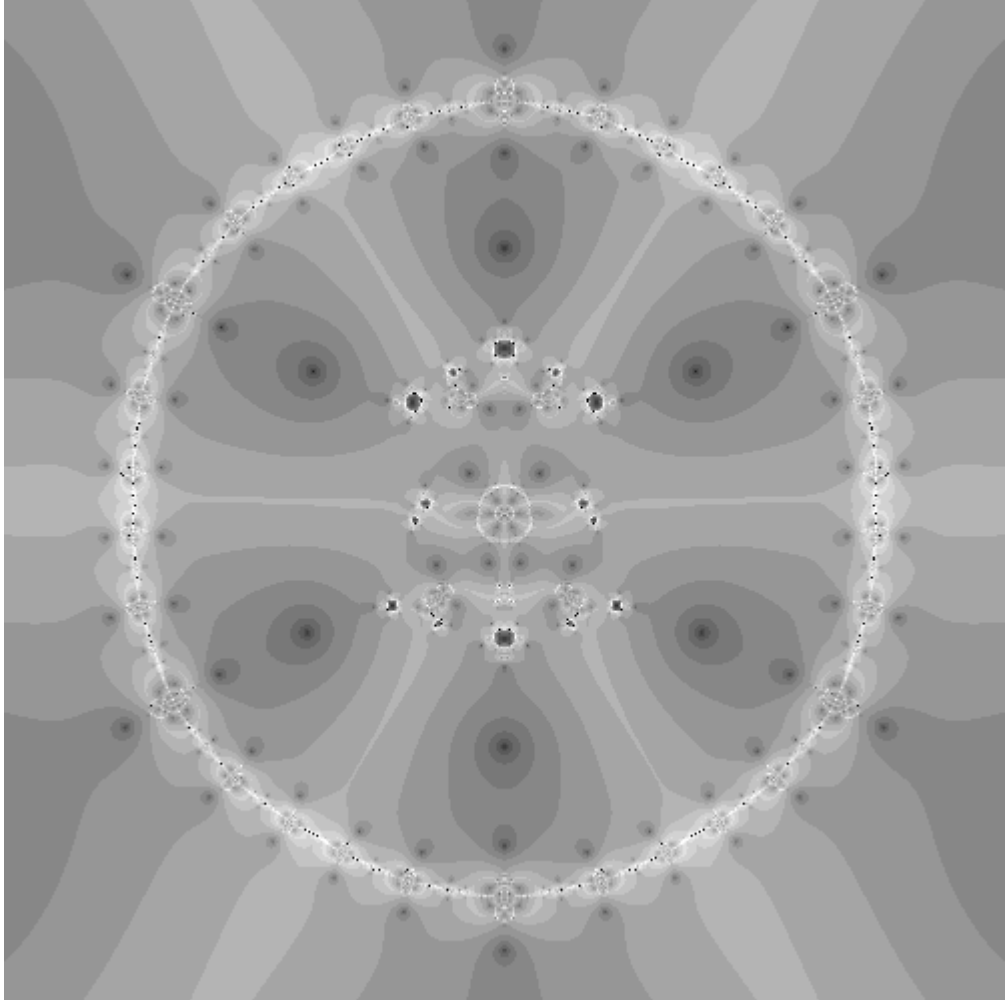


Fig. 4.

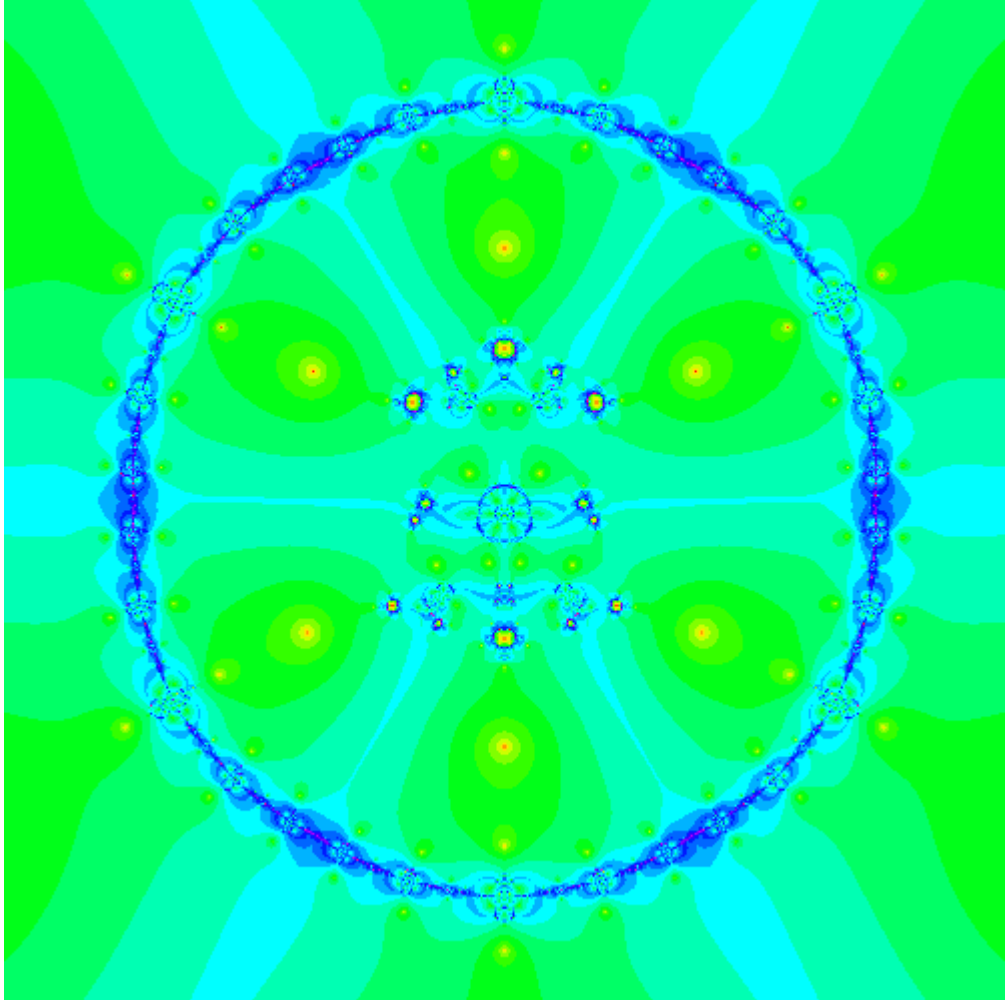


Fig. 5.

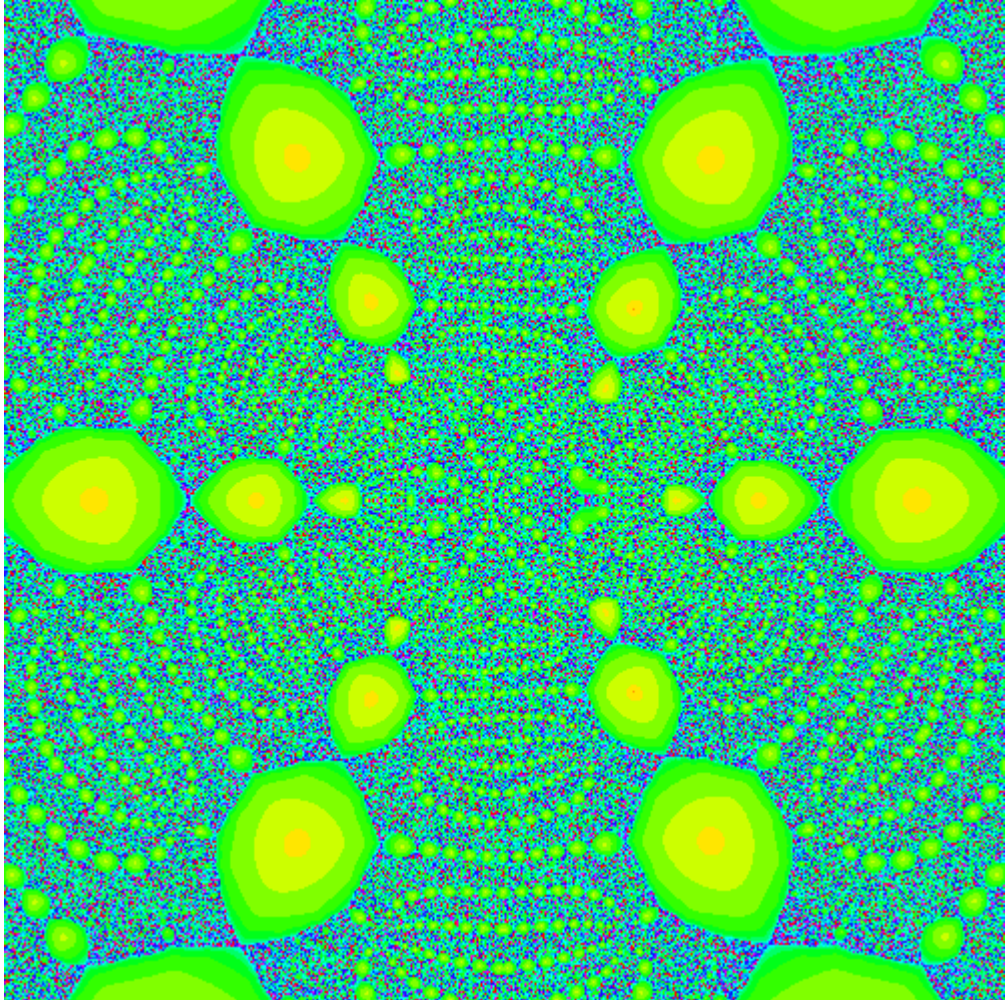


Fig. 6.

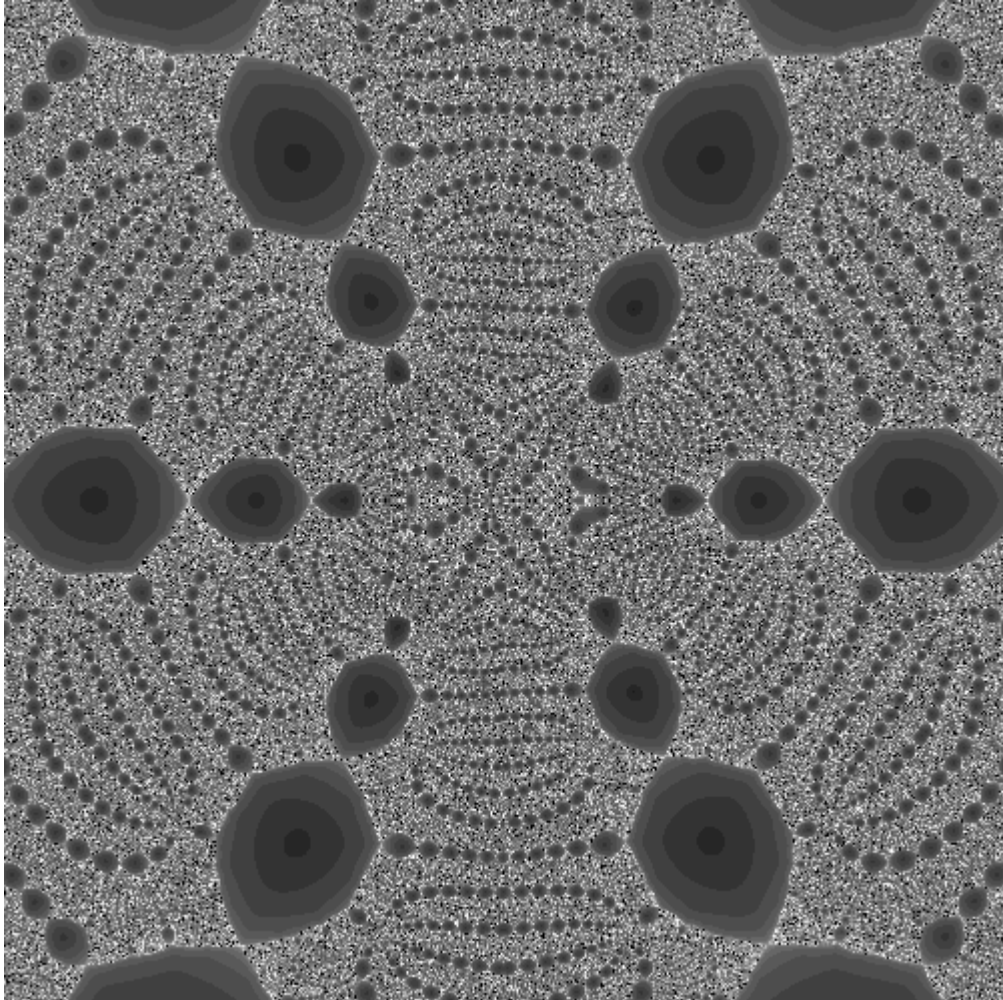


Fig. 7.

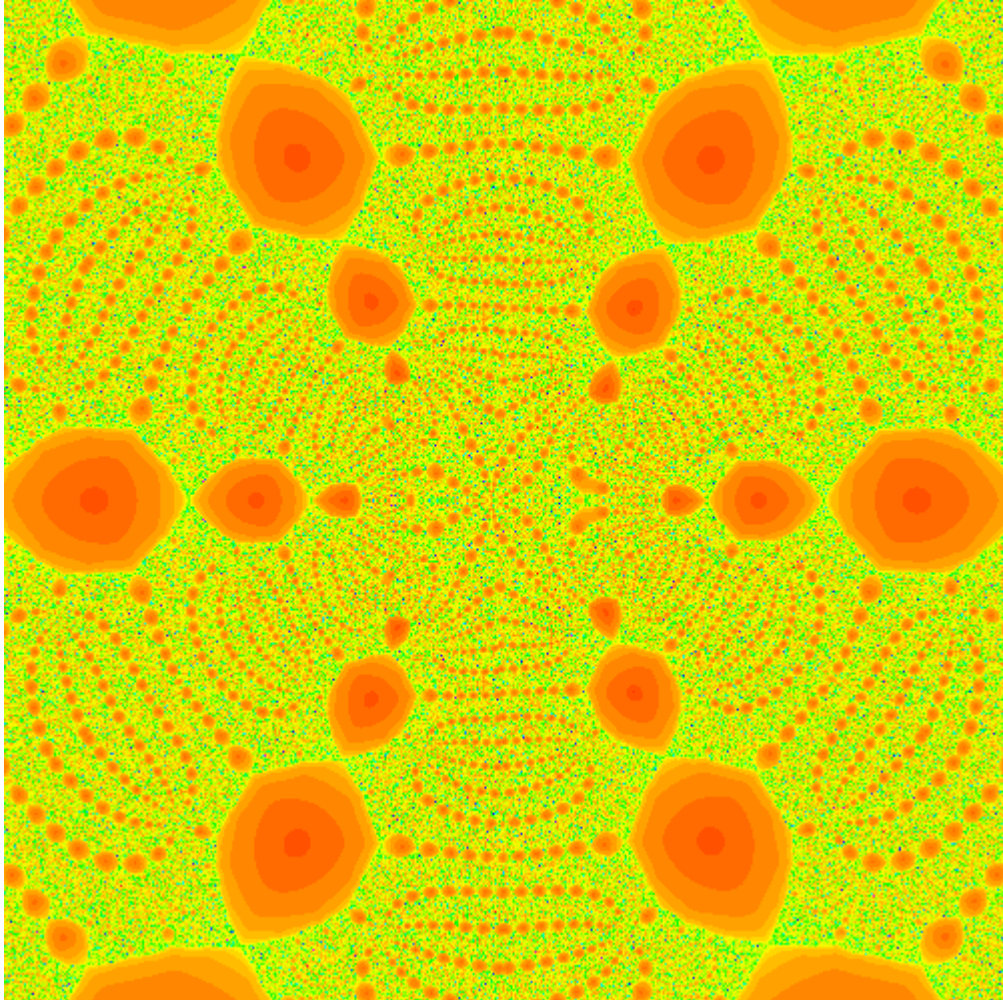


Fig. 8.

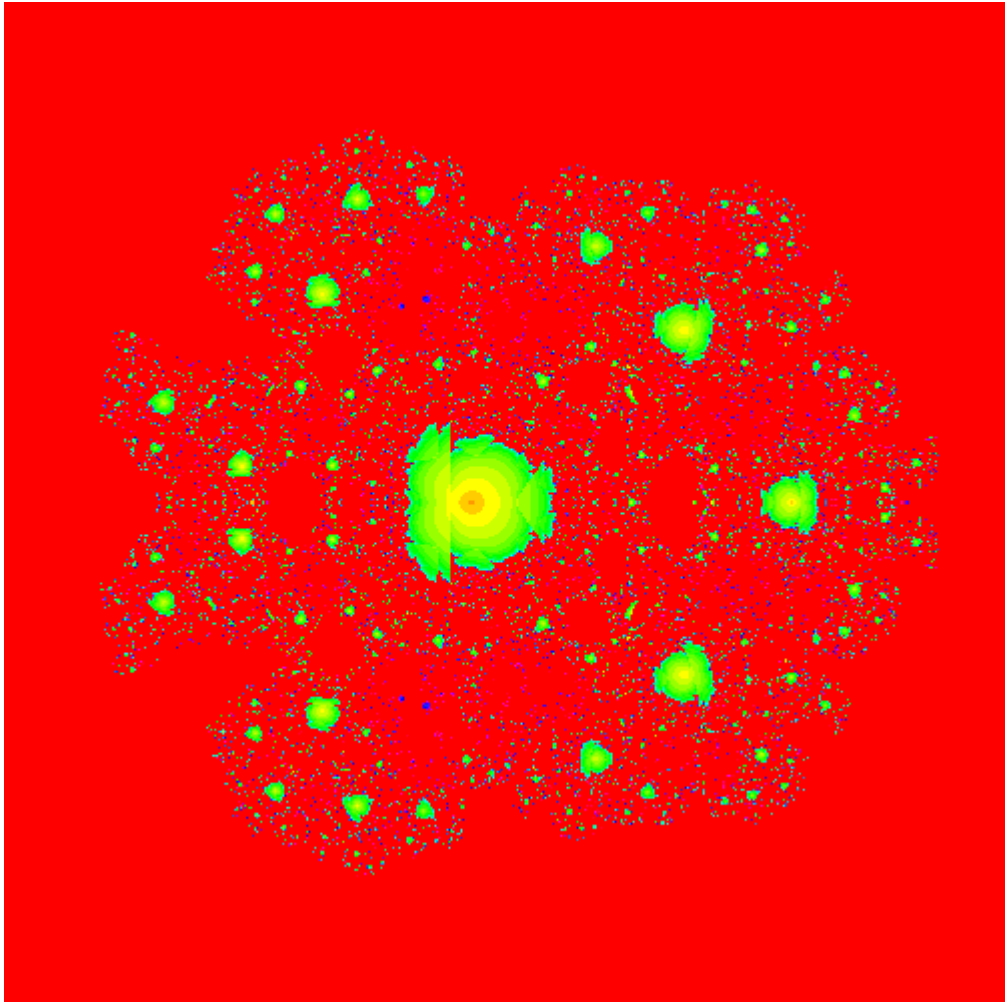


Fig. 9.

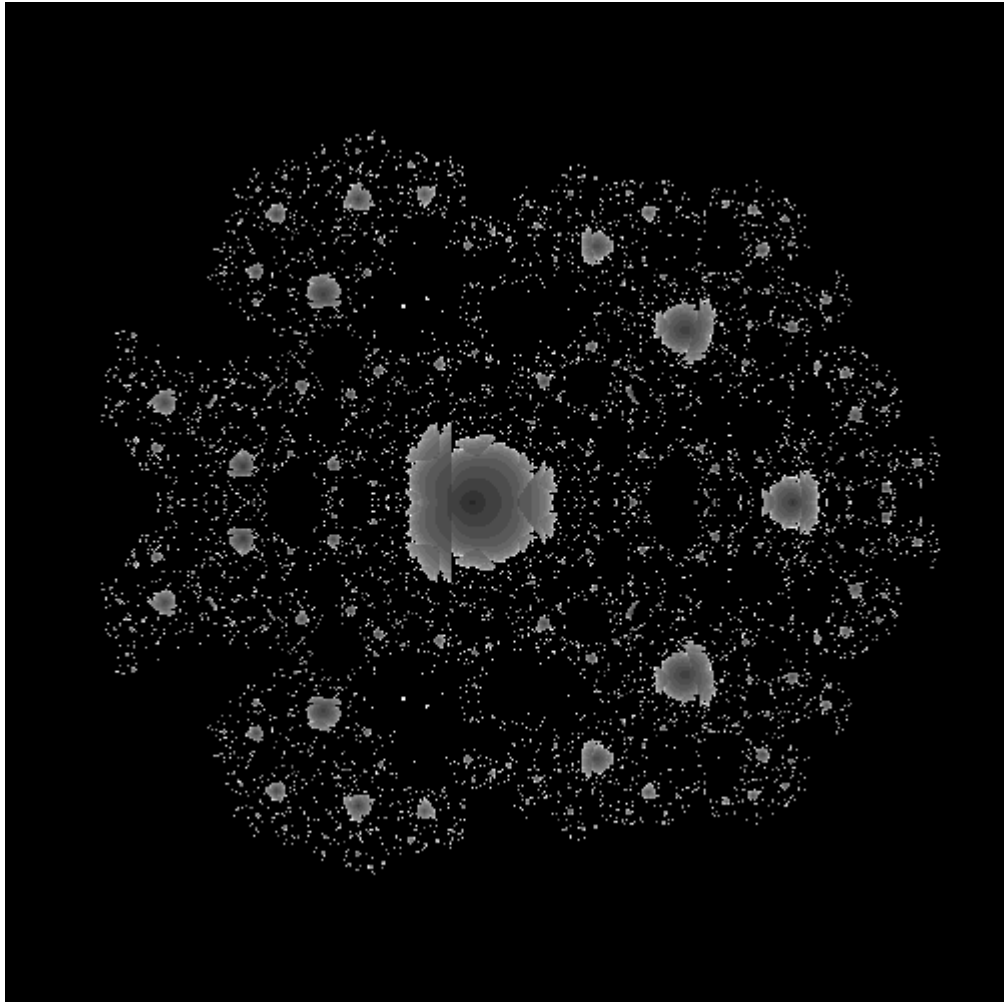


Fig. 10.

References

1. Blanchard, P.: Complex analytic dynamics on the Riemann sphere, Bull. Amer. Math. Soc. (N.S.) 11 (1984), 85-141.
2. Gradshteyn, I.S. and Ryzhik, I.M.: Table of Integrals, Series and Products. 5th ed., ed. Alan Jeffrey. Academic Press, 1994.