Some Insight into Relativity Principle, Covariant Equations, and the Use of Abduction in Physics

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Abstract:

We show relations of the Relativity Principle (RP) to the linear features of space(time), Sec 1. In Sec.2 RP is written in Minkowski's space. Sec. 3 is

devoted to Einstein's relativity principle. The covariant form of equation and the RP in the free Fock space (FFS) are discussed in Sec.4. In Sec.5 are also discussed a possible trace of the quantum features in classical mechanics and possible sources of nonlinearity of basic equations of nature. The final comments are contained in Sec.6.

"Algebra is generous. She often gives more than is asked of her."

Jean le Rond D'Alembert (1717-1783)

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1 Principle of Relativity in Newton's Mechanics and abductive inference

In Wikipedia one can find that:

"In physics, the **principle of relativity** is the requirement that the **equations describing the laws of physics** have the **same form** in all **admissible frames of reference**". Following Galileo, we can explain this sentence using an image of two ships traveling at different speeds and performing on decks the same experiments in the belief that they will proceed in the same way:-).

Let's illustrate the above **principle of relativity (RP)** in the case of Newton's mechanics and the two-particle seen from the decks of two ships of course HMSs! From the center of the first ship S where we are located, we carry out respectively two radius vectors describing the location of two particles: $\vec{r}_1, \vec{r}_2$ and vectors from the center of the another ship S', respectively: $\vec{r}'_1, \vec{r}'_2$, describing the location of the same particles. Let the vector $\vec{R}$ connects two centers. We get:

$$\begin{aligned} \vec{r}'_1 &= \vec{R} + \vec{r}_1 \\ \vec{r}'_2 &= \vec{R} + \vec{r}_2 \end{aligned} \tag{1}$$

The Newton's equations in S are as follows:

$$\begin{aligned} m_1 \ddot{\vec{r}}_1 &= \vec{F}_1[\vec{r}_1, \vec{r}_2] \\ = m_2 \ddot{\vec{r}}_2 &= \vec{F}_2[\vec{r}_1, \vec{r}_2] \end{aligned} \tag{2}$$

where $\vec{F}_1$ is the force acting on the first particle and $\vec{F}_2$ is the force acting on the second particle. Two dots over the vectors represent the second order of the time derivative.

Substituting Eq.1 in Eq.2 we get:

$$\begin{aligned} m_1\{-\ddot{\vec{R}} + \ddot{\vec{r}}'_1\} &= \vec{F}_1[(-\vec{R} + \vec{r}'_1), (-\vec{R} + \vec{r}'_2)] \\ m_2\{-\ddot{\vec{R}} + \ddot{\vec{r}}'_2\} &= \vec{F}_2[(-\vec{R} + \vec{r}'_1), (-\vec{R} + \vec{r}'_2)] \end{aligned} \tag{3}$$

So, if there is no relative acceleration between centers of the ships:

$$\ddot{\vec{R}} = 0 \tag{4}$$

and forces $\vec{F}_1$ and $\vec{F}_2$ do not depend on the vector $\vec{R}$ then the same (identical) equations are satisfied for the both ships. In other words, the RP is fulfilled. But what is most important here is that the RP can be derived from the linearity of space (Euclidean vector space) and observations and/or measurements in the one reference frame only! 'Consider the abstract bare-bones, scheme which Peirce provides for abductive inference', see [9], :

The surprising fact, C, is observed.
But if A were true, C would be a matter of course.
Hence, there is reason to suspect that A is true.
xxxxxxx

Treating the RP as surprising fact, we see from the above considerations that the global linearity (vector space (space with free vectors)) accompanying Newton's physics is a legitimate assumption.

It is also important in all this that in S and S' we can make real observations and measurements which are not usually explicitly stated! Moreover, to ensure the fulfillment of the RP, S and S' may not be inertial systems: It is enough that r.h.s. of Eqs3 do not depend on the vector $\vec{R}$.

2 Relativity Principle (RP) in Minkowski's Mechanics

It looks almost the same as in Newton's mechanics with such difference that the 3D vectors are substituted by 4D vectors and the Newton's time derivatives d/dt are substituted by the proper time derivatives $d/d\tau$. So, so called the generalized Newton's equation, this time for one particle, looks the following:

$$\frac{d}{d\tau}(mu_\nu) = K_\nu, \tag{5}$$

where K_ν- certain 4-vector, known as the Minkowski's force, [1].

3 A possible approach to gravity theory?

It was indeed a completely new approach to the principle of relativity. Albert Einstein concentrates primarily on space, and actually space-time, in which the laws of physics are written. What could possibly be the reason for this? In my opinion, Einstein, under influence of Ernst Mach's influence, wants, according to his deeper understanding the extended relativity principle, that **admissible spacetime is physical space**, that is space where observations and measurements take place. Einstein accomplishes this in his equations by linking the distribution of mass and energy (stress-energy tensor) to the curvature of spacetime. But **physics of space-time** means additionally that space ceases to be a passive actor observing the happening phenomena, but acts on their course and vice versa. In other words, the **principle of action and reaction** takes place, see [2]. This demand is realized in theories in which a space-time metric occures as the dynamical variable interacting with other variables. But it is also possible that an active role of space-time can be substituted by the gravitation field (fourth fundamental interaction) which interact with all other material objects - among other things, affecting the clocks work and other macroscopic properties, see also [5].

We must remember that underlying are linear equations for n-pf, see e.g. author papers, that express our ignorance, and nonlinearity is the result of an approximation to n-pi expressions by 1-pfi which is related to the elimination of μ-scales.

In my opinion, the **covariance of equations** are not a necessary factor in formulation of laws of Nature because Newton's equations are not covariant. The covariant description of equations means that they are reformulated in such a way that by means of one solution one can construct a whole set of solutions with different initial configurations of the system (coordinate free theories). This must indeed complicate the original non-covariant equations and, perhaps, this is a source of problems in constructing the quantum gravity. It is not excluded that the quantum gravity, in spite of excelent successes of Einstein's equations in large scales, does not need them at all! Moreover, it would be very interesting to find simple, equivalent, noncovariant formulations of Einstein's GR, if of course such formulation are possible, see also Sec.6.

4 Covariant description of equations and an analogue expression of Relativity Principle (RP) in the free Fock space (FFS)

> **J. L. Lagrange, "M´ecanique Analytique" (1788)** The reader will find no figures in this work. The methods which I set forth do not require either con- structions or geometrical or mechanical reasonings, but merely algebraic operations subjected to a regular and uniform rule of procedure. Those who are fond of Mathematical Analysis will observe with pleasure Mechanics becoming one of its new branches and they will be grateful to me for having thus extended its domain.

If the initial and / or boundary conditions of the considered equations are treated as random numbers, then the mean values of the solutions and their correlation functions (n-point information (n-pi)) are interesting or, so-called expectation values in the case of quantum theory. These n-pi, denoted by $V(\tilde{x}_{(n)} \equiv V(\tilde{x}_1, \cdots, \tilde{x}_n)$ are generated by the generating vectors

$$|V >= \sum_n \int d\tilde{x}_{(n)} V(\tilde{x}_{(n)}) \hat{\eta}(\tilde{x}_1) \cdots \hat{\eta}(\tilde{x}_n)|0 >,$$

where $\tilde{x}$ contains the space-time variables and other variables describing interior degrees of freedom of particles, see, e.g. [5]. These n-pi are related to the primary quantities fields $\varphi[\tilde{x};\alpha]$, where α denote initial or/and boundary conditions, through averaging procedures:

$$V(\tilde{x}_{(n)}) = \int \delta\alpha\varphi[\tilde{x}_1;\alpha]\cdots\varphi[\tilde{x}_n;\alpha]P[\alpha] \equiv < \varphi(\tilde{x}_1)\cdots\varphi(\tilde{x}_n) >$$

where $P[\alpha]$ is a probability density and $\int \delta\alpha$ may denote the functional integral.

The vectors $|V>$ satisfy the linear equations!:

$$\hat{A}|V >= |\Phi > \tag{6}$$

with given operator $\hat{A}$ and defined, up to zero's component, vector $|\Phi >$, see e.g. [3], [4]. In the **free Fock space** (FFS) the operator $\hat{A}$ is usually right invertible operator. It means that a right invertible operator $\hat{R}$ exists such that

$$\hat{A}\hat{R} = \hat{I} \tag{7}$$

where $\hat{I}$ is the unit operator in the FFS. It means that Eq.6 can be described in an equivalent way as:

$$\hat{R}\hat{A}|V >= \hat{R}|\Phi > \tag{8}$$

expressions b Let us notice that operators

$$\hat{R}\hat{A} \equiv \hat{Q} = \hat{I} - \hat{P} \tag{9}$$

are projectors (idempotence). The projector $\hat{P}$ is called an *initial operator for the operator* $\hat{A}$, see [8].

With the help of these projectors one can describe the general solution to Eq.6 as

$$|V >= \hat{P}|V > +\hat{R}|\Phi > \tag{10}$$

with an arbitrary projection $\hat{P}|V >$.

Amazing Analogies'

By a *symmetry* of the Eq.6 we mean a transformation of its vector solutions, that the transformed vectors satisfy the same equation. Thus, denoting the *symmetry operator* by $\hat{G}$, we should have:

$$\hat{A}\hat{G} = \hat{A} \tag{11}$$

It is easy to see that the transformed vectors denoted by $|V >' = \hat{G}|V >$, where $|V >$satify Eq.6 and $\hat{G}$ is any symmetry operator, satisfy also Eq.6. From Eq.9 and Eq.7 one can see that for the initial projector $\hat{P}$ we have:

$$\hat{A}\hat{P} = 0 \tag{12}$$

expressions b It means that $\hat{P}$ projects on the null space of the operator $\hat{A}$. Hence, the symmetry transformation $\hat{G}$ of Eq.6 can be constructed as follows:

$$\hat{G} = \hat{I} - \hat{P}\hat{T} \tag{13}$$

where $\hat{T}$ is an arbitrary operator acting, like all operators here, in FFS. So, the transformed vector

$$|V>' \equiv \hat{G}|V> = (\hat{I} - \hat{P}\hat{T})|V> = |V> - \hat{P}\hat{T}|V>, \tag{14}$$

for an arbitrary operator $\hat{T}$, satisfies the same Eq.6 as the vector $|V>$. In the case under considexpressions beration the symmetry transformation $\hat{G}$ has infinite number of parameters represented by the operator $\hat{T}$ acting in the linear FFS. It is easy to show with rather mild assumption that $|V>'$ represents arbitray solution to Eq.6 at fixed vector |V>, see AppendixA.

If $\hat{G}$ is an inverse operator then from Eq.6 and Eq.14 the vector $|V^{'}>$ satisfies

$$\hat{A}\hat{G}^{-1}|V>' = |\Phi> \tag{15}$$

or

$$\hat{A}^{'}|V>' = |\Phi>' \tag{16}$$

and where

$\hat{A}^{'} = \hat{G}\hat{A}\hat{G}^{-1}$, $|\Phi>' = \hat{G}|\Phi>$ This is **covariant description** of Eq.6. Since $\hat{G}$ is a group, see AppendixB, any similar transformation of $\hat{A}^{'}$, $|V>'$ and $|\Phi>'$ gives a new equation equivalent to Eq.6.

Because, in general

$$\hat{A}^{'} \equiv \hat{G}\hat{A}\hat{G}^{-1} \neq \hat{A}, \tag{17}$$

expressions b (covariance does not mean invariance).

In this section we have considered the general symmetry associated with the given linear equation, allowing from one solution to create all other by means of symmetry transformations.

The GR symmetry is less(!) general and concerns 'only' different configuration of the system bypassing the initial movements of the system. This leads to creating by Albert Einstein a ***coordinate-free*** approach of GR, [6]. See also [7]. Of course, this is not the only feature of GR. The characteristic feature of this theory is its local Lorentz invariance and the correlation of time-space curvature with the stress-energy tensor. As I wrote in [5], it is not excluded that some of these properties cause difficulties in creating quantum gravity.

Amazing analogies"

The vectors $\vec{r}$ and $\vec{r}'$in Eqs1 with the condition 4 fulfill the same Newton's equations and both of these vectors have similar physical interpretation. This is called the *relativity principle*. The transformation 1 with the condition 4 is called the *'Galilean' symmetry transformation.*

The vectors $|V>, |V>'$ in Eq.14 also satisfy the same equation, but they both do not have to be generating vectors of n-pi. The analogy of *'Galilean' symmetry in the FFS* is described by the operator $\hat{G}$ given by Eq.13 in which the arbitray operator $\hat{T}$ is chosen in such a way that both vectors |V> and |V>' *are physical*: By this we understand that n-pi generated by them are obtained by means of appropriate averaging or smoothing procedures which they do not necessarily have to be related to the transformation of the space-time coordinates. It would be interesting to find such transformations explicitly. However, it is intersting that 'Galilean' symmetrey structure persists even in the case of generating vectors $|V>, |V>'$ which generate multitimes functions what are n-pi.

It can also be suspect that these analogies have their origin in the quantum structure of space-time, which Eddington assumed in his book: Fundamental Theory, see [10].

5 Linearity and nonlinearity in physics

In Wikipedia (29.05.017) you can find the following sentence:

"In physics and systems theory, the **superposition principle**, also known as superposition property, states that, for **all linear systems**, the net response at a **given place** and **time** caused by two or more stimuli **is the sum** of the responses that would have been caused by each stimulus individually."

In order not to complicate the matter, let us agree with the author of this sentence and suppose that the stimuli move at infinite speed like in Newton's mechanics. In this case a stimulus can be a force acting on the particle located at the point $\vec{r}_1$ coming from the second particle located at the point $\vec{r}_2$. In the presence of third particle located at the point $\vec{r}_3$we can say that on the particle located at the point $\vec{r}_1$act two stimuli originating from two particles located in the points $\vec{r}_2$ and $\vec{r}_3$ which is the sum of the responses that would have been caused by each stimulus individually. It is not excluded that the above superposition principle **is a result of the quantum nature** of atoms forming the classical objects.

Where are we dealing here with nonlinearity? Nonlinearity in this example and in a more general situation may arise from the fact that the stimulis and reactions can depend on the positions and the time in which these objects are located. Functions describing gravitational interaction are not linear. There is another manifestation of the quantum properties of the basic components of matter, says Eddington in his Fundamental Theory, where he states that the **curvature of space arises** out of the statistical fluctuations of distribution of a large number of particles, see [10].

Now we will try to clarify the above superposition principle in the case of the electromagnetic field described in the form of linear equation

$$\hat{K}\varphi(\tilde{x}) = \tilde{\varrho(x)} \tag{18}$$

non-local theories where $\hat{K}$ is a linear operator like d'Alambert one and $\tilde{x}$ some variables containing in addition to spatial-temporal variables, discrete variables describing the field components. It seems that the above linearities is the **result of averaging** (e.g. a transition from micro electrodynamics to macro) as well as the quantum nature of the described processes, see [4, 5]. Assuming that ϱ represents stimuli and corresponding responses are represented by the field φ we will show that "the net response at a **given place** and **time** caused by two or more stimuli **is the sum** of the responses that would have been caused by each stimulus individually": For that purpose, let us assume that ϱ is composed with two stimuli:

$$\varrho = \varrho_1 + \varrho_2$$

For every stimulus, we have the same Eq.18:

$$\hat{K}\varphi_{1,2}(\tilde{x}) = \varrho_{1,2}(\tilde{x})$$

Hence, and from linearity of these equations, it is seen that

$$\varphi(\tilde{x}) = \varphi_1(\tilde{x}) + \varphi_2(\tilde{x})$$

satisfies Eq.18♣.

For quantum fields we should substitute the classical equation 18 by the operator one in which fields φ are substituted by operators $\hat{\varphi}$. Also ρ are substituted by the operators $\hat{\varrho}$. So, we consider:

$$\hat{K}\hat{\varphi}(\tilde{x}) = \hat{\varrho}(\tilde{x}) \tag{19}$$

Hence, by going to physical quantities like the expectation values of observables in the right and left hand sides of the equality above, we get:

$$\hat{K} < \Phi|\hat{\varphi}(\tilde{x})|\Phi > = < \Phi|\hat{K}\hat{\varphi}(\tilde{x})|\Phi > = < \Phi|\hat{\varrho}(\tilde{x})|\Phi > \tag{20}$$

where $|\Phi >$ is a state of the system. For two states $|\Phi_{1,2} >$ such that

$$< \Phi_{1,2}|\hat{\varrho}(\tilde{x})|\Phi_{2,1} > = 0 \tag{21}$$

we get, for the superposition state, $|\Phi > = |\Phi_1 > + |\Phi_2 >$ the superposition principle for the expectation values $\varphi(\tilde{x}) = < \Phi|\hat{\varphi}(\tilde{x})|\Phi >$.

Over time, I am more and more inclined to accept the hypothesis that basic equations of nature, which take into account the imperfection of measuring instruments, or the very nature of measurement, are linear. Nonlinearity is a result of the approximation of the infinite series of equations consisting either of the truncation of these chains either by expressing their elements by 1-pi or the higher order n- pi in the case of more strong correlations.

In Quantum Field Theory (QFT), called also second quantization theory, the 'density' operator $\hat{\varrho}$ occuring on the r.h.s. of Eq.19 is expressed by fields which describe the material components of the system (with fractional spins) as well as the fields describing the interaction between them (with integer spins). With supper field $\widehat{\varphi(\tilde{x})}$, in which the particular fields are distinguished by the one of the component of the 'vector' $\tilde{x}$, the Eq.19 can be described as follows:

$$\hat{K}\hat{\varphi}(\tilde{x}) = \hat{\varrho}[\tilde{x}; \hat{\varphi}] \tag{22}$$

where in the quantum electrodynamics, the standard theory of elementary particles and some gravity models, [13], the operator $\hat{\varrho}$depends polynomially on the operators $\hat{\varphi}(\tilde{x})$. So, the Eq.22 are nonlinear. The linear equations are obtained from the above equation when we introduce a more physical quantities called the n-point fuctions (n-pf) or n-point information (n-pi):

$$< 0|\hat{\varphi}(\tilde{x}_1) \cdots \hat{\varphi}(\tilde{x}_n)|0 > \equiv V(\tilde{x}_{(n)}) \tag{23}$$

which satisfy linear equations discussed in previous author papers, e.g, [3]. See also Sec.4. Moreover, for polynomial modes, the operators appearing in the equations satisfied by n-pi 23, in the FFS, are usually right or left invertible what allows for their different fruitful transformations, see e.g. [4].

6 Final remarks

The symmetry of the Eq.6 means that with the help of symmetry transformation, from one particular solution, a whole set of solutions can be generated with different innitial and / or boundary conditions. Postulating the **principle of conservation of dificulties**, one can conclude that for a more symmetrical equation is a more difficult to findsome of its solutions at least.

This makes the covariant Eq.16 to be more complicated than the equations 6 written in non-covarianr form, see 17, in spite of a possibility that in a certain cases Eq.16 can be simpler then Eq.6.

Here we would like to point out that the symmetry considered in Sec.4 is the *most general symmetry* refered to a given equation because by means of such symmetry transformations one can generate, from a one solution, all solutions. In the hierarchy of symmetries, below are symmeries by means of which only some set of solutions can be generated by so-called symmetry transformations appearing in GR, SRT or in Newton's equations They are related to specific demands as the constant velocity of light in any inertial coordinate system, or the independence of physics laws from the reference frames. However, by writing physical laws in a covariant manner, we artificially increases the symmetry of equations thus leading by this to their complications, see Sec.4. Perhaps the reference frames are merely auxiliary quantities as the generating vectors or functions in mathematics. It is not excluded that GR or other good gravity theory can be described in an equivalent or almost equivalent way but not in the covariant way of GR, see also [13].

Following the development of modern physics from Galileo, Newton, Einstein to Bohra, Dirac and others, it is astonishing that the linearity appears in a greater or lesser degree. Taking into account Bohm's and Peat's view that 'a certain continuity is always preserved during a scientific revolution', see [11]; 18p, we are inclined to argue that it is precisely the linearity that belongs to the 'tacit infrastructure of ideas' which will survive in one or another form, [11]; 13p. In particular, we relate here the lack of interactions with the linearity of space and space-time:-)

7 Appendix A

(about an arbitrary solution of linear systems)

We show with rather mild assumption that $|V>'$ given by Eq.14:

$$|V>' \equiv \hat{G}|V>_{fix} = (\hat{I} - \hat{P}\hat{T})|V>_{fix}$$

represents, at fixed solution $|V>_{fix}$ to Eq.6, an arbitray solution to Eq.6. That vector $|V>'$is a solution of Eq.6 it is seen from the property 12 of the initial projector $\hat{P}$ and that $|V>_{fix}$ is a solution. Now we have to show that the projection $\hat{P}|V>'$ can be an arbitrary projection: Because operator $\hat{T}$ is arbitray, we can choose $\hat{T} = \hat{I} - \hat{B}$, where $\hat{B}$ is an arbitrary operator. Hence and from Eq.14 we get,

$$\hat{P}|V>' = \hat{P}\hat{B}|V>_{fix}$$

For a fixed vector $|V>'$ and an arbitrary operator $\hat{B}$ one can expect that the above equation is satisfied♡.

8 Appendix B

(about group symmetry of linear systems)

We show that tranformations given by Eq.13

$$\hat{G} = \hat{I} - \hat{P}\hat{T}$$

can be a group. Takin a product: $\hat{G}_1\hat{G}_2 = \hat{I} - \hat{P}\hat{T}_2 - \hat{P}\hat{T}_1 + \hat{P}\hat{T}_1\hat{P}\hat{T}_2$ we see that it has the structure of the transformation $\hat{G}$. If we assume that operators (parameters) $\hat{T}$ are such that that every $\hat{G} \equiv \hat{G}(\hat{T})$ has the inverse operator then transformations $\hat{G}$ form a group. This is the ful symmetry group of a considered equation.

9 Appendix C

(Excersite: A symmetry in nonlinear systems)

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