The physical nature of the basic concepts of physics

4. The conservation of Kinetic Energy in elastic collisions (i)

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Abstract

The "conservation of energy" is a postulated law that implies the transformation of different forms of "energy" into one another, while the total amount of "energy" remains constant.
In this paper I demonstrate that the conservation of "kinetic energy" in perfectly elastic collisions, as well in its macroscopic form as "kinetic energy of bulk motion" as in its microscopic form as "kinetic energy of internal motion", is a quantitative expression of which the numerical value remains constant when the total amount of motion of the particle system at the given velocity level is physically conserved.

1. The present conservation laws in elastic collisions

In the present physics, the conservation of kinetic energy (K) together with the conservation of linear momentum (**p**), allow us to calculate the final velocities in elastic collisions.
If we take the classical case of an head-on collision of two perfect elastic billiard balls with masses m_1 and m_2, that move to each other along the x-axis with initial velocities $\mathbf{v_{1i}}$ and $\mathbf{v_{2i}}$ (Fig.1) and that exert no influence on each other (except during collision), the final velocities ($\mathbf{v_{1f}}$ and $\mathbf{v_{2f}}$) of the particles after collision, are directly calculated from both conservation laws.

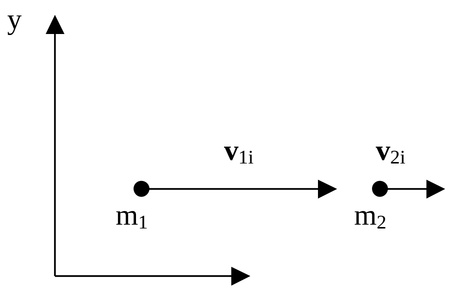

Fig. 1

- The equation of the conservation of the total linear momentum (of any particle system),

$$\mathbf{p} = m_1\mathbf{v_{1i}} + m_2\mathbf{v_{2i}} = m_1\mathbf{v_{1f}} + m_2\mathbf{v_{2f}} = \text{constant}$$

(i) Updated edition of my paper 'Kinetic Energy' December 1991.

- together with the equation of the conservation of total kinetic energy (in elastic particle systems):

$$2K = m_1.v_{1i}^2 + m_2.v_{2i}^2 = m_1.v_{1f}^2 + m_2.v_{2f}^2 = \text{constant}$$

lead to the classical equations for the final velocities (Fig. 2):

$$\mathbf{v_{1f}} = [(m_1 - m_2)\mathbf{v_{1i}} + 2m_2\mathbf{v_{2i}}]/(m_1 + m_2)$$

$$\mathbf{v_{2f}} = [(m_2 - m_1)\mathbf{v_{2i}} + 2m_1\mathbf{v_{1i}}]/(m_1 + m_2)$$

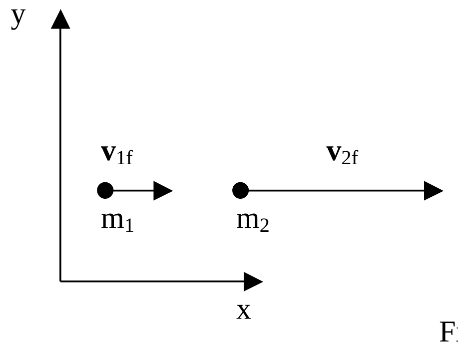

Fig. 2

To understand the physical meaning of these equations, we will take a closer look at both conservation laws.

2. The conservation of translational motion in elastic collisions

To analyze the classic case of an head on collision in one dimension of two perfectly elastic particles with initial velocities $\mathbf{v_{1i}}$ and $\mathbf{v_{2i}}$ we first decompose the initial velocities of both particles into their fundamental components:

- their common/congruent velocity '$\mathbf{v_c}$', and
- their opposite or internal velocities 'q_{1i}' and 'q_{2i}' (Fig. 3).

2.1 The conservation of the congruent translational motion of particle systems

In my paper "Part 1: The physical nature of linear momentum" I have demonstrated that the CM-velocity is in fact the common or congruent velocity component ($\mathbf{v_c}$) of all the particles of a particle system, which means that it cannot be affected by the motions of the individual particles and that it is therefore bound to remain constant.

$$\mathbf{v_c} = (m_1\mathbf{v_{1i}} + m_2\mathbf{v_{2i}})/(m_1 + m_2) = \text{constant}$$

This leads to the fact that in any collision the total amount of congruent translational motion, which is in fact the resultant velocity of all the basic particles with unit mass (and which is generally indicated as the linear momentum) is unaffected by the collision and is therefore said to be 'conserved' in magnitude as well as in direction:

$$\mathbf{p} = (m_1 + m_2)\mathbf{v_c} = m_1\mathbf{v_{1i}} + m_2\mathbf{v_{2i}} = \mathbf{p_1} + \mathbf{p_2} = \text{constant}$$

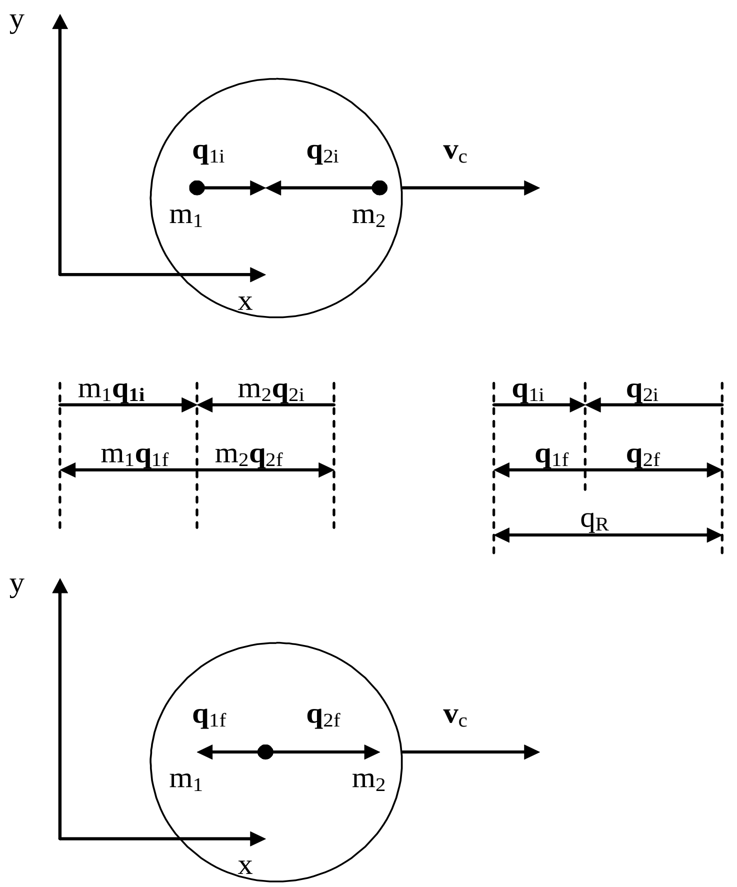

Fig. 3

2.2 The conservation of the internal translational motion in elastic collisions

The former equation of the congruent or common velocity of both particles allows us to calculate their internal velocities as the remaining velocity components:

$$\mathbf{q_{1i}} = (\mathbf{v_{1i}}\text{-}\mathbf{v_c}) = [m_2/(m_1+m_2)].(\mathbf{v_{1i}} - \mathbf{v_{2i}})$$

$$\mathbf{q_{2i}} = (\mathbf{v_{2i}}\text{-}\mathbf{v_c}) = [m_1/(m_1+m_2)].(\mathbf{v_{2i}} - \mathbf{v_{1i}})$$

These are the velocities with which the two perfectly elastic particles will physically collide with each other. It follows from the fact that the linear momentum (i.e. the total amount of congruent translational motion) of the internal velocities is by definition equal to zero, that the internal linear momentums with which the two objects collide are always equal but opposite:

$$\mathbf{p_q} = m_1\mathbf{q_{1i}} + m_2\mathbf{q_{2i}} = 0$$

that: $\quad m_1\mathbf{q_{1i}} = -m_2\mathbf{q_{2i}}$

- On the one hand, the linear momentum, which was initially zero, must remain zero:

  $$\mathbf{p_q} = m_1\mathbf{q_{1i}} + m_2\mathbf{q_{2i}} = m_1\mathbf{q_{1f}} + m_2\mathbf{q_{2f}} = 0$$

  or: $m_1\mathbf{q_{1i}} = -m_2\mathbf{q_{2i}}$ and $m_1\mathbf{q_{1f}} = -m_2\mathbf{q_{2f}}$

  This means that in an elastic collision, the internal linear momentums of the colliding particle systems remain equal and opposite. Since both particles are however physically hindered by each others presence, they cannot proceed their course.

- On the other hand, the present textbooks of physics tell us that a collision is ‘elastic’ if the total kinetic energy of the colliding particles is the same before and after the collision. In the light of my paper Part 3 about the true physical nature of work and kinetic energy, this means that in elastic collisions no translational motion can be absorbed or dissipated, so that the total amount of internal translational motion at the given velocity levels in each direction and consequently the magnitudes of the internal linear momentums, must be conserved.

  $$m_1q_{1f} = m_1q_{1i} = m_2q_{2i} = m_2q_{2f}$$

Both conditions lead to the fact that the equal but opposite internal linear momentums of the colliding particles must inevitably conserve their magnitudes, while they simultaneously reverse their directions (Fig. 3):

$$m_1\mathbf{q_{1f}} = -m_1\mathbf{q_{1i}} = m_2\mathbf{q_{2i}} = -m_2\mathbf{q_{2f}}$$

so that: $m_1\mathbf{q_{1f}} = m_2\mathbf{q_{2i}}$

and $m_2\mathbf{q_{2f}} = m_1\mathbf{q_{1i}}$

This means that from the viewpoint of motion, it is like nothing happens and both equal and opposite amounts of translational motion just proceed their way, or in other words, the individual masses cannot proceed their way, but the amount of internal translational motion (the momentum flows) in both directions does and is unaffected by the elastic collisions.
This allows us to conclude that in elastic collisions, the internal translational motion is conserved, as well in magnitude as in direction.

2.3 The final velocities

The final velocities in the given reference frame are easily obtained by re-adding the invariable "common" velocity of the binary particle system to the final internal velocities of the particles:

$$\mathbf{v_{1f}} = \mathbf{v_c} + \mathbf{q_{1f}} = \mathbf{v_c} - \mathbf{q_{1i}} = [(m_1 - m_2)\mathbf{v_{1i}} + 2m_2\mathbf{v_{2i}}]/(m_1 + m_2)$$

$$\mathbf{v_{2f}} = \mathbf{v_c} + \mathbf{q_{2f}} = \mathbf{v_c} - \mathbf{q_{2i}} = [(m_2 - m_1)\mathbf{v_{2i}} + 2m_1\mathbf{v_{1i}}]/(m_1 + m_2).$$

which are the classical equations for the final velocities in elastic collisions (Fig. 2).

For an elastic collision between two identical particles ($m_1 = m_2$), this gives us:
$\mathbf{v_{1f}} = \mathbf{v_{2i}}$ and $\mathbf{v_{2f}} = \mathbf{v_{1i}}$
and it is like nothing happened and both (identical) particles proceed their way.

It is important to underline the fact that I have obtained these classical equations for the final velocities, solely on the basis of the conservation of the translational motion of the particles:

- their external or congruent translational motion
- their internal or opponent translational motions

I will come back to the conservation of these external and internal amounts of motion in section 2.5.

Although my deduction of the final velocities in a two-dimensional elastic collision, on the sole basis of the physical conservation of external and internal translational motion is extremely simple and obvious, it was in 1991 when I first submitted it for publication, totally un known and therefore rejected by the Flemish engineers association (KVIV).
In 2014 I discovered that my approach was in fact nothing new, because it was already published in 1688 by the Dutch scientist Christiaan Huygens [1]!

2.4 The conservation of the internal momentum flow

To conserve equilibrium at each moment, the equal but opposite internal linear momentums of both colliding particles must remain equal and opposite at each moment:

$$m_1.(d\mathbf{q_1}/dt) = - m_2.(d\mathbf{q_2}/dt)$$

This means that the transfer of momentum flow to particle 2 is at each moment equal but opposite to the transfer of momentum flow to particle 1.

Since: $\mathbf{q_i} = (\mathbf{v_i} - \mathbf{v_c})$ and $\mathbf{v_c}$ = constant

this means that this conclusion is valid in any chosen reference frame

$$m_1(d\mathbf{v_1}/dt) = - m_2(d\mathbf{v_2}/dt)$$

Since "force" is defined as the as the momentum transfer per unit time, this equal but opposite momentum transfer can be written as:

$$\mathbf{F_1} = \mathbf{Q_{M1}} = - \mathbf{Q_{M2}} = - \mathbf{F_2} .$$

which gives us Newton's third law of motion: *"Whenever a body exerts a force on another body, the latter exerts a force of equal magnitude and opposite direction on the former".*
Our logical deduction of Newton's equation allows us to conclude that Newton's third law is a logical consequence of the conservation of translational motion between colliding bodies in their internal reference frame.

Newton's third law leads us to the important conclusion that from the viewpoint of the colliding bodies, collisions always take place in the internal reference frame, so that they are always symmetric and the forces are always equal and opposite.

2.5 The conservation of the velocity gap in elastic collisions

In section 2.2 we have seen that in elastic collisions, the internal linear momentums are conserved in magnitude and in direction (the conservation of internal translational motion).

$$m_1\mathbf{q_{1f}} = m_2\mathbf{q_{2i}}$$

$$m_2\mathbf{q_{2f}} = m_1\mathbf{q_{1i}}$$

An important consequence of this conservation of the internal linear momentums is that the internal velocities conserve their magnitudes, but reverse their directions:

$\mathbf{q_{1f}} = -\mathbf{q_{1i}}$ or: $q_{1f} = q_{1i}$

and $\mathbf{q_{2f}} = -\mathbf{q_{2i}}$ or: $q_{2f} = q_{2i}$

The consequence of this is that also the relative internal velocity conserves its magnitude but reverses its direction:

$$(\mathbf{q_{1i}} - \mathbf{q_{2i}}) = -(\mathbf{q_{1f}} - \mathbf{q_{2f}})$$

Since: $\mathbf{q_{1i}} = (\mathbf{v_{1i}} - \mathbf{v_c})$

and $\mathbf{q_{2i}} = (\mathbf{v_{2i}} - \mathbf{v_c})$

this means that

$$(\mathbf{v_{1i}} - \mathbf{v_{2i}}) = -(\mathbf{v_{1f}} - \mathbf{v_{2f}})$$

This leads to the classic conclusion that in an elastic collision the relative velocity of the colliding particles reverses during the collision that is, it conserves its magnitude but changes its sign. In the present textbooks, this reversal of the relative velocity in elastic collisions is considered as a consequence of the conservation of energy [2].
It is important to realize that this means that in the present physics, a bare physical fact such as the conservation of the speed with which material objects physically collide with each other, is regarded as the consequence of a postulated law.

I have hereby however demonstrated that, the reversal of the relative velocity in elastic collisions is a direct consequence of conservation of the amount of translational motion in each direction.

$m_1\mathbf{q_{1f}} = m_2\mathbf{q_{2i}}$ and $m_2\mathbf{q_{2f}} = m_1\mathbf{q_{1i}}$

In section 2.2, I came to the conclusion that from the viewpoint of motion it is like nothing happens and both equal but opposite internal linear momentums proceed their way.
This means that from the viewpoint of the conservation of motion, the reversal of the relative velocity is in fact a mathematical consequence of the physical conservation of the "velocity gap" between both particles:

$$(\mathbf{v_{1i}} - \mathbf{v_{2i}}) = (\mathbf{v_{2f}} - \mathbf{v_{1f}}) = \Delta\mathbf{v}$$

This velocity gap between two colliding particles is in fact of the same nature as the velocity gap between the runners in a relay race, where the first runner runs with a velocity "**v**" and the second runner stands still, waiting for the first runner to arrive. After the handing over of the baton, the first runner stops and the second runner runs with a velocity "**v**". In this case also, it is mathematically correct to state that the relative velocity has conserved its magnitude but has reversed its sign, but physically this has no sense because from the point of view of the amount of motion, it is like nothing has happened and the motion continues its way. This means that in analogy with our statement about the conservation of internal translational motion, we can conclude that in elastic collisions, the velocity gap between colliding bodies remains invariable.

It is very important to realize that the velocity gap between material particles is a hard physical fact, because it is as a matter of fact the impact velocity with which they physically collide with each other and which is the basis of the temperature scale (ii). This leads to the important conclusion that the velocity and consequently the linear momentum of a physical body in a reference frame that is linked to any other physical object, must also be real physical (iii).

2.6 The conservation of external and internal kinetic energy in elastic collisions

The fact that in elastic collisions, the conservation of the kinetic "energy" is a mathematical expression of the conservation of the total amount of motion, can also be illustrated by considering the equation of the total amount of momentum flow (Q_{ML}) of a colliding bi-particle system and by decomposing the velocities ($\mathbf{v_1}$ and $\mathbf{v_2}$) of the particles into their internal ($\mathbf{q_1}$ and $\mathbf{q_2}$) and their coherent ($\mathbf{v_c}$) velocity components:

$$Q_{ML} = m_1.\mathbf{v}_{1i}^2 + m_2.\mathbf{v}_{2i}^2$$

$$Q_{ML} = m_1(\mathbf{v_c} + \mathbf{q_{1i}})^2 + m_2(\mathbf{v_c}+\mathbf{q_{2i}})^2$$

$$Q_{ML} = (m_1 + m_2)v_c^2 + (m_1q_{1i}^2 + m_2q_{2i}^2) + 2\mathbf{v_c}(m_1.\mathbf{q_{1i}} + m_2.\mathbf{q_{2i}})$$

In the centre of mass reference frame of any collision, the total internal linear momentum is by definition zero and must remain zero:

$$(m_1\mathbf{q_{1i}} + m_2\mathbf{q_{2i}}) = (m_1\mathbf{q_{1f}} + m_2\mathbf{q_{2f}}) = 0$$

So that:

$$Q_{ML} = (m_1 + m_2)v_c^2 + (m_1q_{1i}^2 + m_2q_{2i}^2)$$

In the specific case of elastic collisions, the magnitudes of the internal velocities are conserved:

$q_{1i} = - q_{1f}$ and $q_{2i} = - q_{2f}$

So that:

(ii) See my paper "The true nature of temperature and thermal energy".
(iii) The physical nature of velocity will be revealed in my paper "The true nature of velocity".

$Q_{ML} = (m_1 + m_2)v_c^2 + (m_1q_{1f}^2 + m_2q_{2f}^2)$

$Q_{ML} = (m_1 + m_2)v_c^2 + (m_1q_{1f}^2 + m_2q_{2f}^2) + 2\mathbf{v_c}(m_1\mathbf{q_{1f}} + m_2\mathbf{q_{2f}})$

$Q_{ML} = m_1(\mathbf{v_c} + \mathbf{q_{1f}})^2 + m_2(\mathbf{v_c} + \mathbf{q_{2f}})^2$

$Q_{ML} = m_1.\mathbf{v_{1f}}^2 + m_2.\mathbf{v_{2f}}^2$

In this way I have demonstrated that the conservation of the total amount of momentum flow "Q_{ML}" is the physical basis for the conservation of the kinetic energy in elastic collisions.

$Q_{ML} = m_1.\mathbf{v^2_{1i}} + m_2.\mathbf{v^2_{2i}} = m_1.\mathbf{v^2_{1f}} + m_2.\mathbf{v^2_{2f}} = 2K = \text{constant}$

$K = Q_{ML}/2$ constant

The previous equation of the conservation of the total amount of momentum flow can also be written as:

$Q_{ML} = (m_1 + m_2)v_c^2 + (m_1 q_{1i}^2 + m_2q_{2i}^2) = (m_1 + m_2)v_c^2 + (m_1q_{1f}^2 + m_2q_{2f}^2)$

So that:

$K = (m_1 + m_2)v_c^2/2 + (m_1q_{1i}^2 + m_2q_{2i}^2)/2 = (m_1 + m_2)v_c^2/2 + (m_1q_{1f}^2 + m_2q_{2f}^2)/2$

In Hans Ohanians's textbooks on physics [3]:

- the equation: $(m_1 + m_2)v_c^2/2$
  is defined as the kinetic "energy" associated with the translational motion of the center of mass of the of the bi-particle system. We have hereby demonstrated that the numerical value of this equation remains constant because of the physical conservation of the congruent translational velocity of the bi-particle system.
  I have hereby demonstrated that this congruent velocity v_c and its corresponding kinetic energy are relative characteristics that depend on the chosen reference frame.

- the equation:
  $(m_1.q_{1i}^2 + m_2.q_{2i}^2)/2 = (m_1.q_{1f}^2 + m_2.q_{2f}^2)/2 = [m_1m_2/2(m_1 + m_2)](v_1 - v_2)^2 = \text{constant}$
  is defined as the "internal" kinetic energy that is associated with the translational motions relative to the centre of mass of the bi-particle system. We have hereby demonstrated that this equation remains constant because of the physical conservation of the magnitudes of the internal velocities during elastic collisions.
  I have hereby demonstrated that this internal kinetic energy is an absolute, physical characteristic, that doesn't depend on the chosen reference frame and

2.7 The conservation of the total amount of kinetic energy in elastic collisions

In our strictly logical development we have demonstrated that the final velocities of elastic collisions can be obtained in a natural way:

- on the one hand by the general principle of the invariability of the congruent/external translational motion, which is mathematically expressed by the conservation of the resultant linear momentum:

$$m_1\mathbf{v_{1i}} + m_2\mathbf{v_{2i}} = m_1\mathbf{v_{1f}} + m_2\mathbf{v_{2f}}$$

so that: $m_1 (\mathbf{v_{1i}} - \mathbf{v_{1f}}) = m_2 (\mathbf{v_{2f}} - \mathbf{v_{2i}})$

- on the other hand by the invariability in elastic collisions, of the physical internal translational motion, which is mathematically expressed by the conservation of the velocity gap:

$$\Delta\mathbf{v} = (\mathbf{v_{1i}} - \mathbf{v_{2i}}) = (\mathbf{v_{2f}} - \mathbf{v_{1f}})$$

So that: $(\mathbf{v_{1i}} + \mathbf{v_{1f}}) = (\mathbf{v_{2f}} + \mathbf{v_{2i}})$

The multiplication of both equations gives us:

$$m_1 (\mathbf{v_{1i}} - \mathbf{v_{1f}}).(\mathbf{v_{1i}} + \mathbf{v_{1f}}) = m_2 (\mathbf{v_{2f}} - \mathbf{v_{2i}}).(\mathbf{v_{2f}} + \mathbf{v_{2i}})$$

$$m_1 (\mathbf{v^2_{1i}} - \mathbf{v^2_{1f}}) = m_2 (\mathbf{v^2_{2f}} - \mathbf{v^2_{2i}})$$

so that:

$$m_1.\mathbf{v^2_{1i}} + m_2.\mathbf{v^2_{2i}} = m_1.\mathbf{v^2_{1f}} + m_2.\mathbf{v^2_{2f}} = 2K = Q_{ML} = \text{constant}$$

which demonstrates once more that the conservation of the total amount of momentum flow is the physical basis for the conservation of the kinetic energy in elastic collisions:

$$K = Q_{ML}/2 = \text{constant}$$

3. Elastic collisions in two and three dimensions

The classic calculation of the final velocities of elastic collisions in two (or three) dimensions on the basis of the two conservation laws is made impossible by the fact the equation of the kinetic energy is not a vector equation, so that there is no unique solution,

- in a 2-dimensional collision, for the four, and
- in a 3-dimensional collision, for the six

vector components, and the objects can move in a variety of directions with a variety of speeds [4]. This is caused by the fact that the collisions are not necessarily head-on collisions of point-like billiard balls. This means that if we want to solve these type of collisions, we have to treat them in detail as non head-on collisions between perfect billiard balls. In that case, the outcome of the collision is not only determined by the intersection angle of the velocities of the colliding particles, but also by the relative position of their point of contact when they collide. Taking this into account, these type of collisions can be solved by decomposing the initial velocity vectors of the perfect billiard balls into two vector components. In the case of a 2-dimensional collision, this means:

- a velocity component along the line that connects the two centers of the billiard balls (which we call the x-axis) and,
- a velocity component along a line that is perpendicular to the x-axis and that passes through the contact point between both billiard balls (and which we call the y-axis).

- The velocity components along the x-axis will act as an head on collision and can be treated as a one-dimensional collision between point like particles (see section 2).
- The velocity components along the y-axis will in the case of ideal frictionless billiard balls, remain unchanged by the collision.
- The composition of these two velocity vector components of each billiard ball will automatically give us their final velocities.

In the classic case of billiard balls that have equal masses and where one of the billiard balls is at rest (in the reference plane of the billiard table), the velocity component of the launched billiard ball along the x-axis will be completely transmitted to the billiard ball that was at rest and the velocity component of the launched billiard ball along the y-axis will remain unchanged, so that both billiard balls are perpendicular to one another [5].

4. Conclusion: The conservation of motion in elastic particle systems

In this paper I have demonstrated that the conservation of linear momentum and of kinetic energy in elastic particle systems are both based on the physical conservation of the translational motion in elastic collisions.

- The conservation of linear momentum is a mathematical expression of the physical conservation of the congruent translational velocity ($\mathbf{v_c}$) with which all the individual particles of the particle system move with the same speed in the same direction.
  The conservation of this type of congruent motion is evident, because it is not affected by the internal collisions in the particle system.

- The conservation of the kinetic energy is a mathematical expression of the physical conservation of the isotropic, 'thermal' translational velocities of the individual unit particles, which are evenly distributed over all possible directions so that frequent collisions between the particles will take place:
  - Between collisions, the motion/velocity of each particle and therefore its linear momentum, remains necessarily constant
  - During each elastic collision, the velocity gap between the colliding particles is physically is physically conserved, so that the amount of motion of the colliding particles is physically conserved

From this we can conclude that in perfectly elastic particle systems the internal collisions do not modify the amount of motion or i.e. the momentum flow and that therefore in these systems, the total amount of translational motion as well as the average velocity level of the particles remains constant.

The reason for the fact that this principle of the conservation of motion has not yet been formulated as a general principle is twofold:

- In the reference frame of the observer, the numerical values of the magnitudes of the velocities do not in any way remain constant, and neither is their sum, nor the sum of the magnitudes of their linear momentums, so that it is not obvious at first sight that motion is conserved.

- The concept of 'kinetic energy' as been formulated simultaneously with the concepts of ‘force’ and ‘work’ on the basis of Newton’s second law of motion:

$$W = \int F.ds = \int m.a.ds = \int m.v.dv = m.v^2/2 = K$$

so that it was obvious to use the principle of the conservation of ‘kinetic energy’, and not that of the conservation of congruent and internal motion to calculate the final velocities in elastic collisions.

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