On the Evidence of the Number of Colours in Particle Physics

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Abstract

It is commonly believed (and as well reflected in current textbooks in particle physics) that the R ratio in e^+e^- scattering and $\pi^0 \to \gamma\gamma$ decay provide strong evidences of the three colours of the Quantum Chromodynamics group $SU(3)_c$. This is well documented in current literature. However, here we show that with a better understanding of the structure of the electric charge in the Standard Model of particle physics at hand, one rejects the second evidence as given above but continues to accept the first one. Thus $\pi^0 \to \gamma\gamma$ decay is not a proof of three colours anymore. This fact is well known. However unfortunately some kind of inertia has prevented this being taught to the students. As such the textbooks and monographs should be corrected so that more accurate information may be transmitted to the students.

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Quantum Electrodynamics is built around the Abelian group $U(1)_{em}$. Its counterpart for strong interactions is Quantum Chromodynamics built around the non-Abelian group $SU(3)_c$. But why the number 3 for colours here?

It is commonly believed that there are two well known evidences confirming that there are three colours [1,2]. The first one comes from e^+e^- scatteriing data and the other one from neutral pion decay into two photons.

The first one is that the following ratio for quarks without colour is [1-2],

$$R = \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma\ (e^+e^- \rightarrow\ \ \mu^+\mu^-)}\ \sim \sum_q {e_q}^2 \tag{1}$$

where R is the cross-section ratio of $\sigma(e^+e^- \rightarrow\ \ all\ the\ hadrons)$ and $\sigma\ (e^+e^- \rightarrow\ \ \mu^+\mu^-)$, i.e., to a muon pair. This is equal to the sum of the square of all the charges of quarks which may be allowed to exist at the center-of-mass energy available to the system.

At low energies only v, d, s quarks would be needed. Then the above equation gives $\frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{2}{3}$. Note that there is no colour degree of freedom here. Experimentally R in this region however it is not 2/3 as predicted above, but actually ~ 2 [1, p. 216] [2, p. 165] Thus colour shows up as one sees that multiplication by an extra colour factor of 3 would match the experimental number. At higher enough energy (above 3 GeV) of the $c\bar{c}$ threshold, the quarks are u, d, s, c. Thus there is an extra factor of 4/9. Without colour the ratio would be 10/9, again in disagreement with the experimental number, and again an extra factor of 3 from the colour fixes the discrepancy. Similarly for energy above the the b-quark threshold. Thus we are forced to include the number of colours as $N_c = 3$ in the above expression as $N_c \sum_q {e_q}^2$. Thus the corrected expression with colour is:

$$R = \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma\ (e^+e^- \rightarrow\ \ \mu^+\mu^-)}\ \sim N_c(\sum_q {e_q}^2) \tag{2}$$

The second support in favour of $N_c = 3$ is believed to be found in the study of $\pi^0 \rightarrow \gamma\gamma$ decay. The decay rate is [1,2,3]

$$\Gamma\left(\pi^0 \rightarrow \gamma\gamma\right) = \ N_c^2(Q_u^2 - Q_d^2)^2 \frac{\alpha^2 m_{\pi 0}^2}{64\ \pi^3 F_\pi^2} \tag{3}$$

where Q_u and Q_d are the $u-$ and $d-$ quark charges, N_c is the number of colours, m_{π^0} is the neutral pion mass, $\alpha = \frac{e^2}{4\pi}$ and F_π, the pion decay constant $\sim$ 91MeV.

The experimental value of the decay rate is $\sim$ 7.8eV [1,2,3]. Take quark charges as $Q_u = \frac{2}{3}$ and $Q_d = -\frac{1}{3}$. If there were no colours ($N_c = 1$), then from the above formulae one obtains the decay rate of 0.84 eV. This is much too low a value. Thus one is forced to include $N_c = 3$ and then the fit is good. This is taken as a standard proof of the evidence of 3-colours in particle physics and is well documented in the current literature [1-8].

The Standard Model which is the most successful model of particle physics [1,2] has the group structure $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. The subgroup structure $SU(2)_L \otimes U(1)_Y$ (without the QCD part) is the Glashow-Salam-Weinberg electro-weak part. The electric charge of u- and d- quark in the Standard Model is always taken as 2/3 and -1/3 respectively. And this is what was used in the above two expressions in support of three colours. Note that these charges are pure electro-weak part quantity- that is it has no knowledge of colour in it. And thus in both the above evidences of colour, the colour factor $N_c = 3$ always sat outside the correspponding expressions of the charges in the right-hand-side in eqns. (2) and (3). Hence lack of any dependence of colour in the electric charges of quarks and their occurrence outside on the right-hand-side in eqns. (2) and (3), led to the claim of these being evidence of three colours in particle physics.

However, it has been shown that, in line with the full group stucture for arbitary number of colours $SU(N)_c \otimes SU(2)_L \otimes U(1)_Y$ (also for $N_c = 3$ as well), and having the same theoretical structure as the above electro-weak model, and with the same Englert-Brout-Higgs mechanism of spontaneous suymmetry breaking etc., one obtains a more general definition of electric charges [9] as,

$$Q_u = Q_c = Q_t = \frac{1}{2}(1 + \frac{1}{N_c}) \tag{4}$$

$$Q_d = Q_s = Q_b = \frac{1}{2}(-1 + \frac{1}{N_c}) \tag{5}$$

The Standard Model with group structure $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, the electric charge of quarks intrinsically know of the colour degree of freedom. It was also shown that the correct charge does not merely take the static values of 2/3 and -1/3 (i.e., independent of any colour), but can differ from these values for arbitrary number of colours [9].

For $N_C = 3$ this yields the correct empirically determined charges of 2/3 and -1/3, However it is important to realize that there exists a significant colour component in it. It has been shown that these colour-dependent charges, and not the static (colour-independent) 2/3, -1/3 charges, are the correct ones to use for SU(N_c) QCD theory [9]. Note that the conventional Standard Model (SM) with given charges of 2/3 and -1/3 values, the spontaneous symmetry breaking generates masses. While in the new model in addition to the masses the spontaneous symmetry breaking also generates the charges as well [9]. This is a major difference with respect to the conventional Standard Model. As such this new structure should be considered an improvement over the conventional Standard Model and should be taken as an extension of it. To make this distinction clear and to avoid confusion one may call it the Quantized Charge Standard Model (QCSM).

Here we use these new colour dependent charges to see how the results in eqn. (2) and (3) may be affected.

In equation (2) for example, for energy above c-quark threshold, above we found that the external colour included result and with static charges 2/3 for u- and c-quarks and -1/3 for d- and s-quarks, the experimenatally matched result was 10/3. Now with the new colour dependent charges the right-hand-side of eqn.(2) is

$$\begin{aligned} & N_c({Q_u}^2 + {Q_d}^2 + {Q_s}^2 + {Q_c}^2) \\ = N_c & \left[2\left\{\frac{1}{2}\ \left(1+\frac{1}{N_c}\right)\right\}^2 + 2\left\{\frac{1}{2}\ \left(-1+\frac{1}{N_c}\right)\right\}^2 \right]^2 \\ & = N_c(1+\frac{1}{{N_c}^2}) \end{aligned} \tag{6}$$

which for $N_c = 3$ is 10/3. Thus both the static charges 2/3 and -1/3 and the new colour dependent charges, match the experiement for 3 colours. Therefore this may be taken as a genuine evidence for three colours in particle physics.

However as to the expression eqn. (3), which gave evidence of three colours with static charges 2/3 and -1/3 (as shown above), things are quite different when correct colour dependence of electric charge eqns.(4) and (5) are used. The right-handside in eqn. (3) now reads:

$$N_c{}^2(Q_u{}^2 - Q_d{}^2)^2 = N_c{}^2\left[\left\{\frac{1}{2}\left(1+\frac{1}{N_c}\right)\right\}^2 - \left\{\frac{1}{2}\left(-1+\frac{1}{N_c}\right)\right\}^2\right]^2 = 1 \quad (7)$$

And hence overall there is no N_c- dependence left in the decay rate of $\pi^0 \to \gamma\gamma$ and the subsequent result matches the experiment well. So when correct colour dependent electric charges for quarks are taken, the decay rate is actually independent of colour degrees of freedom.

This fact that the above pion decay rate is not a proof of three colours. is there in literaure [10,11,12] and well known. However a look at text books in particle physics does not reflect this truth. And the false statement that the above pion decay is an evidence of colour has become a "folklore" in particle physics and is reflected in current literature. One may pick up any text book or monograph in particle physics. As a sample of this, here we just quote a few well known books ([3-8], [1] p. 182, [2]. p. 165).

The above word **folklore** points to the Relativistic Social Constructionism, the Strong Programme in sociology of science. One is reminded of what Pickering [13] had said, "There is no reason for outsiders to show the present HEP (High Energy Physics) world-view any more respect" (p. 143) as "the world of HEP was socially produced" (p. 406). As scientists we reject this shocking assessment of HEP. But we have to keep our house in order to counter such allegations and debunking of physics.

We have shown that $\pi^0 \to \gamma\gamma$ decay is not a proof of three colours anymore. It is unfortunate that the text books are continuing to teach this falsehood to students. The purpose of this paper is to make scientists and students aware of the above fact so that text-books may be corrected and proper understanding of what colour is, and how nature displays it or does not display it, should be taught to students. This is a serious issue as wrong fundamental definitions are known to hamper progress in science.

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