

A proof of Fermat's Last Theorem \\ \\ BY RAMASWAMY KRISHNAN \\ B7/203 VIJAYN PARK THANE INDIA-400615 \\ email:- ramasa421@gmail.com \\ \\ SYNOPSIS:- This proof is based on an assumption that value of an infinite series can not be obtained from finite number of terms of the series. For all possible factors of $(x+y-z)$ which are not factors of x or y or z , three infinite series can be developed, two convergent and one divergent. In all the three cases the value of the infinite series can be obtained by considering only a finite number of terms. \\ This gives the value for $(x+y-z) = \{ p \}^{\alpha} \{ p \}_1 \{ p \}_2 \{ p \}_3$ thus proving Fermat's last theorem. \\ Proof:- \\ If $x^p + y^p = z^p$ where 'p' is a prime number has solution in integers it is well known that integers $\{ p \}_1, \{ p \}_2, \{ p \}_3, \{ q \}_1, \{ q \}_2, \{ q \}_3$ and α exists such that $x = \{ p \}_1 \{ q \}_1$, $y = \{ p \}_2 \{ q \}_2$, $z = \{ p \}_3 \{ q \}_3$ and $z-y = (\{ p \}_1)^p$, $z-x = (\{ p \}_2)^p$, $x+y = (\{ p \}_3)^p$ if p is prime to x, y, z and if p is a divisor of one of them say z then $z = \{ p \}^{\alpha} \{ p \}_3 \{ q \}_3$; $x+y = \{ p \}^{p\alpha-1} (\{ p \}_3)^p$. \\ $x+y-z \equiv 0 \pmod{\{ p \}_1 \{ p \}_2 \{ p \}_3}$ is obvious. It is easy to prove that $\{ q \}_1 \equiv \{ q \}_2 \equiv \{ q \}_3 \equiv 1 \pmod{\{ p \}^2}$ and therefore $x+y-z \equiv 0 \pmod{\{ p \}^3}$ \\ thus $x+y-z \equiv 0 \pmod{\{ p \}^{\alpha} \{ p \}_1 \{ p \}_2 \{ p \}_3}$ and $\alpha \geq 3$. Further HCF of $(x^2 + xy + y^2), (z^2 - zx + x^2), (z^2 - zy + y^2)$ is $\{ p \}_4$ then $x+y-z \equiv 0 \pmod{\{ p \}_4}$ if $p \equiv 5 \pmod{6}$ and $x+y-z \equiv 0 \pmod{\{ p \}_4^2}$ if $p \equiv 1 \pmod{6}$. In addition $x+y-z$ and $\frac{(x+y)^2 - x^2 - y^2}{xy(x+y)(x^2 + xy + y^2)}$ or $\frac{(x+y)^2 - x^2 - y^2}{xy(x+y)(x^2 + xy + y^2)}$ can have a common factor say $\{ p \}_5$. because it can be concluded that $x+y-z = \{ p \}^{\alpha} \{ p \}_1 \{ p \}_2 \{ p \}_3 \{ p \}_4^l \{ p \}_5$

where $l=1$ or 2 . \ \ Let us assume that p is prime to x, y, z . $z \equiv x+y \pmod{p^\alpha}$ --- So we can take $z = (x+y) + k_1 p^\alpha$; $\therefore z^p \equiv (x+y)^p + p(x+y)^{p-1} k_1 p^\alpha \pmod{p^{2\alpha+1}}$ --- (1) \ from (1) $k_1 p^\alpha \equiv \frac{1}{p} \cdot \frac{x^p + y^p - (x+y)^p}{(x+y)^{p-1}} \pmod{p^{2\alpha+1}}$ --- (2) \ this can be continued further $k_2 p^{2\alpha}, k_3 p^{3\alpha}, k_4 p^{4\alpha}, \dots, k_r p^{\beta}$, --- up to ∞ \ Let $Q = \frac{x^p + y^p - (x+y)^p}{p(x+y)^p} \equiv 0 \pmod{p^\alpha}$ and $\not\equiv 0 \pmod{p^{\alpha+1}}$ \ because $x+y-z \equiv 0 \pmod{p^\alpha}$ --- (3) \ $\frac{1}{p} C_r = (1-p)(1-2p)(1-3p)\dots(1-rp+1)$ --- (4) \ then $k_r p^\beta \equiv \frac{1}{p} C_r Q^r (x+y) \pmod{p^{\beta+\alpha}}$ where if $(r+1)! \equiv 0 \pmod{p^\gamma}$ then $\beta + \alpha = \frac{(r+1)\alpha}{\gamma}$ --- (5) \ therefore $z \equiv (x+y) + \sum_{i=1}^r \frac{1}{p} C_i k_i \pmod{p^{\beta+\alpha}}$ --- (6) \ Basically eqn (6) is the expansion of $\frac{z}{x+y} = \frac{1 + \frac{x^p + y^p - (x+y)^p}{(x+y)^p}}{1 + \frac{x^p + y^p - (x+y)^p}{(x+y)^p}}$ \ Now if $z \equiv \{z\}_r \pmod{p^{\beta+\alpha}}$ and where $\{z\}_r < p^{\beta+\alpha}$ the eqn (6) will generate an infinite sequence $\{z\}_1, \{z\}_2, \{z\}_3, \{z\}_4, \dots$ \ For a given x and y if z is irrational Then $\{z\}_1 < \{z\}_2 < \{z\}_3 < \{z\}_4 < \dots$ to ∞ \ But however if z is an integer then $\{z\}_1 < \{z\}_2 < \{z\}_3 < \{z\}_4 < \dots < \{z\}_r = z = \{z\}_{r+1} = \{z\}_{r+2} = \dots$ --- (7) \ If the last term for $\{z\}_r$ $k_r p^\beta < (x+y)$. \ therefore the value of an infinite series can be obtained by using finite number of terms. \ As this is not possible either z is irrational or p is a divisor of x or y or z . \ Similar

series can be developed using z and x and z and y which are given by

$$\frac{x}{z-y} = \frac{1}{1 + \frac{z^p - y^p}{z-y}}$$

$$= \frac{1}{1 + \frac{z^p - y^p}{z-y}} = \frac{1}{1 + \frac{z^p - y^p}{z-y}}$$

and

$$\frac{y}{z-x} = \frac{1}{1 + \frac{z^p - x^p}{z-x}}$$

If $y < x$ then $2x > z$ or $x > (z-x)$

$$\frac{z^p - x^p}{z-x} = \frac{z^p - x^p}{z-x} = (z-x)^{p-1} + (z-x)^{p-2}x + \dots + x^{p-1}$$

$$\frac{x}{z-x} = \frac{1}{(z-x)^{p-1} + (z-x)^{p-2}x + \dots + x^{p-1}}$$

Therefore the terms in the series developed for 'y' will tend to $+\infty$ or $-\infty$. Therefore given z and x with $2x > z$ 'y' has to be irrational or 'p' is a divisor of x, y, z, p_4, p_5 are prime to each other. So if q^r is a factor either p_4 or p_5 and $x + y - z$ then what is applicable to 'p' is also applicable to 'q'. Therefore $r=0$ or $p_4 = 1$ and $p_5 = 1$ hence $(x+y-z) = p_1^{alpha} p_2 p_3$. Therefore $p^{p/alpha - 1} p_3^p - p_1^p - p_2^p = 2p^{alpha} p_1 p_2 p_3 - (8)$. As eqn (8) has no solution Fermat's last theorem stands proved.
