Review of C_{0101} Weyl tensor as an aid to reviewing the Penrose Weyl Tensor Conjecture near start of Inflation. i.e. that no need to suppose C_{ijkl} is always zero at start of inflation

Andrew Walcott Beckwith

Physics Department, Chongqing University, College of Physics, Chongqing University Huxi Campus, No. 44 Daxuechen Nanlu, Shapinba District, Chongqing 401331, People's Republic of China

Rwill9955b@gmail.com; abeckwith@uh.edu

Abstract

We review, using our prior work the C_{0101} Weyl tensor as an aid to reviewing the Penrose Weyl Tensor Conjecture near start of Inflation. Our supposition is that this C_{0101} Weyl tensor will not vanish, and this is in tandem with a non- zero initial entropy. In doing so, we make use of two representations of the scale factor, one with $\dot{a} \sim aH$, and another in terms of $\dot{a} \sim \gamma t^{\gamma-1}$, with the time, scaled as $t \sim \frac{\Delta t}{t_{Planck}}$. The second representation of the derivative of $\dot{a}$ will yield a C_{0101} not equal to zero due to Δt initial time step not being zero, i.e. a quantization of time, whereas the first representation of $\dot{a} \sim aH$ depends upon the initial H value assumed and picked. We then afterwards, say something as to Malek's doctoral thesis rendition of C_{ijkl} , for n general dimensions as a way to buttress our conclusion, that indeed C_{ijkl} is non-zero at start of inflation provided certain assumptions are made. Which in turn affects entropy production at the start of the universe. Note . in the case of a Pre Planckian imaginary time Δt , i.e. to have C_{0101} real valued, but nonzero, we have to restrict γ = 1, as a way of making sense of what happens if we do use purely imaginary time. If not, then the Weyl tensor could be complex valued. If we have real time for Δt , we have far less restrictive conditions, as is given in our document. i.e. This idea of use of a much larger initial value of the cosmological constant in line with [16] leads to a closed form solution for the C_{0101} **if** we have real time for Δt . We conclude with our buildup to the future works section by the end of our conclusion, which has that a Pre Planckian Δt may be linked to the inverse of the Planckian value of the initial Hubble parameter. With future input from reference [13] of our paper.

Key words: Weyl tensor, Penrose Weyl Tensor conjecture, entropy

I. Introduction

We will initiate our discussion by referencing [1] which has a treatment of higher dimensional cosmology and in particular has a layout of non-zero Weyl Tensor contributions, and we start off with [2]. Which has

$$
\begin{aligned}
R_{\lambda\mu\nu\kappa} &= \frac{1}{(\breve{n}-2)} \cdot \left(g_{\mu\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu} \right) \\
&+ \frac{R}{(\breve{n}-1)(\breve{n}-2)} \cdot \left(g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu} \right) + C_{\lambda\mu\nu\kappa}
\end{aligned}
\tag{1}
$$

If we go to [1], page 28, the first non-zero Weyl tensor we observe can be written as C_{0101} and we will write this out as, if $\breve{n}$ is a formal Space-time dimension.

$$
C_{0101} = R_{0101} + \frac{2R_{01}}{(\breve{n}-2)} - \frac{R}{(\breve{n}-1)(\breve{n}-2)} \tag{2}
$$

Several of the other non-zero Weyl Tensors are [1]

$$
\begin{aligned}
C_{01ij} &= R_{01ij} + \frac{2R_{01}}{(\breve{n}-2)} - \frac{R}{(\breve{n}-1)(\breve{n}-2)} \\
C_{01ij} &= R_{01ij} \\
C_{011i} &= R_{011i} - \frac{2R_{1i}}{(\breve{n}-2)} \\
C_{1i1j} &= R_{1i1j} - \frac{(R_{11}\delta_{ij})}{(\breve{n}-2)} \\
C_{1ijk} &= R_{1ijk} + \frac{(R_{1k}\delta_{ij} - R_{1j}\delta_{ik})}{(\breve{n}-2)}
\end{aligned}
\tag{3}
$$

We will formally analyze Eq. (2) using our Pre Planckian to Planckian transformation, and make some comments about several of Eq. (3), but our main takeaway from this is that Eq. (2) will not vanish for reasons we will discuss, at the start of expansion of the Universe. Keep in mind, this is to go for a re do of a supposition by Penrose, that the Weyl tensor must vanish in the neighborhood of a presumed space-time singularity, as is given in [3,4]

II. Filling in terms for R_{0101}, R_{01}, and R for C_{0101}

We will be taking some values from [5] with our substitutions

$$
\begin{aligned}
R_{00} &= -\frac{3\ddot{a}}{a} \\
R_{11} &= \frac{a\ddot{a} + 2\dot{a}^2 + 2\underline{k}}{1 - \underline{k}r^2} \\
R_{22} &= r^2 \cdot \left(a\ddot{a} + 2\dot{a}^2 + 2\underline{k} \right) \\
R_{33} &= r^2 \cdot \left(a\ddot{a} + 2\dot{a}^2 + 2\underline{k} \right) \cdot \sin^2 \theta \\
R &= \frac{6}{a^2} \cdot \left(a\ddot{a} + \dot{a}^2 + \underline{k} \right)
\end{aligned} \tag{4}
$$

And then we go to the Ricci Curvature Tensor using the following, namely, for a simple metric [2]

$$
\begin{aligned}
g_{00} &= 1 \\
g_{11} &= \frac{-a^2}{1 - \underline{k}r^2} \\
g_{22} &= a^2 r^2 \\
g_{33} &= a^2 r^2 \sin^2 \theta
\end{aligned} \tag{5}
$$

And by [2]

$$
R_{\lambda\mu\nu k} = g_{\lambda\nu} R_{\mu k} - g_{\lambda k} R_{u\nu} - g_{u\nu} R_{\lambda k} + g_{uk} R_{\lambda\nu} - \frac{1}{2} \cdot \left(g_{\lambda\nu} g_{\mu k} - g_{\lambda k} g_{\mu\nu} \right) \cdot R \tag{6}
$$

And by [6],we use

$$
\begin{aligned}
&Either \\
&\dot{a} \sim aH_{early-universe} \\
&or \\
&\dot{a} \sim \gamma t^{\gamma - 1} \\
&\& \\
&t \sim \frac{\Delta t}{t_{Planck}}
\end{aligned} \tag{7}
$$

And in analyzing the above, we will be considering two candidates for $H_{early-universe}$ in the later part of the manuscript. I.e. centered as in [6,7] either a quantum bounce for which $H_{early-universe}$ is zero, or dependent upon zero. The second choice for the time derivative of the scale factor, will contain an extensive discussion of Δt , with one being a very weird case, a case where Δt is purely imaginary, as in [8], whereas the other cases for Δt which is restricted to real values of Δt . In any case, we restrict our attention of Eq. (6) to when we are looking at

$$R_{0101} = \left[g_{\lambda\nu} R_{\mu k} - g_{\lambda k} R_{uv} - g_{uv} R_{\lambda k} + g_{uk} R_{\lambda\nu} - \frac{1}{2} \cdot \left(g_{\lambda\nu} g_{\mu k} - g_{\lambda k} g_{\mu\nu} \right) \cdot R \right]_{\substack{\lambda=\nu=0, \\ \mu=k=1}}$$

$$= g_{00} R_{11} - g_{01} R_{10} - g_{10} R_{01} + g_{11} R_{00} - \frac{1}{2} \cdot \left(g_{00} g_{11} - g_{01} g_{10} \right) \cdot R \qquad (8)$$

$$= g_{00} R_{11} + g_{11} R_{00} - g_{00} g_{11} \cdot \frac{R}{2}$$

So, then we will be examining the behavior of

$$C_{0101} = g_{00} R_{11} + g_{11} R_{00} - g_{00} g_{11} \cdot \frac{R}{2} - \frac{R}{\left(\breve{n}-1\right)\left(\breve{n}-2\right)} \qquad (9)$$

A factor to keep in mind, is that in the case of the Causal barrier construction, as alluded to in [6,7,8] we are making the following substitution, namely for the interior bubble of quantum bo0unce space time, to almost at the causal barrier [9]

$$\begin{aligned} & g_{00}\big|_{\mathrm{Pr}e-Planck} = g_{00} + \delta g_{00} \\ & \& \\ & \delta g_{00} = a_{\min}{}^{2} \phi_{\inf} << 1 \end{aligned} \qquad (10)$$

This for the interior of a causal bounce bubble, with two regimes of space-time delineated by the first in the interior of a bubble of space time, using what is given by [10] as

$$\begin{aligned} & a \approx a_{\min} t^{\gamma} \\ & \Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln\left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\} \\ & \Leftrightarrow V \approx V_0 \cdot \exp\left\{ -\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi(t) \right\} \end{aligned} \qquad (11)$$

The Causal barrier described by

$$\begin{aligned} & g_{tt} \sim \delta g_{tt} \approx a_{\min}^{2} \phi_{initial} << 1 \\ & \xrightarrow[\mathrm{Pr}e-Planck \to Planck]{} \delta g_{tt} \approx a_{\min}^{2} \phi_{Planck} \sim 1 \\ & \Leftrightarrow \left(\frac{R_c\big|_{initial} \sim c \cdot \Delta t}{l_{Planck}} \right) \sim \vartheta(1)\Big|_{Planck} \end{aligned} \qquad (12)$$

The exterior region defined by [6,7,11] by

$$
\begin{aligned}
&g^{00} = -1 \\
&g^{11} = \frac{a^2}{1 - k(Curvature)\cdot r^2} \\
&g^{22} = a^2 \cdot r^2 \\
&g^{33} = a^2 \cdot r^2 \sin^2\theta \\
&R_{00} = -3\frac{\ddot{a}}{a} \\
&R_{ij} = \frac{3}{a^2}\cdot\left(a\cdot\ddot{a} + 2\dot{a}^2 + k(Curvature)\right)\cdot g_{ij}
\end{aligned}
\tag{13}
$$

The main difference between the regimes of Eq.(13) for space-time outside the quantum bounce, and Causal barrier, and inside is in the following sort of very strange situation, namely

$$
\begin{aligned}
&g_{00}\big|_{\text{Pre}-Planck} = (g_{00} = 1) + (\delta g_{00} = a_{\min}{}^2\phi_{\inf} << 1) \\
&\xrightarrow[Past-Causal-barrier]{} (g_{00} = 1)
\end{aligned}
\tag{14}
$$

Keeping this in mind, we will next be examining the behavior of Eq. (9), i.e. while examining the behavior of how the time step, may be either real valued or complex valued, as given by C_{0101} in the interior of the presumed bubble of space-time. In the complex time case we will be examining what is given in the case where the time, below is actually Δt , and this comes from [8] which uses inputs from Padmanabhan,[10] and Eq. (11)

$$
\begin{aligned}
&(t^2)^2 - (1 + m^2(t^2))\cdot\left(\frac{\gamma\cdot(3\gamma-1)\cdot V\left(n-1\dim\right)}{16\pi^2 G^2 V_0 E}\right) = 0 \\
&\& A = \frac{\gamma\cdot(3\gamma-1)V\left(n-1\dim\right)}{16\pi^2 G^2 V_0 E} \\
&\Rightarrow (t^2) = \frac{m^2 A}{2}\cdot\left(1 \pm \sqrt{1 + \frac{1}{m^4 A}}\right) \\
&\&(t^2) \sim -\frac{1}{4m^2}
\end{aligned}
\tag{15}
$$

Where the square of the minimum time, as specified above, is our Δt , provided that m, is a pre universe mass, and we presume then that the energy, E, so specified is probably related to a multiverse generalization of the Penrose Cyclic Conformal cosmology, as given in [12]. If this does not hold, we will probably be content to do the presumably real value iteration of time as given in [6] which will be of the form

$$(\Delta t)^2 \sim \cdot \left(\frac{a_{initial} \cdot \gamma}{12\pi G \cdot \left(1 + 2V_0 \cdot \gamma^2 \cdot \frac{(3\gamma - 1)}{32\pi}\right)} \right) A_1$$

$$A_1 = \frac{\left(1 - a_{initial}{}^2 \cdot \sqrt{\frac{\gamma}{4\pi G}} \cdot \left(\left[\sqrt{\frac{8\pi G \cdot V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right] \Big/ \sqrt{\frac{8\pi G \cdot V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t \right)\right)}{\left(a_{initial}{}^2 / 3\right) + a_{initial}{}^2 \cdot \gamma \cdot (\gamma - 1) + 2 a_{initial}{}^2 \gamma^2} \tag{16}$$

i.e. to solve for Δt would involve a transcendental nonlinear root finder scheme, but this could be matched against an earlier result which was represented in [7] as

$$\Delta t \cdot \left| \left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right) - \frac{\left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right)^2}{2} + \frac{\left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right)^3}{3} - \ldots \right| \tag{17}$$

$$\approx \left(\sqrt{\frac{\gamma}{\pi G}} \right)^{-1} \frac{48\pi\hbar}{a_{\min}^2 \cdot \Lambda}$$

All three of these cases will be discussed, with limiting values assigned, and with consequences of all three of their solutions laid out.

In addition a linkage of the causal barrier idea , to work done by Licata, Manpoor and Corda[13] on Torsion will be explicitly brought up partly as a future works project, and also brought up as a way to interpret C_{0101} as not vanishing even at the start of cosmological expansion.

III. Basic treatment as to imaginary time Δt and its influence as to non-zero values of C_{0101}

First, we shall begin with Δt being imaginary in the Pre Planckian regime. If we have imaginary time, this corresponds to

$$\begin{gathered} \dot{a} \sim \gamma t^{\gamma - 1} \xrightarrow[t \to \pm \frac{i}{2m}]{} \gamma \cdot \left(\pm \frac{i}{2m \cdot t_{Planck}} \right)^{\gamma - 1} \\ \& \\ \dot{a}^2 \sim \gamma^2 t^{2\gamma - 2} \xrightarrow[t \to \pm \frac{i}{2m}]{} \gamma^2 \cdot \left(\pm \frac{i}{2m \cdot t_{Planck}} \right)^{2\gamma - 2} \end{gathered} \tag{18}$$

$$a\ddot{a}+2\dot{a}^2+2\underline{k}\sim\gamma t^{\gamma}+2\gamma^2 t^{2\gamma-2}+2\underline{k}$$
$$\xrightarrow[t\to\pm\frac{i}{2m}]{}\gamma\cdot\left(\pm\frac{i}{2m\cdot t_{Planck}}\right)^{\gamma}+2\gamma^2\cdot\left(\pm\frac{i}{2m\cdot t_{Planck}}\right)^{2\gamma=2}+2\underline{k} \tag{19}$$

We will then eliminate all non-zero terms. We then get a simple expression for the Weyl Tensor, i.e.

$$C_{0101}=R_{0101}-\frac{6\cdot\left(a\ddot{a}+\dot{a}^2+\underline{k}\right)}{a^2\left(\breve{n}-1\right)\left(\breve{n}-2\right)} \tag{20}$$

Then, we obtain

$$C_{0101}=g_{00}R_{11}+g_{11}R_{00}-g_{00}g_{11}\cdot\frac{R}{2}-\frac{R}{\left(\breve{n}-1\right)\left(\breve{n}-2\right)}$$
$$=\left(g_{00}+\delta g_{00}\right)\left(R_{11}-g_{11}\cdot\frac{R}{2}\right)+g_{11}R_{00}-\frac{6\cdot\left(a\dot{a}+2\dot{a}^2+2\underline{k}\right)}{a^2\left(\breve{n}-1\right)\left(\breve{n}-2\right)} \tag{21}$$

Using, $g_{00}+\delta g_{00}=1+a_{\min}{}^2\phi_{\inf}$ and $g_{11}=\frac{-a^2}{1-\underline{k}r^2}$ and $R_{11}=\frac{a\ddot{a}+2\dot{a}^2+2\underline{k}}{1-\underline{k}r^2}$ and also having $R=\frac{6}{a^2}\cdot\left(a\ddot{a}+\dot{a}^2+\underline{k}\right)$ and $R_{00}=-\frac{3\ddot{a}}{a}$

$$\left(R_{11}-g_{11}\cdot\frac{R}{2}\right)=\frac{a\ddot{a}+2\dot{a}^2+2\underline{k}}{1-\underline{k}r^2}+\left(\frac{6\cdot\left(a\ddot{a}+2\dot{a}^2+2\underline{k}\right)}{1-\underline{k}r^2}\right)\sim\left(\frac{7\cdot\left(a\ddot{a}+2\dot{a}^2+2\underline{k}\right)}{1-\underline{k}r^2}\right) \tag{22}$$

$$g_{11}R_{00}=\left(\frac{-a^2}{1-\underline{k}r^2}\right)\cdot\left(-\frac{3\ddot{a}}{a}\right)=\frac{3\ddot{a}a}{1-\underline{k}r^2} \tag{23}$$

Then

$$C_{0101}=g_{00}R_{11}+g_{11}R_{00}-g_{00}g_{11}\cdot\frac{R}{2}-\frac{R}{\left(\breve{n}-1\right)\left(\breve{n}-2\right)}$$
$$=\left(1+a_{\min}{}^2\phi_{\inf}\right)\cdot\left(\frac{a\ddot{a}+2\dot{a}^2+2\underline{k}}{1-\underline{k}r^2}-\frac{3\ddot{a}a}{1-\underline{k}r^2}\right)-\frac{6\ddot{a}a}{\left(1-\underline{k}r^2\right)\cdot\left(\breve{n}-1\right)\left(\breve{n}-2\right)}+\frac{3\ddot{a}a}{1-\underline{k}r^2} \tag{24}$$

If , here n = 3, we have that we can obtain, in the Pre Planckian era, a nonzero C_{0101}

$$
\begin{aligned}
C_{0101} &= g_{00}R_{11} + g_{11}R_{00} - g_{00}g_{11}\cdot\frac{R}{2} - \frac{R}{(\breve{n}-1)(\breve{n}-2)} \\
&= \left(1+a_{\min}{}^{2}\phi_{\inf}\right)\cdot\left(\frac{a\ddot{a}+2\dot{a}^{2}+2\underline{k}}{1-\underline{k}r^{2}} - \frac{3\ddot{a}a}{1-\underline{k}r^{2}}\right) - \frac{6\ddot{a}a}{\left(1-\underline{k}r^{2}\right)\cdot(\breve{n}-1)(\breve{n}-2)} + \frac{3\ddot{a}a}{1-\underline{k}r^{2}} \\
&\xrightarrow[\breve{n}\to 3]{} \left(1+a_{\min}{}^{2}\phi_{\inf}\right)\cdot\left(\frac{2a\ddot{a}+2\dot{a}^{2}+2\underline{k}}{1-\underline{k}r^{2}}\right) \\
&\xrightarrow[t\to\pm\frac{i}{2m}]{} \left(\frac{1+a_{\min}{}^{2}\phi_{\inf}}{1-\underline{k}r^{2}}\right)\cdot\left(\gamma\cdot\left(\pm\frac{i}{2m\cdot t_{Planck}}\right)^{\gamma} + 2\gamma^{2}\cdot\left(\pm\frac{i}{2m\cdot t_{Planck}}\right)^{2\gamma-2} + 2\underline{k}\right)
\end{aligned}
\tag{25}
$$

If we insist, upon, up to the Causal barrier having C_{0101} real valued, this then puts the restriction of γ equal to 1

i.e. this is, to put it mildly, highly restrictive, but doable. i.e. to have C_{0101} real valued, but nonzero, we have to restrict γ = 1, as a way of making sense of what happens if we do use purely imaginary time. If not, then the Weyl tensor could be complex valued.

IV. Removal of imaginary time Δt, i.e. the different cases of real Δt and C_{0101} behavior; i.e. Eq. (17) when the Cosmological constant is very large.

We will be examining what can be done when we look at Eq. (17) when we have an enormous cosmological constant, i.e. $\Lambda_{initial} >> \Lambda_{Today}$ when applied to Eq. (17) leads to, to first order, the following

$$
\begin{aligned}
&\Delta t\cdot\left|\left(\sqrt{\frac{8\pi G V_{0}}{\gamma\cdot(3\gamma-1)}}\cdot\Delta t-1\right) - \frac{\left(\sqrt{\frac{8\pi G V_{0}}{\gamma\cdot(3\gamma-1)}}\cdot\Delta t-1\right)^{2}}{2} + \frac{\left(\sqrt{\frac{8\pi G V_{0}}{\gamma\cdot(3\gamma-1)}}\cdot\Delta t-1\right)^{3}}{3} - \ldots\right| \\
&\approx \left(\sqrt{\frac{\gamma}{\pi G}}\right)^{-1}\frac{48\pi\hbar}{a_{\min}^{2}\cdot\left(\Lambda_{initial} >> \Lambda_{today}\right)} \sim \varepsilon^{+} \\
&\Leftrightarrow \left(\sqrt{\frac{8\pi G V_{0}}{\gamma\cdot(3\gamma-1)}}\cdot\Delta t-1\right) \sim \varepsilon^{+} \\
&\Leftrightarrow \Delta t \sim \sqrt{\frac{\gamma\cdot(3\gamma-1)}{8\pi G V_{0}}} + \varepsilon^{+}
\end{aligned}
\tag{26}
$$

I.e. this assumes Quintessence, i.e. that $\Lambda_{initial} >> \Lambda_{today}$ [14], [15], and this is in line with the situation if the initial cosmological constant is say up to 10^ 122 times larger, than today, which is not outside the values of the Field Theoretic derived cosmological constant, as specified, say in [16]

Then we look at

$$a\ddot{a}+2\dot{a}^{2}+2\underline{k}\sim\gamma t^{\gamma}+2\gamma^{2}t^{2\gamma-2}+2\underline{k}$$
$$\xrightarrow[\Delta t\to\left(\sqrt{\frac{\gamma\cdot(3\gamma-1)}{8\pi GV_0}}+\varepsilon^{+}\right)]{}\gamma\cdot\left(\frac{\left(\sqrt{\frac{\gamma\cdot(3\gamma-1)}{8\pi GV_0}}+\varepsilon^{+}\right)}{t_{Planck}}\right)^{\gamma}+2\gamma^{2}\cdot\left(\frac{\left(\sqrt{\frac{\gamma\cdot(3\gamma-1)}{8\pi GV_0}}+\varepsilon^{+}\right)}{t_{Planck}}\right)^{2\gamma-2}+2\underline{k} \tag{27}$$

&

$$\dot{a}^{2}\sim\gamma^{2}t^{2\gamma-2}\xrightarrow[\Delta t\to\left(\sqrt{\frac{\gamma\cdot(3\gamma-1)}{8\pi GV_0}}+\varepsilon^{+}\right)]{}\gamma^{2}\cdot\left(\frac{\left(\sqrt{\frac{\gamma\cdot(3\gamma-1)}{8\pi GV_0}}+\varepsilon^{+}\right)}{t_{Planck}}\right)^{2\gamma-2}$$

We will relabel this as leading to

$$C_{0101}=g_{00}R_{11}+g_{11}R_{00}-g_{00}g_{11}\cdot\frac{R}{2}-\frac{R}{(\breve{n}-1)(\breve{n}-2)}$$
$$=\left(1+a_{\min}{}^{2}\phi_{\inf}\right)\cdot\left(\frac{a\ddot{a}+2\dot{a}^{2}+2\underline{k}}{1-\underline{k}r^{2}}-\frac{3\ddot{a}a}{1-\underline{k}r^{2}}\right)-\frac{6\ddot{a}a}{\left(1-\underline{k}r^{2}\right)\cdot(\breve{n}-1)(\breve{n}-2)}+\frac{3\ddot{a}a}{1-\underline{k}r^{2}}$$
$$\xrightarrow[\breve{n}\to 3]{}\left(1+a_{\min}{}^{2}\phi_{\inf}\right)\cdot\left(\frac{2a\ddot{a}+2\dot{a}^{2}+2\underline{k}}{1-\underline{k}r^{2}}\right)$$
$$\xrightarrow[\Delta t\to\left(\sqrt{\frac{\gamma\cdot(3\gamma-1)}{8\pi GV_0}}+\varepsilon^{+}\right)]{}\left(\frac{1+a_{\min}{}^{2}\phi_{\inf}}{1-\underline{k}r^{2}}\right)\cdot\Xi$$

&

$$\Xi=\left(\gamma\cdot\left(\frac{\left(\sqrt{\frac{\gamma\cdot(3\gamma-1)}{8\pi GV_0}}+\varepsilon^{+}\right)}{t_{Planck}}\right)^{\gamma}+2\gamma^{2}\cdot\left(\frac{\left(\sqrt{\frac{\gamma\cdot(3\gamma-1)}{8\pi GV_0}}+\varepsilon^{+}\right)}{t_{Planck}}\right)^{2\gamma-2}+2\underline{k}\right) \tag{28}$$

This idea of use of a much larger initial value of the cosmological constant in line with [16] leads to a closed form solution for the C_{0101}

v. Reviewing what happens if the cosmological constant is very small in Eq.(17)

In that case, we have a situation for which we see the following

$$\Delta t \approx \left(\sqrt{\frac{\gamma}{\pi G}}\right)^{-1} \frac{48\pi\hbar}{a_{\min}^{2}\cdot\Lambda} + H.O.T. \qquad (29)$$

I.e. very nonlinear. The term H.O.T. refers to higher order terms, and we will then be using a root finder , with a very, very large term for γ . I.e. close to infinite.

The chance this is going to be acceptable is low. Needless to say, it means we will be looking at a root finder for Eq. (17) [17] [18]

In doing so, we would re write Eq. (28) as

$$\begin{aligned}
C_{0101} &= g_{00}R_{11} + g_{11}R_{00} - g_{00}g_{11}\cdot\frac{R}{2} - \frac{R}{(\breve{n}-1)(\breve{n}-2)} \\
&= \left(1 + a_{\min}{}^{2}\phi_{\inf}\right)\cdot\left(\frac{a\ddot{a} + 2\dot{a}^{2} + 2\underline{k}}{1-\underline{k}r^{2}} - \frac{3\ddot{a}a}{1-\underline{k}r^{2}}\right) - \frac{6\ddot{a}a}{\left(1-\underline{k}r^{2}\right)\cdot(\breve{n}-1)(\breve{n}-2)} + \frac{3\ddot{a}a}{1-\underline{k}r^{2}} \\
&\xrightarrow[\breve{n}\to 3]{} \left(1 + a_{\min}{}^{2}\phi_{\inf}\right)\cdot\left(\frac{2a\ddot{a} + 2\dot{a}^{2} + 2\underline{k}}{1-\underline{k}r^{2}}\right) \\
&\xrightarrow[\Delta t \to \left[\left(\sqrt{\frac{\gamma}{\pi G}}\right)^{-1}\frac{48\pi\hbar}{a_{\min}^{2}\cdot\Lambda} + H.O.T.\right]]{} \left(\frac{1 + a_{\min}{}^{2}\phi_{\inf}}{1-\underline{k}r^{2}}\right)\cdot\tilde{\Xi}
\end{aligned}$$

&

$$\tilde{\Xi} = \left(\gamma\cdot\left(\frac{\left[\left(\sqrt{\frac{\gamma}{\pi G}}\right)^{-1}\frac{48\pi\hbar}{a_{\min}^{2}\cdot\Lambda} + H.O.T.\right]}{t_{Planck}}\right)^{\gamma} + 2\gamma^{2}\cdot\left(\frac{\left[\left(\sqrt{\frac{\gamma}{\pi G}}\right)^{-1}\frac{48\pi\hbar}{a_{\min}^{2}\cdot\Lambda} + H.O.T.\right]}{t_{Planck}}\right)^{2\gamma-2} + 2\underline{k}\right) \qquad (30)$$

VI. What can be said about Torsion in this situation? See Fabbri [19], and [20]. I.e. Torsion in this setting is linked to Eq. (30) of our document. No Quinessence (change in cosmological constant) over time.

As indicated in [19] and [20] , quote

The emerging picture is that both cosmological constant and mass generation are due to symmetry breaking through a dynamical scalar whose vacuum depends on the fermion density: within fermions the scalar has non-trivial vacuum and dynamical breakdown of symmetry occurs, but neither cosmological constant nor masses would appear without fermionic distributions

End of quote

I.e. symmetry breaking is the idea which is indicated, in this situation, in [19] on page 52, there is a mechanism for generation of both the cosmological constant, and mass. Also seen in Formula (8)of [20], but in truth, what is referenced here is a symmetry breaking which we do think may be akin to an analysis of what happens if we examine our Eq. (29) which has an explicit value of the cosmological constant, Λ, the time step Δt , which then is linked, in part to Eq. (30)

I.e. we should keep in mind that Eq.(30) is a highly symbolic representation for what would be, in fact, a nonlinear equation which would be, if nonzero (as we would expect) reaffirmation of what was brought up by both (19], and [20] about a deep scaling between the removal of singularities, via torsion, and a partial refutation of the Penrose conjecture, [4]

Note, if the equation to consider for a non-linear Weyl tensor is Eq.(28), the direct linkage to a Cosmological constant is lost. i.e. for our methodology to be consistent with respect to Torsion [21] , we need to have the cosmological constant to remain invariant. If we have quintessence, it appears that Torsion no longer applies.

VII. Conclusion: Referral to the [13] reference. I.e. we refer to Torsion as linked to Eq. [30]. Other consequences?

From [13] we have the following quote, namely from the conclusion, we cite here that

. In fact, the Rastall theory seems to be in agreement with observational data on the Universe age and also on the Hubble parameter [26]. (note the reference [26] here is for what is part of our reference [13])

End of quote

Aside from the use of Torsion which we have already covered, and which is also a consequence of [21] , we should also make reference as to [6, 7, 22]

$$a_{\min} \sim \alpha_0 \cdot \left(\frac{\alpha_0}{2\tilde{\lambda}} \cdot \left(\sqrt{\alpha_0{}^2 + 32\pi\mu_0\omega \cdot B_0^2} - \alpha_0 \right) \right)^{1/4}$$
$$\alpha_0 = \sqrt{\frac{4\pi G}{3\mu_0 c^2}} B_0 \qquad (31)$$
$$\tilde{\lambda} = \frac{\Lambda_{Einstein} c^2}{3}$$

i.e. the Magnetic field, B, is covered in [6,7] in terms of a current, which we derive, but that derivation is also linkable to an earlier result. This should be compared to an earlier relationship given by Beckwith at [6,7] which has, if $a_{\min} \sim 10^{-55} \sim a_{bounce}$

$$a_{bounce} \sim \Delta t \cdot \sqrt{\frac{12\pi G \cdot k(curvature)}{\gamma}} \cdot \sqrt{1 + 2V_0 \cdot \gamma^2 \cdot \frac{(3\gamma - 1)}{32\pi}} \qquad (32)$$

The relevance to Torsion, and a choice of Δt is then also interconnected with our choice of an appropriate linkage between Δt and the Hubble parameter, as seen by H (initial) which may be due to inflation, enormous. i.e. what we will be examining will be, as subsequently modified by considerations in [13] a phenomenological linkage we will explore which we will be calling

$$\Delta t \sim 1 / H(Hubble)$$
$$\& H(Hubble) w.r.t.reference[13] \qquad (33)$$

i.e. strange as it may seem, a Pre Planckian Δt may be linked to the inverse of the Planckian value of the initial Hubble parameter.

This supposition will be explored in future articles, and in conjunction with [13], plus [19],[20], [21]

VIII. **Acknowledgements**

This work is supported in part by National Nature Science Foundation of China grant No. 1137527

Bibliography

[1] Málek , T. , Ph.D. thesis, defended under the original title "General Relativity in Higher Dimensions"; https://arxiv.org/abs/1204.0291

[2] Weinberg, S. " Gravitation and Cosmology, Principles, and Applications of the General Theory of Relativity" , John Wiley, and Sons, 1972, New York, New York, USA

[3] Gron, O. and Hervik, S."The Weyl Curvature Conjecture", https://arxiv.org/pdf/gr-qc/0205026.pdf

[4] Penrose , R. (1979). "Singularities and Time-Asymmetry". In S. W. Hawking; W. Israel. *General Relativity: An Einstein Centenary Survey*. Cambridge University Press. pp. 581–638.

[5] Carroll, S. https://ned.ipac.caltech.edu/level5/March01/Carroll3/Carroll8.html

[6] Beckwith, A., "How to Determine a Jump in Energy Prior to a Causal Barrier, with an Attendant Current, for an Effective Initial Magnetic Field. in the Pre Planckian to Planckian Space-Time", http://vixra.org/abs/1707.0250 , Accepted for publication by JHEPGC, with Christian Corda, as head editor, July 2017

[7] Beckwith, A., "How to Determine Initial Starting Time Step with an Initial Hubble Parameter H = 0 After Formation of Causal Structure Leading to Investigation of the Penrose Weyl Tensor Conjecture", http://vixra.org/abs/1706.0110 , Accepted for publication by JHEPGC, with Christian Corda as head editor, July 2017

[8] Beckwith, A., Setting Time as Purely Imaginary in Regime of Space-Time Before Causal Barrier and Planckian Space-Time", http://vixra.org/abs/1708.0064

[9] Beckwith, A. (2016) Gedanken Experiment for Refining the Unruh Metric Tensor Uncertainty Principle via Schwarzschild Geometry and Planckian Space-Time with Initial Nonzero Entropy and Applying the Riemannian-Penrose Inequality and Initial Kinetic Energy for a Lower Bound to Graviton Mass (Massive Gravity). *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 106-124. doi: 10.4236/jhepgc.2016.21012.

[10] Padmanabhan, T. "An Invitation to Astrophysics" World Scientific Series in Astronomy and Astrophysics: Volume 8, 2006, Singapore, Republic of Singapore,

[11] Huang, K. " A Superfluid Universe", World Scientific, Singapore, Republic of Singapore, 2017

[12] Beckwith, A.W." Analyzing Black Hole Super-Radiance Emission of Particles/Energy from a Black Hole as a Gedankenexperiment to Get Bounds on the Mass of a Graviton" ,Hindawi Publishing Corporation Advances in High Energy Physics Volume 2014, Article ID 230713, 7 pages http://dx.doi.org/10.1155/2014/230713

[13] Licata, I. H. Moradpour, H., Corda, C. ,"The commutator algebra of covariant derivative as general framework for extended gravity. The Rastall theory case and the role of the torsion",

https://arxiv.org/abs/1706.06863

[14] *Zeng, Xiao-Xiong; Chen, De-You; Li, Li-Fang (2014). "Holographic thermalization and gravitational collapse in the spacetime dominated by quintessence dark energy". Physical Review D.* ***91*** *(4): 046005. Bibcode:2015PhRvD..91d6005Z. arXiv:1408.6632. doi:10.1103/PhysRevD.91.046005.*

[15] *Hu, Wayne (2005). "Crossing the phantom divide: Dark energy internal degrees of freedom". Physical Review D.* ***71*** *(4): 047301. Bibcode:2005PhRvD..71d7301H. arXiv:astro-ph/0410680 . doi:10.1103/PhysRevD.71.047301.*

[16] Peskins, M. E. and Schroeder, D. "An introduction to Quantum Field Theory, Perseus books, Menlo Park, California, 1995, USA

[17] http://math.niu.edu/~dattab/MATH435.2013/ROOT_FINDING

[18] http://www.karenkopecky.net/Teaching/eco613614/Notes_RootFindingMethods.pdf

[19] Fabbri, L. , https://lpsc-indico.in2p3.fr/Indico/event/1474/material/slides/0.pdf

[20] Fabbri, L, "A simple assessment on the hierarchy problem", https://arxiv.org/pdf/1504.04191.pdf

[21] Penrose, R., Foundations of Physics March 1983, Volume 13, Issue 3, pp 325–339

[22] Camara, C.S., de Garcia Maia, M.R., Carvalho, J.C. and Lima, J.A.S. (2004) Nonsingular FRW Cosmology and Non Linear Dynamics. Arxiv astro-ph/0402311 Version 1, Feb 12, 2004