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WHAT IF THE GRAVITON HAD A NEGATIVE MASS?

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ABSTRACT: Gravitons are the as-yet undiscovered particles that convey gravity. "Massive gravity" – the idea that gravitons may have a tiny positive mass m_g - has recently been revived. The equations of massive gravity contain the quantity m_g^2 but not m_g itself. Yet, the possibility that m_g could be negative has never been explored. This paper shows that negative mass gravitons can 1) explain the actual mechanism of gravity in terms of Newtonian mechanics, 2) be used to derive Milgrom's constant in Modified Newtonian Dynamics (MOND), a theory that obviates the need for Dark Matter, 3) identify Dark Energy as being equivalent to the momentum flux of negative mass gravitons, 4) suggest why a vacuum has a far lower density than what quantum field theory predicts.

1] INTRODUCTION

"Massive gravity" theories have had a revival in recent years [1,2]. According to [2], the cosmological constant Λ is simply the inverse of the squared Compton wavelength of the graviton, i.e.,

$$\Lambda = 1.174 \times 10^{-52}\ \mathrm{m}^{-2} = [h/m_g c]^{-2} = [m_g c/h]^2 \qquad [1]$$

where h = Planck constant, c = light speed, m_g = graviton mass.

Equation 1 gives $m_g = 2.395 \times 10^{-68}$ kg.

The equations of massive gravity found in [1-4] contain the quantity m_g^2, but not m_g itself. Since $m_g^2 = (-m_g)^2$, gravitons with a tiny negative mass $-m_g$ will still be compatible with various massive gravity theories, and with general relativity (GR) [5]. *Yet, the possibility that m_g could be negative has never been explored.*

So, this paper examines the consequences of the graviton having a tiny negative mass, $-m_g$.

2] THOUGHT EXPERIMENT: NEGATIVE MASS GRAVITONS

A Sun emits gravitons in all directions. A small fraction of this graviton flux is intercepted by the sunlit side of a planet. If these gravitons had a positive mass or momentum, the planet will be gravitationally *repelled* by the Sun. If the planet accelerated these gravitons and ejected them out of its dark side, then it would indeed be gravitationally attracted towards the Sun. However, this is unlikely, since a massive graviton's velocity $v = c$, practically speaking.

If the gravitons had a negative mass and/or negative momentum, the solar graviton flux striking the planet's sunlit side will actually attract the planet towards the Sun. Similarly, the fraction of the graviton flux from the planet that is intercepted by the Sun will produce an equal and opposite force on the Sun, as required by Newton's 3rd law.

3] DERIVING NEWTONIAN GRAVITY FROM GRAVITONS

In Section 2, both Sun and planet each absorb a momentum flux of $-nm_g v$, i.e., each absorbs gravitons at the rate of n gravitons/second.

Let N_s = number per second of gravitons emitted by the Sun. N_s gets spread over an area of $4\pi r^2$, where r = distance between Sun and planet.

$N_s \;\alpha\; m_s$, the Sun's mass [2]

The negative pressure imposed on the planet by the graviton flux from the Sun is $-N_s m_g v/4\pi r^2$.

The graviton flux from the planet imposes a negative pressure $-N_p m_g v/4\pi r^2$ on the Sun.

$N_p \;\alpha\; m_p$, the planet's mass [3]

The probability p_p of solar gravitons interacting with the planet is $p_p = n/N_s$. p_p will be proportional to the negative pressure, i.e.,

$p_p \;\alpha\; -N_s m_g v/4\pi r^2 \;\alpha\; -m_s m_g v/4\pi r^2$ [4]

p_s = probability of planetary gravitons interacting with the Sun; $p_s = n/N_p$.

$p_s \;\alpha\; -N_p m_g v/4\pi r^2 \;\alpha\; -m_p m_g v/4\pi r^2$ [5]

The joint probability of gravitons interacting with both the Sun and the planet is $p_p p_s$.

The attractive pressure P between Sun and planet will be proportional to $p_p p_s$, i.e.,

$P \;\alpha\; p_p p_s \;\alpha\; N_s N_p\, m_g^2 v^2/16\pi^2 r^4 \;\alpha\; m_s m_p m_g^2 v^2/(4\pi r^2)^2$ [6]

Equation 6 morphs into Newton's gravitation law, i.e.,

$$P \times 4\pi r^2 = \text{force } F = G\, m_s\, m_p / r^2 \qquad [7]$$

if the other constants and another constant, $8\pi\rho_v$, are absorbed into the gravitational constant G.

$$G = m_g^2 v^2 c^2/8\pi\rho_v h^2 = m_g^2 c^4/8\pi\rho_v h^2 = \Lambda c^2/8\pi\rho_v \qquad [8]$$

where 8π = GR constant, ρ_v = vacuum density = 6.29 x 10^{-27} kg/m^3.

4] GRAVITONS AND MODIFIED NEWTONIAN DYNAMICS (MOND)

Since $F = -nm_g v$, the graviton flux n between the Sun and Earth can be calculated to be: n = 4.94 x 10^{81} gravitons/second. This means that every second, some 10^{36} gravitons pass through 1 square femtometer of the Earth's surface area. Since 1 fm is about the size of an atomic nucleus, the probability of an outgoing graviton colliding or interfering with an incoming graviton should be non-negligible, even if we assume that the graviton radius r_g is around 10^{-19} femtometers (see equation 13, below).

Let q = probability of "non-interference" between gravitons.

This should be inversely proportional to g, the gravitational field intensity. The reduction in the value of g, i.e., Δg, due to "gravitonic" interference, may be written as

$$\Delta g/g \quad \alpha \quad q \quad \alpha \quad 1/g \; (\approx 1/[g + \Delta g] = 1/g_m) \qquad [9]$$

$$\Delta g / g = a_m / g_m \qquad [10]$$

where

R = radius of a gravitating body, or the distance from it
M = mass of a gravitating body,
and a_m , g_m are defined below

The true value of gravitational acceleration should be: $g_m = g(1 + \Delta g/g) = g(1 + a_m/g_m)$, so we get

$$g_m = [GM/R^2]\,[1 + a_m/g_m] \qquad [11]$$

Equation 11 is identical to the original Modified Newtonian Dynamics (MOND) equation, where a_m = Milgrom's constant $\approx 1.2 \times 10^{-10}$ m/s^2 $\approx c^2(3\Lambda/5)^{0.5}/2\pi$, according to [6].

Using equation 1, we get

$$a_m = -(3/5)^{0.5} m_g c^3/2\pi h = m_g c^3/8.112h \approx m_g c^3/8h \quad [12]$$

To estimate the radius r_g of the graviton, one can interpret a_m as the acceleration due to gravity on the surface of a graviton. If

$$a_m = Gm_g/r_g^2 \quad [13]$$

one gets $r_g = 1.154 \times 10^{-19}$ femtometers.

Since the Planck length l_p is:

$$l_p = [hG/2\pi c^3]^{0.5} \quad [14]$$

we get, from the above 3 equations, the relationship

$$r_g^2 = 2\pi \times 8.112 \times l_p^2 \approx 16\pi l_p^2 \quad [15]$$

The constant 16π appears in many GR equations [7]. Since l_p depends on G, it is not unreasonable to expect r_g to be related to l_p.

5] GRAVITONS AND TRYON'S ZERO-ENERGY UNIVERSE

If negative mass gravitons are being emitted at a rate of $-E_u$ kg/s, by the Observable Universe, such a "graviton luminosity" will exert a negative pressure, $-P_u$, on the Universe, causing it to expand.

This implies that Dark Energy could merely be due to a flux of negative mass gravitons leaving the Observable Universe.

A force balance yields:

$$-P_u \times 4\pi R_u^2 = -E_u c \quad [16]$$

where R_u = co-moving distance = radius of the Observable Universe = 4.4×10^{26} m. Note that due to the expansion of the Universe, $R_u > R_h$. The Hubble Radius $R_h = ct_u$ where t_u (= 4.32×10^{17} seconds = 13.7 billion years) is the age of the Universe.

Using equation 8 and the Dark Energy equation of state [8], we can write

$$-P_u = -\rho_v c^2 = -\Lambda c^4/8\pi G \quad [17]$$

From equations 16 and 17, one obtains

$$-E_u = -\rho_v c \times 4\pi R_u^2 = -\Lambda c^3/8\pi G \times 4\pi R_u^2 = -4.59 \times 10^{36} \text{ kg/s} \quad [18]$$

The total negative mass, $-M_{tot,g}$, of all gravitons within the Universe can be roughly estimated, by assuming that the average residence time in the Universe of a graviton equals half the age of the Universe. Then,

$$-M_{tot,g} = -E_u \times 0.5t_u = -3.37 \times 10^{54} \text{ kg} \qquad [19]$$

The latest measurements [9] show that the Universe is flat and has a density which is practically the same as the critical density ρ_{crit}, which can be calculated as

$$\rho_{crit} = 3/(8\pi G t_u^2) = 9.22 \times 10^{-27} \text{ kg/m}^3 \qquad [20]$$

The mass M_u of the Universe estimated as

$$M_u = \rho_{crit} \times 4\pi R_u^3/3 = 3.29 \times 10^{54} \text{ kg} \qquad [21]$$

is very close to $M_{tot,g}$ estimated using equation 19. With a better estimate of the graviton residence time in equation 19, it is possible that M_u will exactly equal $M_{tot,g}$, which will imply that the Universe has a net mass of exactly zero!

This will agree with Tryon's [10] hypothesis that the Universe has zero energy and mass, after he added the positive mass-energy of the Universe to its gravitational potential energy (which is negative, according to Newtonian mechanics).

If we equate M_u with $M_{tot,g}$, then we will get, from equations 18 and 21, the result

$$\Lambda = 2R_u/(ct_u)^3 \approx 2/(ct_u)^2 = 1.19 \times 10^{-52} \text{ m}^{-2} \qquad [22]$$

Other authors [2,6] have obtained equation 22 from dimensional and numerical considerations, because the value of Λ obtained from equation 22 is practically the same as the indirectly measured value [10] of Λ (i.e., 1.174×10^{-52} m^2).

6] GRAVITONS AND THE VACUUM CATASTROPHE

In any vacuum, virtual particle pairs, usually electron-positron pairs [11], spontaneously appear and disappear, every 10^{-21} seconds [12], within a volume of a few cubic picometers [13]. Quantum Field Theory says that the vacuum of Outer Space should have [14] a density ρ_v of 10^{19} to 10^{96} kg/m^3, whereas measurements give $\rho_v = 6.29 \times 10^{-27}$ kg/m^3! This enormous discrepancy is called [15] the Vacuum Catastrophe.

The graviton number density, ρ_{num}, in the Universe can be computed as:

$$\rho_{num} = (-M_{tot,g} / -m_g) \div 4\pi R_u^3/3 = 3.9 \times 10^5 \text{ gravitons per cubic picometer.} \qquad [23]$$

These gravitons, which are real particles, are occupying the space where virtual electron-positron pairs are supposed to form. A virtual electron-positron pair needs [14] a volume of approximately 1 cubic picometer to form, (because the Compton wavelength of an electron is ~1 picometer). Thus, it is likely that gravitons suppress virtual particle formation in a vacuum. This could be a possible explanation for the Vacuum Catastrophe.

On the other hand, if the Observable Universe is emitting gravitons into regions beyond it, then these 'lost' gravitons could get replenished by spontaneously forming in the vacuum. Since r_g is much smaller than the electron radius, gravitons occupy far less volume than electrons do, so their formation may not get suppressed by the gravitons that are already present in the vacuum. This is permitted by Heisenberg's Energy-Time Uncertainty principle [11,12], which, in this context, can be written as:

$$|-m_g c^2| \times t_g = h/2\pi \qquad [24]$$

where t_g is the lifetime of a graviton. From equation 24, we get $t_g = 4.9 \times 10^{16}$ seconds, which is about one-tenth the present age t_u of the Universe. Thus, gravitons can have very long lifetimes, due to their tiny mass.

So it is possible that gravitons could spontaneously form in the Universe, to replenish the gravitons being emitted by the Universe.

CONCLUSIONS

1) Negative mass gravitons present an easy-to-understand mechanism of how gravity actually propagates. Positive mass gravitons cannot do this.
2) The MOND equation [7] can be derived from ideas involving massive gravitons.
3) MOND's Milgrom constant may have a physical interpretation as the acceleration due to gravity on the surface of a graviton.
4) Because of negative mass gravitons, it may be possible that the net mass of the Universe is zero, as Tryon [10] suggested.
5) Negative mass gravitons will exert a negative pressure (Dark Energy) on the Universe, causing its expansion.
6) The high graviton number density in the vacuum may suppress the formation of virtual particles, thereby explaining the Vacuum Catastrophe.

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