From Memristor theory:
For the Missing Links in Electrical and Mechanical System

(The first draft)

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Abstract

The memristor is a kind of resistor with memory characteristic, not new fourth circuit element. The relationship between electric charge and magnetic flux linkage represents the resistor. The linear resistor has linear relationship between charge and magnetic flux, and the non-linear resistor has nonlinear relationship between them. The memristor has non-volatile and memory characteristic as well as non-linear characteristic. In addition, from the viewpoint of the mechanical system instead of the electrical system, it can be seen that the relationship between the momentum and the displacement represents the drag coefficient. This implies that there is a new term associated with consumption, not the momentum mv we usually think of. Generally, total energy is expressed as the sum of kinetic energy and potential energy. This may be due to stereotypes, and it has to be corrected because we have not considered the concept of consumption enough. It may be too far ahead, but these ideas may be the key to solving many contradictions arising from relativity, quantum mechanics, and dark energy.

1. What is the memristor?

In 1971, L. Chua proposed the fourth fundamental passive circuit element – so called memristor - that connects electric charge and magnetic flux linkage [1]. This concept begins to get a lot of attention after a group of researchers at HP labs announce the paper entitled 'The missing memristor found' in 2008 [2].

In classical electrical circuit elements, there are three fundamental passive elements – inductor, resistor, and capacitor – except for the ambiguous ones such as diode. These can be related with four circuit variables such as charge, current, voltage, and magnetic flux $(\mathrm{V}, \mathrm{I}, \mathrm{Q}, \Phi)$. Inductor connects magnetic flux and current $(\Phi = \mathrm{LI})$, resistor connects voltage and current $(\mathrm{V} = \mathrm{RI})$, and capacitor connects voltage and charge $(V = Q/C)$. However, Chua proposed the fourth missing fundamental passive element that relates charge and magnetic flux $(\Phi = MQ)$ in 1971. The fourth missing element is called as memristor [1, 2]. A diagram of fundamental circuit variables are represented in figure 1. But is it really correct? Is there really missing element? And what is the memristor?

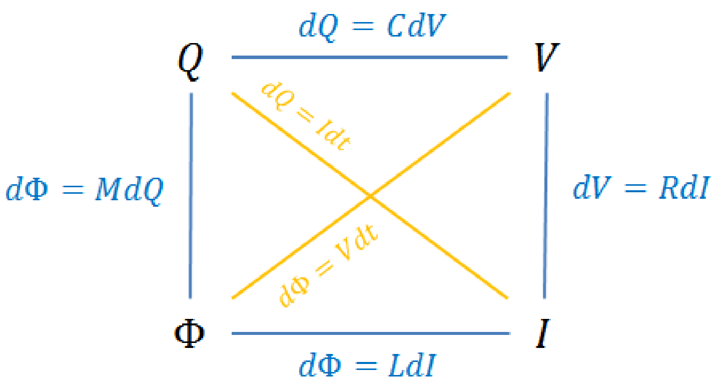

Figure1. The four fundamental passive circuit elements suggested by L. O. Chua: resistor, inductor, capacitor and memristor

In the linear circuit, the coefficient of fundamental passive elements is constant. There are only three elements such as resistance R, inductance L, and capacitance C. If memristance M is actually exists, it must be distinguishable from the others because memristor is the fourth fundamental element according to the memristor's original definition [1, 2]. For example, memristance must be distinct from resistance as a linear element. However, according to their claim, memristance is equivalent with resistance when memristance is constant. In other worlds, the memristor is just resistor when M is constant $(M = R)$. This means that the resistor has the linear relationship between charge and magnetic flux linkage.

$$\Phi = RQ \tag{1}$$

If so, what is the memristor in non-linear circuit? Is the non-linear resistor? What is the difference between memristance and non-linear resistance? In non-linear relationship, memristance may be non-linear resistance. Of course, the memristor remember change of resistance by input signal. But that is the definition of memristor, not memristance. There is no significant difference between memristance and resistance. It is difficult to tell the difference between the two. For example, we can say: "charge-dependent memristance $M(Q)$ is equal to charge-dependent resistance $R(Q)$." What is different? If someone wants to use the term "memristance" while leaving the term "resistance", he needs to clarify it.

Likewise, the difference between impedance and memristance should be explained. If the memristance depends on the frequency of input signal, the memristance may be similar to the impedance Z. The impedance Z is expressed by the following equation.

$$Z = R + jX\ [\Omega] \tag{2}$$

The term j is complex number and the term X is reactance. The impedance is variable resistance according to the frequency of input signal in alternating current (AC). Inductive reactance and capacitive reactance is variable according to the frequency of input signal ($X_L = \omega L,\ X_c = 1/\omega C$). What is the difference between these elements? In alternating current (AC), there is no significant difference between memristance and impedance. (Of course, the reactance of memristor can be different according to the frequency of input signal.)

In short, the problem is that the definition of memristance is not clear enough to distinguish it from other terms. The memristor can be defined as a kind of resistor with memory characteristics, but the memristance is not yet clearly defined. If you can't find a way to express the characteristic of memristance in the form of a formula like the impedance, it means memristance hasn't yet been clearly defined. We can't completely deny the concept of memristance, but we must clarify this point if it is accepted well.

In this paper, I assume that memristance is resistance in DC and impedance in AC. If memristance is equal to resistance, memristance M and resistance R can be expressed by the following equations.

$$M = R = \frac{\mathrm{d}\Phi}{\mathrm{dQ}} \tag{3}$$

Also, Magnetic flux is the time integral of voltage. Assuming that the voltage follows ohm's law when R is constant, the flux is

$$\Phi = \int V dt = R \int I dt = RQ \tag{4}$$

In the variable resistor, the resistance R is variable. When R is variable over time, the voltage can be expressed by the following equation.

$$V = \frac{d\Phi}{dt} = R\frac{dQ}{dt} + Q\frac{dR}{dt} \tag{5}$$

On the other hands, in linear resistor, the resistance R is constant. When R is constant ($dR/dt = 0$), the voltage is expressed by the following equation.

$$V = \frac{d\Phi}{dt} = R\frac{dQ}{dt} = RI \quad (6)$$

This equation is ohm's law. The relationship between charge and magnetic flux means a resistance (or resistor).

In the memristor, the most important thing is memory characteristic that remember the change of memory state by electric stimulus. The 'memristor' is the abbreviation of memory resistor. That is to say, the memristor must remember the changed resistance by electric stimulus. Otherwise, it is not appropriate to call it 'memristor'. The memristor may have at least two or more memory states, and the resistance of memristor can be variable when the voltage is applied above threshold value. This variable characteristic is similar with variable resistor. However, the memristor differs from a typical variable resistor in that it remembers and maintains the changed resistance state even after the power is turned off.

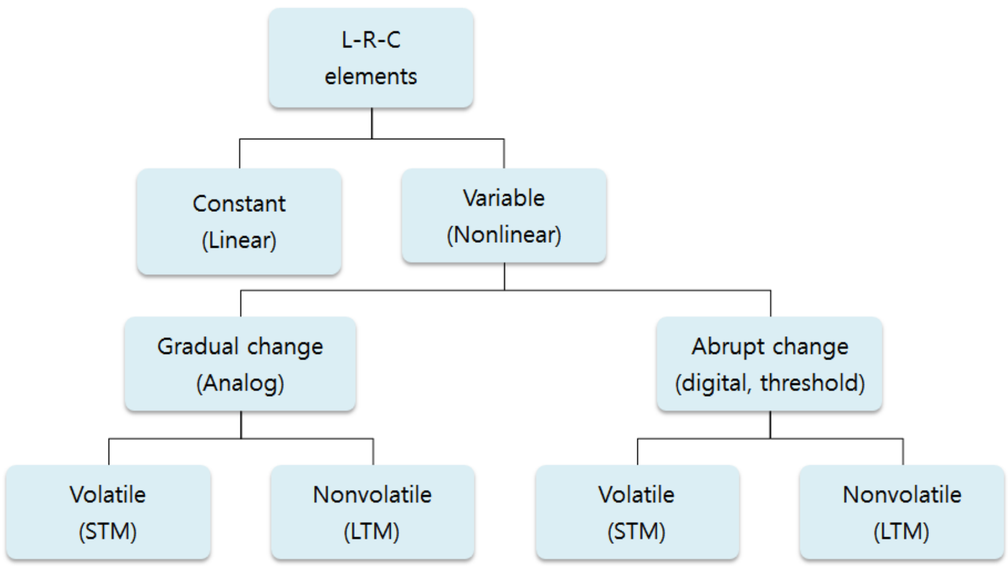

Figure2. The temporary classification of fundamental circuit elements according to their characteristics

For the sake of arguments, the temporary classification of fundamental circuit element is represented in Figure2. (This is arbitrary categorization, so if there is a better way, you can modify it to the form you think is appropriate.) First, LRC elements are classified as linear element with constant coefficient and nonlinear element with variable coefficient. Also, LRC elements can change gradually or abruptly. Gradual change of elements remind of the analog system, while abrupt change of elements remind of the digital system or threshold effect such as switch. In point of view of memory, the device can be divided by volatile memory or nonvolatile memory. The volatile memory means short term memory or simple variable device, whereas the nonvolatile memory means long term memory which last for a long time, or permanently. The nonvolatile memory can be related with the mem-elements or mem-devices, which was proposed by M. Di Ventra, V. Pershin, and L. Chua [3]. The mem-elements, such as memristor, mem-inductor and mem-capacitor, can be defined as circuit elements with memory. The states of these devices are changed by electrical stimulus and remain changed even after the power is turned off. The mem-devices can have hysteretic loop due to their nonvolatile characteristics. However, as I mentioned earlier, it is not appropriate to use new terms, such as memristance, without special reason. If you want to use the new terms such as "mem-inductance", you should be able to define "mem-inductance" in the form of formula like impedance. Otherwise, it is not practical. (But, if we can clearly define mem-elements that are distinct from fundamental circuit elements, we can also create a new term such as "mem-impedance".)

2. Modifying the diagram of fundamental circuit elements

In 2015, Vongher and Meng claim that the resistive switching memory is not the real memristor because the real memristor requires magnetism [4]. Their claims can seem to deny the memristor at first sight. But although they deny HP's claim, this argument is based on the belief that the memristance exist and it is different from the resistance. They seem to believe that there is or will be a "fourth" passive element. At least it seems to clear that they are discussing the "fourth" element connecting charge and magnetic flux. However, there is no guarantee that the thing connecting charge and magnetic flux is the "fourth" element. This may be trivial, but the conclusion may be different.

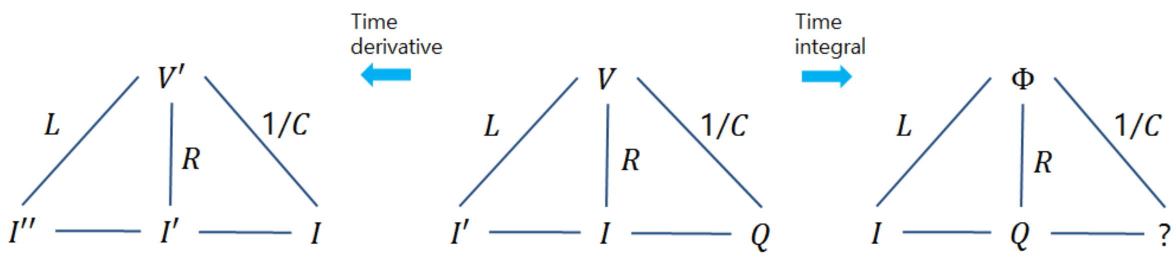

Figure3. The diagrams of fundamental circuit elements

In this paper, I propose the modified diagrams of circuit elements under assumption that memristance is resistance. The diagrams for fundamental circuit elements are represented in Figure3. The fundamental circuit elements are consists of three types that already existing in electrical circuit. There is no new fundamental element. Like the conventional circuits, the inductor connects the voltage and the change of current ($V = LI'$), the resistor connects the voltage and the current ($V = RI$), and the capacitor connects the voltage and the charge ($V = Q/C$).

However, in addition to the diagram for the relationship between voltage and elements, we can add two more diagrams by differentiating or integrating the voltage over time. In the diagrams of Figure3, the voltage prime represents the time derivative of the voltage, and the magnetic flux represents the time integration of the voltage. Also, each link represents the fundamental circuit elements such as inductor, resistor and capacitor. Some of these relationships are what we already know. For example, the capacitor connects the voltage prime and the current. ($V' = I/C$). What does this relationship mean? That means the displacement current in the capacitor which occurs by the electromagnetic wave. The displacement current was first proposed by James Clerk Maxwell in 1800s in order to explain current flow in the capacitor. Generally, the direct current (DC) doesn't flow in capacitor except leakage current. While the alternating current (AC) such as sinusoidal wave or pulse can flow through the capacitor even though the capacitor has very high resistance in DC. But this odd phenomenon doesn't mean new fundamental circuit element. In other worlds, the displacement current is intrinsic property of the capacitor itself. Likewise, the relationship between flux and charge ($\Phi = RQ$) means intrinsic property of the resistor itself, not new fundamental elements such as memristance.

The relationship between flux and charge means the resistance R. If so, what is the physical meaning of charge Q and magnetic flux Φ in the resistor? In linear resistor, the charge Q means the amount of charge that pass through the resistor, and the flux Φ means the amount of dissipated magnetic flux while the charged particle is passing through the resistor. So, the physical meaning in resistor is somewhat different with the inductor. The inductor is related with the generation of magnetic flux when current flows. When the charged particles are moving through the device, the charged particles have always the magnetic flux that proportional to the magnitude of current and inductance. On the other hands, the moving particles continually consume the magnetic flux due to the resistance. The consumed magnetic flux is mainly converted into the form of thermal energy or electromagnetic wave. When the current is zero, the dissipation of magnetic flux must be zero except that resistance is variable because the charge Q in the linear resistor means the amount of charge that pass through the device, not the accumulated charge in the capacitor. The accumulated charge is related with potential rather than dissipation. Considering these things, the magnetic flux by each element can be expressed by the following equations.

$$\Phi_L = LI \quad (7) \qquad \Phi_R = RQ \quad (8) \qquad \Phi_C = \int \frac{Q}{C} dt \quad (9)$$

Ideally, each equation means the magnetic flux in passive device such as inductor, resistor and capacitor, but the physical meaning in each term is slightly different. In the ideal inductor, the magnetic flux is the generated magnetic flux when current flow in the inductor ($\Phi_L = LI$). Unlikely, the flux in ideal resistor means the dissipated flux that proportional to the amount of passed charge ($\Phi_R = RQ$). The magnetic flux in ideal capacitor is the potential magnetic flux by the accumulated charge in the device ($\Phi_C = \int \frac{Q}{C} dt$).

Of course, the ideal devices never exist in real. For example, the real resistor has always some capacitance and inductance although the influence of other elements is negligible. Other devices are also the same. Thus, the total voltage and magnetic flux in real device can be expressed by the following equations.

$$V = V_L + V_R + V_C = LI' + RI + Q/C \quad (10)$$

$$\Phi = \Phi_L + \Phi_R + \Phi_C = LI + RQ + \int \frac{Q}{C} dt \quad (11)$$

Likewise, the time derivative of voltage in real resistor can be expressed by the following equation.

$$V' = V'_L + V'_R + V'_C = LI'' + RI' + I/C \quad (12)$$

These equations are second order differential equations with constant coefficient.

	Passive elements					
	Ideal			Real		
	Inductor	Resistor	Capacitor	Inductor	Resistor	Capacitor
Time derivative	$V' = LI''$	$V' = RI'$	$V' = I/C$	$V' = LI'' + RI' + I/C$		
↕	$V = LI'$	$V = RI$	$V = Q/C$	$V = LI' + RI + Q/C$		
Time integral	$\Phi = LI$	$\Phi = RQ$	$\Phi = \int \frac{Q}{C} dt$	$\Phi = LI + RQ + \int \frac{Q}{C} dt$		

Figure4. The difference of equation between ideal and real devices in the passive elements

Figure4 shows the difference of equation between ideal and real in the fundamental passive circuit elements. In ideal resistor, the inductance and the inverse of capacitance are ignored because those values are close to zero. Unlikely, the real devices have all passive elements regardless of the type of the devices. In fact, whether it is really real or ideal does not matter because the formula depends on the given condition. In order to simplify equations, we can choose the appropriate formula depending on the situation.

3. The mathematical similarity between mechanical system and electrical system, and the derivation of force from ohm's law.

In the mechanical system such as Body/Spring/Oil (BSO) system [4], the displacement x corresponds to the charge Q in electrical system, the velocity v corresponds to the current I, and the acceleration a corresponds to the rate of change of current dI/dt. Similarly, the mass m corresponds to the inductance L in electric circuits, the drag coefficient b corresponds to the resistance R, and the spring constant k corresponds to the inverse of capacitance $1/C$. The elements such as mbk can be regarded as fundamental elements in mechanical system. The force F corresponds to the voltage V. The relationship between two systems is represented in Figure5.

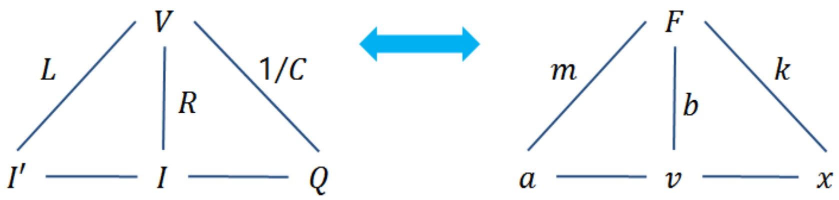

Figure5. The similarity in mechanical and electrical system

Both two systems have the similar differential form. The total force that considers all fundamental elements in mechanical system can be expressed by the following equation.

$$F = ma + bv + kx = mx'' + bx' + kx \tag{13}$$

The total voltage in electrical system with all passive elements can be expressed by the following equation.

$$V = LI' + RI + \frac{Q}{C} = LQ'' + RQ' + \frac{1}{C}Q \tag{14}$$

They are similar mathematically, but also it can be directly derived from the other. For example, the resistive force can be derived from ohm's law under assumption that uniform field is applied. The reverse derivation is also possible.

$$F = -q\mathcal{E} = q\frac{V}{l} = \frac{q}{l}RI = \frac{q}{l}\left(\frac{ml}{\tau nq^2 A}\right)(nqAv) = \frac{m}{\tau}v = bv \tag{15}$$

(where $\mathcal{E}$ is the electric field, l is the lenth of resistor, A is the area of resistor, q is the charge of electron, τ is the time constant called collision or relaxation time, b is the drag coefficient)

$$F = bv \tag{16}$$

In the above equation (13), the term ma means Newton's law ($F_m = ma$), which is the accelerating force that changes the velocity of object. The term bv means Stokes' law ($F_b = bv$), which is the resistive force that maintains the velocity of object against the resistance. The term kx means Hook's law ($F_k = kx$), which is the kind of elastic force by the transformation of object or space.

$F_m = ma$ $(Newton's\ law)$ (17) $F_b = bv$ $(Stokes'law)$ (18) $F_k = kx$ $(Hook's\ law)$ (19)

Of course, these physical laws are only applicable at limited range. For example, Newton's law can't be applied when the speed is close to the speed of light or when there is resistance to space. Other laws of physics also can't be applied beyond a limited range. Stokes' law does not apply when the speed is too fast. Likewise, Hook's law can't be applied when the elastomer is deformed too much. These fundamental laws are applicable only when mbk elements are constant. The mbk elements are fundamental passive elements in mechanical system.

Most conductors with low resistance follow Ohm's law at a limited range of current. When applying voltage, the conductors reach the steady state ($dv/dt = 0$) very quickly because the electrons have very low mass. In the steady state, the electrons moves with average velocity called the terminal drift velocity. $\left(v_d = -\mu\mathcal{E} = -\frac{q\tau}{m}\mathcal{E} = \frac{\tau}{m}F = \frac{F}{b}\right)$. After reaching at the steady state, the resistive term is dominated. Thus, Stokes' law ($F_b = bv_d$) is applicable in conductor with high conductivity. Considering both mass and resistance elements, the equations for the electric force can be expressed by the followings.

$$F_b = bv = \frac{m}{\tau}v = -q\mathcal{E} - ma \tag{20}$$

or

$$F_m = ma = m\frac{dv}{dt} = -q\mathcal{E} - bv \tag{21}$$

$$\rightarrow\ F_m = F_{applied} - F_b \tag{22}$$

Then, rearranging equation, the applied total electric force can be written by the following.

$$F_{applied} = -q\mathcal{E} = F_m + F_b = ma + bv \tag{23}$$

The equation (23) is 1st order differential equation with constant coefficient. In order to simplify the equation, the spring constant is not considered. The solution equation (23) is represented as the following forms.

$$a = \frac{F_{applied}}{m}, \qquad (acceleration\ at\ initial\ state) \tag{24}$$

$$a = \frac{F_{applied}}{m} e^{-\frac{t}{\tau}}, \qquad (acceleration\ at\ transient\ state) \tag{25}$$

$$v = \frac{F_{applied}}{b}\left(1 - e^{-\frac{t}{\tau}}\right), \qquad (velocity\ at\ transient\ state) \tag{26}$$

$$v_d = \frac{F_{applied}}{b}\ , \qquad (velocity\ at\ steady\ state) \tag{27}$$

$$x = \tau v_d\left(\frac{t}{\tau} + e^{-\frac{t}{\tau}} - 1\right), \qquad (displacement) \tag{28}$$

If the applied force is gravitational force (F=mg),

$$a = g, \qquad (acceleration\ at\ initial\ state) \tag{29}$$

$$a = ge^{-\frac{t}{\tau}}, \qquad (acceleration\ at\ transient\ state) \tag{30}$$

$$v = \tau g\left(1 - e^{-\frac{t}{\tau}}\right), \qquad (velocity\ at\ transient\ state) \tag{31}$$

$$v = \tau g = v_d\ , \qquad (velocity\ at\ steady\ state) \tag{32}$$

$$x = \tau v_d\left(\frac{t}{\tau} + e^{-\frac{t}{\tau}} - 1\right), \qquad (displacement) \tag{33}$$

These formulas can be also applied to the falling motion in which the air resistance acts under the assumption that the drag coefficient b is constant.

4. Dissipating elements (Damping elements) – loss of momentum and energy

Mathematically, Ohm's law is similar with Stokes' law. Voltage V corresponds to force F. Resistance R corresponds to drag coefficient b, and current I corresponds to velocity v. Likewise, magnetic flux Φ corresponds to momentum p, and charge Q corresponds to displacement x. Voltage and magnetic flux in resistive term are expressed by following equations.

$$V_R = RI \tag{34}$$

$$\Phi_R = RQ \tag{35}$$

Thus, momentum and force in resistive term can be expressed by following equations.

$$F_b = bv \tag{36}$$

$$p_b = bx \tag{37}$$

The term p_b is the consumption of momentum by resistance. The consumption of momentum p_b is proportional to the displacement x. In other words, the relationship between momentum and displacement represents the drag coefficient b. If b is constant regardless of velocity v or displacement x,

$$b = \frac{dp}{dx} \tag{38}$$

As in the electrical system, the total momentum in mechanical system can be expressed as:

$$p = p_m + p_b + p_k = mv + bx + \int kxdt \tag{39}$$

Assuming that the potential element of momentum is insignificant (when $p_k = 0$),

$$p = p_m + p_b = mv + bx \tag{40}$$

Generally, the momentum p is expressed as mv, but this is incomplete. In the original equation, the resistive term was not sufficiently considered. Until now, the momentum by the resistive term has been neglected because of the stereotypes that momentum is, of course, the product of mass and velocity. The term p_m means the momentum of the object that move at velocity v with mass m. On the other hands, the term p_b means the consumption of momentum by resistance when the object moves displacement x. Considering resistance, each term can be expressed by the follows. Assuming that initial velocity and initial momentum are zero,

$$p_m = mv = mv_d\left(1 - e^{-\frac{t}{\tau}}\right) \tag{41}$$

$$p_b = bx = b\tau v_d\left(\frac{t}{\tau} + e^{-\frac{t}{\tau}} - 1\right) = mv_d\left(\frac{t}{\tau} + e^{-\frac{t}{\tau}} - 1\right) \tag{42}$$

Thus, total amount of momentum is expressed by the following equation. (when $p_k = 0$)

$$p = p_m + p_b = \frac{t}{\tau}mv_d = bv_d t = mgt \tag{43}$$

Likewise, Energy can be considered similarly. The total energy E can be expressed by the following equations.

$$E = K.E + D.E + P.E \tag{44}$$

(Where $K.E$ is kinetic energy, $D.E$ is dissipating energy, $P.E$ is potential energy)

Assuming that potential energy and rest energy are ignored,

$$E = K.E + D.E = \int ma\,dx + \int bv\,dx \tag{45}$$

Kinetic energy and dissipating energy can be expressed by the following equations.

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}mv_d{}^2\left(1 - e^{-\frac{t}{\tau}}\right)^2 \tag{46}$$

$$D.E = mv_d{}^2\left(\frac{t}{\tau} + e^{-\frac{t}{\tau}} - 1\right) - \frac{1}{2}mv_d{}^2\left(1 - e^{-\frac{t}{\tau}}\right)^2 = bv_d x - \frac{1}{2}mv^2 \tag{47}$$

Thus,

$$E = K.E + D.E = bv_d x = mgx \tag{48}$$

We can also see one fact from the above equations.

$$F = bv_d = mg \tag{49}$$

5. One more thing: about total energy ($\mathbf{T.E} = \boldsymbol{\gamma}\mathbf{m}\boldsymbol{c}^2$) of the relativity theory

The sum of kinetic energy and dissipating energy is,

$$E = bv_d x = m{v_d}^2\left(\frac{t}{\tau} + e^{-\frac{t}{\tau}} - 1\right) \quad (50)$$

Assuming that gamma γ is t/τ and drift velocity v_d is c,

$$E = K.E + D.E = (\gamma - 1 + e^{-\gamma})mc^2 = \gamma mc^2 - mc^2 + e^{-\gamma}mc^2 \quad (51)$$

Rearranging the equation about total energy ($\mathrm{T.E} = \gamma \mathrm{m}c^2$) in relativity theory,

$$\mathrm{T.E} = \gamma mc^2 = K.E + D.E + mc^2 - e^{-\gamma}mc^2 \quad (52)$$

However, the form of this equation is slightly different with thing of relativity. In this equation, the gamma γ is zero when time t is zero. So, let's make a different assumption.

if $\gamma = \frac{t}{\tau} + e^{-\frac{t}{\tau}}$,

$$E = K.E + D.E = (\gamma - 1)mc^2 = \gamma mc^2 - mc^2 \quad (53)$$

In this equation, $\gamma = 1$ when $\mathrm{t} = 0$. (When t=0, v=0). Also, it is consistent with the form of the formula in relativity. Then, rearranging the equation,

$$\mathrm{T.E} = \gamma mc^2 = K.E + D.E + mc^2 \quad (54)$$

Generally, the total energy is considered as the sum of kinetic energy and potential energy. However, we didn't consider enough missing elements such as dissipating energy in total energy. Considering all elements such as dissipating energy and rest energy ($\mathrm{R.E}$), the total energy can be expressed by the follows.

$$\mathrm{T.}E = K.E + D.E + P.E + R.E \quad (55)$$

I think that the missing elements such as dissipating energy may be related to dark energy and dark matter that we don't know yet. The dissipating elements may act as a repulsive force against gravity, or it may play a role in promoting expansion of the universe due to the increase of temperature.

Of course, the above equations are slightly different with things of relativity although they are similar. In the above equations, there is assumption that time τ is constant. On the other hand, in the relativity, time τ is variable. Nevertheless, I think they may be key that finding new answer about universe including the relativity.

6. Conclusion

In chapter 1, I mentioned that the most important thing is memory characteristic that remember the change of state by electric stimulus. According to the original definition, the memristor was the fourth fundamental circuit element, forming a non-linear relationship between charge and magnetic flux. However, the memristor is not the "fourth" fundamental element. The linear resistor has linear relationship between charge and magnetic flux, and the non-linear resistor or variable resistor have non-linear relationship between charge and magnetic flux. If so, what is the memristor? The memristor is a kind of (variable) resistor with memory characteristic. The memristor has variable characteristic but also nonvolatile characteristic. The memristor remembers the change of resistance depending on the past electric stimulus. There is no significant difference between resistance and memristance. If you are looking for something else, it is that the state of resistance changes by external stimuli and maintains the changed state.

Nevertheless, from Memristor theory, we can get some hints about new relationships that we are missing between variables in circuit elements. For example, the resistance connects electric charge and magnetic flux. The resistor represents dissipating elements. Furthermore, despite of the departure from the classical perspective, we can also relate to the relativity and the quantum mechanics by considering the missing fundamental elements, because time variable τ is related to probability and statistics as well as the relativity. If we study deeper, a theory that unifies relativity and quantum mechanics may be possible. Of course, my guess might be wrong. But isn't it worth a little thought?

7. *Talking about Inertia and Newton's law

Inertia is the resistance of any physical object to any change in its state of motion. According to Newton's first law of motion, in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a force [5-7]. Is it right? The answer is correct, but it is applied only if there is no resistance. There is no guarantee that the resistance of space is zero. Why? It is because that the resistance of space may be close to zero, rather than the resistance of space being zero. If the laws of nature are applied to a huge scale, not a small scale, the reality may be quite different. Also, the law of inertia has always preconditions. "If there is no friction or resistance" or "If there is no effect of gravity". The laws of inertia and Newton's law are valid, but they are limited to specific condition.

Aristoteles believed that the object would move as long as forces was applied them. Although there may be some errors in his thoughts, it is reasonable from the point of view of resistance. When there is resistance to motion in any space, the object will eventually stop unless the force is applied to the object. This can be expressed as Stokes' law.

Likewise, the similar logic can be applied to Hook's law in spring and elastic bodies. It is static. I think it may be related with potential such as gravitational force and electric force as a source.

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