Factoring any Second Order
Homogeneous Linear Ordinary Differenial Equation

The following theorem demonstrates that any Second Order Homogeneous Linear Ordinary Differenial Equation may be factored via two linear differential operators.

Theorem I.1: Any Second Order Homogeneous Linear Ordinary Differenial Equation may be factored via two linear differential operators.

Proof:

From the reduction of order formula:

$$y_1'' + Py_1' + Qy_1 = 0 \Rightarrow y_2'' + Py_2' + Qy_2 = 0 \quad , \quad \left(y_2 = y_1 \int y_1^{-2} e^{-\int P dx} dx \right)$$

Now, under the transformation: $y_1 = e^{\int s dx} \Leftrightarrow s = (\log y_1)'$:

$$y_2 = e^{\int s dx} \int e^{-2\int s dx} e^{\int P dx} dx = e^{\int s dx} \int e^{-\int (2s+P) dx} dx$$

So, let: $g = -s$

$$\Rightarrow y_2 = e^{-\int g dx} \int e^{-\int (-2g+P) dx} dx$$

Define: $h \equiv P - g \Rightarrow P = h + g$

$$\Rightarrow y_2 = e^{-\int g dx} \int e^{-\int (-2g+h+g) dx} dx = e^{-\int g dx} \int e^{\int (g-h) dx} dx$$
$$= e^{-\int g dx} \int e^{\int g dx} \left(e^{-\int h dx} \right) dx$$
$$\Rightarrow y_2 e^{\int g dx} = \int e^{\int g dx} \left(e^{-\int h dx} \right) dx$$
$$\Rightarrow \left(y_2 e^{\int g dx} \right)' = e^{\int g dx} \left(e^{-\int h dx} \right)$$
$$\Rightarrow e^{-\int g dx} \left(y_2 e^{\int g dx} \right)' = e^{-\int h dx}$$
$$\Rightarrow y_2' + g y_2 = e^{-\int h dx}$$
$$\Rightarrow (D+g) y_2 = e^{-\int h dx}$$
$$\Rightarrow (D+h)(D+g) y_2 = (D+h) e^{-\int h dx} = -h e^{-\int h dx} + h e^{-\int h dx} = 0$$
$$= (D+h)(y_2' + g y_2)$$
$$= Dy_2' + D(g y_2) + h y_2' + h g y_2$$
$$= y_2'' + g' y_2 + g y_2' + h y_2' + h g y_2$$
$$= y_2'' + (g+h) y_2' + (g' + hg) y_2$$
$$= y_2'' + P y_2' + (-s' - (P-g)s) y_2$$
$$= y_2'' + P y_2' + (-s' - (P+s)s) y_2$$
$$= y_2'' + P y_2' + (-s' - s^2 - Ps) y_2$$

But:

$$0 = y_1'' + Py_1' + Qy_1 = (s y_1)' + P(s y_1) + Q y_1$$
$$= s' y_1 + s^2 y_1 + P s y_1 + Q y_1$$
$$= (s' + s^2 + Ps + Q) y_1$$
$$\Rightarrow Q = -s' - s^2 - Ps$$
$$\Rightarrow 0 = y_2'' + Py_2' + Qy_2 = (D+h)(D+g) y_2 \quad , \quad \left(P = h + g \quad , \quad Q = g' + hg \right)$$

alternatively written:

$$\Rightarrow 0 = y_2'' + Py_2' + Qy_2 = [D + (P+s)](D-s) y_2 \quad , \quad (Q = -s' - s^2 - Ps)$$

or:

$$\Rightarrow 0 = y_2'' + Py_2' + Qy_2 = \left(P + \left[D + \frac{y_1'}{y_1} \right] \right) \left[D - \frac{y_1'}{y_1} \right] y_2 \quad , \quad \left(Q = -\frac{y_1''}{y_1} - P \frac{y_1'}{y_1} \right)$$

□

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Not with convincing arguments, but with mathematically rigorous proofs.
That's how mathematics is taught, from high school through graduate school.
So, why are articles in peer-reviewed magazines written like articles in ordinary media?
I implore mathematicians, physicists and engineers to stop paying for garbage; and put your money where it ought to go, to where the real work with real results are.

AND THAT'S NOT ALL!!! :

The second order particular solution formula may be used to derive Abel's Wronskian formula.

So since I have extended the particular solution formula to any order; an analog to Abel's Wronskian formula may be, likewise, extended.

BUT THERE'S MORE!!! :

The reduction of order formula extends beyond the second order. Thus, by induction, any Homogeneous Linear Ordinary Differenial Equation of Any Order may be factored via linear differential operator factors.

These conjectures will be proven in future publications (if someone else doesn’t prove them first, which I encourage).

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