Traveling Salesman Problem Solved with Zero Error
Data Ordering and Route Construction Approach

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

The traveling salesman can determine by hand, with zero or negligible error, the shortest route from home base city to visit once, each of three cities, 10 cities, 20 cities, 100 cities, or 1000 cities, and return to the home base city. The general approach to solving the different types of NP problems is the same, except that sometimes, specific techniques may differ from each other according to the process involved in the problem. In the salesman problem, the first step is to arrange the data in the problem in increasing order, since one's interest is in the shortest distances. The main principle here is that the shortest route is the sum of the shortest distances such that the salesman visits each city once and returns to the starting city. The approach in this paper is different from the author's previous approach (viXra:1505.0167) in which the needed distances not among the least ten distances were added to the least ten distances before route construction began. In this paper, one starts with only the least ten distances and only if a needed distance is not among the set of the least ten distances, would one consider distances greater than those in the set of the ten least distances. The shortest route to visit nine cities and return to the starting city was found in this paper. It was also found out that even though the length of the shortest route is unique, the sequence of the cities involved is not unique. The approach used in this paper can be applied in workforce project management and hiring, as well as in a country's workforce needs and immigration quota determination. Since an approach that solves one of these problems can also solve other NP problems, and the traveling salesman problem has been solved, all NP problems can be solved, provided that one has an open mind and continues to think. If all NP problems can be solved, then all NP problems are P problems and therefore, P is equal to NP. The CMI Millennium Prize requirements have been satisfied.

Preliminaries

Given: The distances between each pair of cities.

Required : To find the shortest route to visit each of the cities once and return to the starting city. It is assumed that there is a direct route between each pair of cities.

Note

1. Number of distances required to travel to each city once and return equals the number of cities involved in the problem.

2 The symbol $C_{1,2}$ can mean the distance from City 1 to City 2.

The distance $C_{1,2}$ = the distance $C_{2,1}$.

Used as a sentence, $C_{1,2}$ can mean, from City 1, one visits City 2.

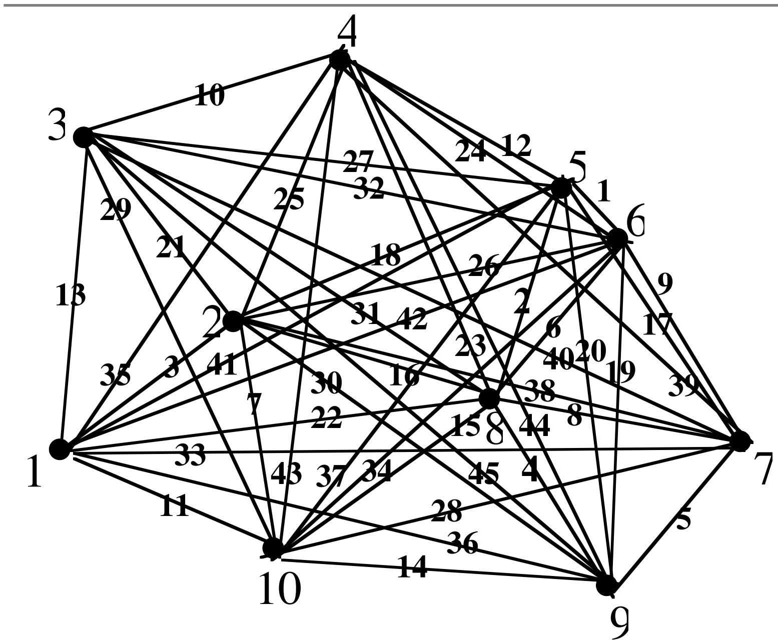
3. C_1 is the home base (starting city) of the traveling salesman.

4. $C_{1,2}(3)$ shows that the numerical value of $C_{1,2}$ is 3.

Determining the Shortest Route

Example From City 1, a traveling salesman would like to visit once each of **nine** other cities, namely, City 2, City 3, City 4, City 5, City 6, City 7, City 8, City 9, City 10; and return to City 1. Determine the shortest route**.**

As it was in the author's previous solutions of NP problems, the first step is to arrange the distances in this problem in increasing order. The main principle in this paper is that the shortest route is the minimum sum of the shortest distances such that the salesman visits each city once and returns to the starting city.



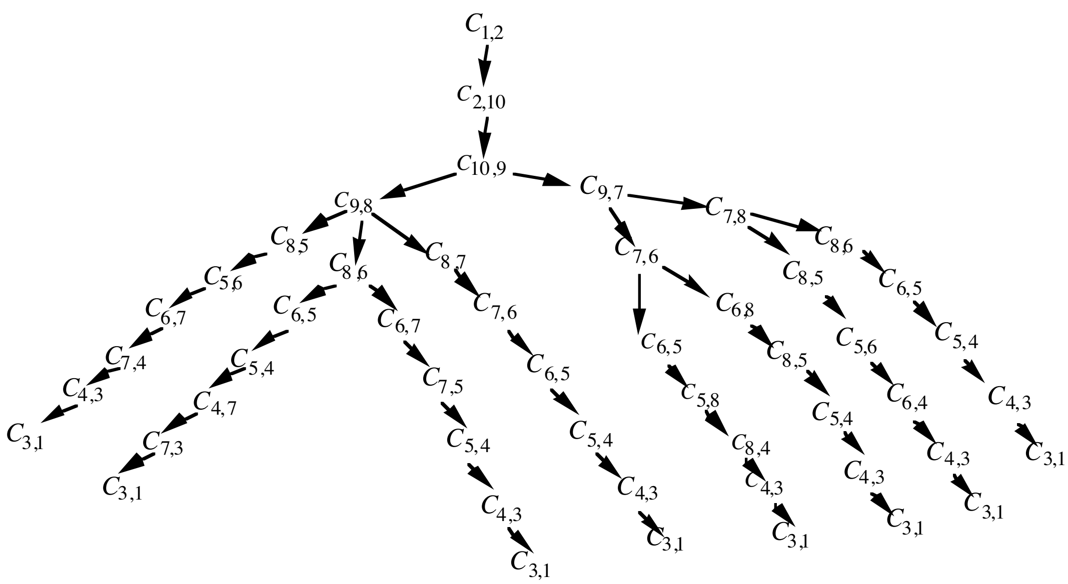
Distances Between Each Pair of Cities

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
$C_{1,2}$ **3**	$C_{2,3}$ **21**	$C_{3,4}$ **10**	$C_{4,5}$ **12**	$C_{5,6}$ **1**	$C_{6,7}$ **9**	$C_{7,8}$ **8**	$C_{8,9}$ **4**	$C_{9,10}$ **14**
$C_{1,3}$ **13**	$C_{2,4}$ **25**	$C_{3,5}$ **27**	$C_{4,6}$ **24**	$C_{5,7}$ **17**	$C_{6,8}$ **6**	$C_{7,9}$ **5**	$C_{8,10}$ **15**	
$C_{1,4}$ **35**	$C_{2,5}$ **18**	$C_{3,6}$ **32**	$C_{4,7}$ **39**	$C_{5,8}$ **2**	$C_{6,9}$ **19**	$C_{7,10}$ **28**		
$C_{1,5}$ **41**	$C_{2,6}$ **26**	$C_{3,7}$ **40**	$C_{4,8}$ **23**	$C_{5,9}$ **20**	$C_{6,10}$ **34**			
$C_{1,6}$ **42**	$C_{2,7}$ **38**	$C_{3,8}$ **31**	$C_{4,9}$ **44**	$C_{5,10}$ **37**				
$C_{1,7}$ **33**	$C_{2,8}$ **16**	$C_{3,9}$ **45**	$C_{4,10}$ **43**					
$C_{1,8}$ **22**	$C_{2,9}$ **30**	$C_{3,10}$ **29**						
$C_{1,9}$ **36**	$C_{2,10}$ **7**							
$C_{1,10}$ **11**								

Step A: Arrange the numerical values of the distances in increasing order

$C_{5,6}$ **1**	$C_{5,8}$ **2**	$C_{1,2}$ **3**	$C_{8,9}$ **4**	$C_{7,9}$ **5**	$C_{6,8}$ **6**	$C_{2,10}$ **7**	$C_{7,8}$ **8**	$C_{6,7}$ **9**
$C_{3,4}$ **10**	$C_{1,10}$ **11**	$C_{4,5}$ **12**	$C_{1,3}$ **13**	$C_{9,10}$ **14**	$C_{8,10}$ **15**	$C_{2,8}$ **16**	$C_{5,7}$ **17**	$C_{2,5}$ **18**
$C_{6,9}$ **19**	$C_{5,9}$ **20**	$C_{2,3}$ **21**	$C_{1,8}$ **22**	$C_{4,8}$ **23**	$C_{4,6}$ **24**	$C_{2,4}$ **25**	$C_{2,6}$ **26**	$C_{3,5}$ **27**
$C_{7,10}$ **28**	$C_{3,10}$ **29**	$C_{2,9}$ **30**	$C_{3,8}$ **31**	$C_{3,6}$ **32**	$C_{1,7}$ **33**	$C_{6,10}$ **34**	$C_{1,4}$ **35**	$C_{1,9}$ **36**
$C_{5,10}$ **37**	$C_{2,7}$ **38**	$C_{4,7}$ **39**	$C_{3,7}$ **40**	$C_{1,5}$ **41**	$C_{1,6}$ **42**	$C_{4,10}$ **43**	$C_{4,9}$ **44**	$C_{3,9}$ **45**

Tree Diagram for the Route Construction



Possible Routes

R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**
$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**
$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**
$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**
$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**
$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{8,6}$ **6**	$C_{8,7}$ **8**	$C_{7,8}$ **8**	$C_{7,8}$ **8**	$C_{7,6}$ **9**	$C_{7,6}$ **9**
$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{6,7}$ **9**	$C_{7,6}$ **9**	$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{6,5}$ **1**	$C_{6,8}$ **6**
$C_{6,7}$ **9**	$C_{5,4}$ **12**	$C_{7,5}$ **17**	$C_{6,5}$ **1**	$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{5,8}$ **2**	$C_{8,5}$ **2**
$C_{7,4}$ **39**	$C_{4,7}$ **39**	$C_{5,4}$ **12**	$C_{5,4}$ **12**	$C_{6,4}$ **24**	$C_{5,4}$ **12**	$C_{8,4}$ **23**	$C_{5,4}$ **12**
$C_{4,3}$ **10**	$C_{7,3}$ **40**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**
$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**
102	**139**	**95**	**81**	**87**	**79**	**87**	**81**

Since there are ten cities, ten distances are needed for the salesman to visit each of nine cities once and return to City 1. For the departure from City 1, the first subscript of the distance from City 1 is 1, and for the return to City 1, the second subscript of the last distance is 1. One will select ten distances, one at a time, to obtain ten well-connected distances to allow the salesman to visit each city once and return to City 1. Ideally, if one were able to use only the least ten distances for the route construction, one would have surely, constructed the shortest route, since, numerically, one would have found the sum of the least ten distances.

A $C_{5,6}$ or $C_{6,5}$ **1**	$C_{4,5}$ or $C_{5,4}$ **12**	$C_{4,8}$ or $C_{8,4}$ **23**	$C_{6,10}$ or $C_{10,6}$ **34**
B $C_{5,8}$ or $C_{8,5}$ **2**	$C_{1,3}$ or $C_{3,1}$ **13**	$C_{4,6}$ or $C_{6,4}$ **24**	$C_{1,4}$ or $C_{4,1}$ **35**
C $C_{1,2}$ or $C_{2,1}$ **3**	$C_{9,10}$ or $C_{10,9}$ **14**	$C_{2,4}$ or $C_{4,2}$ **25**	$C_{1,9}$ or $C_{9,1}$ **36**
D $C_{8,9}$ or $C_{9,8}$ **4**	$C_{8,10}$ or $C_{10,8}$ **15**	$C_{2,6}$ or $C_{6,2}$ **26**	$C_{5,10}$ or $C_{10,5}$ **37**
E $C_{7,9}$ or $C_{9,7}$ **5**	$C_{2,8}$ or $C_{8,2}$ **16**	$C_{3,5}$ or $C_{5,3}$ **27**	$C_{2,7}$ or $C_{7,2}$ **38**
F $C_{6,8}$ or $C_{8,6}$ **6**	$C_{5,7}$ or $C_{7,5}$ **17**	$C_{7,10}$ or $C_{10,7}$ **28**	$C_{4,7}$ or $C_{7,4}$ **39**
G $C_{2,10}$ or $C_{10,2}$ **7**	$C_{2,5}$ or $C_{5,2}$ **18**	$C_{3,10}$ or $C_{10,3}$ **29**	$C_{3,7}$ or $C_{7,3}$ **40**
H $C_{7,8}$ or $C_{8,7}$ **8**	$C_{6,9}$ or $C_{9,6}$ **19**	$C_{2,9}$ or $C_{9,2}$ **30**	$C_{1,5}$ or $C_{5,1}$ **41**
I $C_{6,7}$ or $C_{7,6}$ **9**	$C_{5,9}$ or $C_{9,5}$ **20**	$C_{3,8}$ or $C_{8,3}$ **31**	$C_{1,6}$ or $C_{6,1}$ **42**
J $C_{3,4}$ or $C_{4,3}$ **10**	$C_{2,3}$ or $C_{3,2}$ **21**	$C_{3,6}$ or $C_{6,3}$ **32**	$C_{4,10}$ or $C_{10,4}$ **43**
$C_{1,10}$ or $C_{10,1}$ **11**	$C_{1,8}$ or $C_{8,1}$ **22**	$C_{1,7}$ or $C_{7,1}$ **33**	$C_{4,9}$ or $C_{9,4}$ **44**
			$C_{3,9}$ or $C_{9,3}$ **45**

One will always begin and concentrate on the distances in the box with thicker borders, and one will call this box the Royal box. If necessary, one will move up to the distances outside the Royal box to select distances, one distance at a time, with higher numbers. The approach here is different from the author's previous approach (viXra:1505.0167) in which the needed distances not among the least ten distances were added to the least ten distances before route construction began. Here, one starts with only the least ten distances and only if a needed distance is not among the set of the least ten distances, would one consider distances greater than those in the set of the least ten distances.

Solution Use the above tree diagram and the possible routes to follow the solution steps. One will always begin the selection of the distances in the Royal box. In the royal box ,$C_{1,2}$ (in box C) is the only distance with subscript 1, and it will be the starting (departure) distance

Step 1: Begin with first city distance $C_{1,2}$ (from box C, above).

Note: $C_{1,2}$ means distance from City 1 to City 2. (From City 1, salesman visits City 2.)

Step: 2: Since the second subscript of $C_{1,2}$ is 2, the first subscript of the next distance will be 2. Inspect the boxes in the Royal box to pick a distance whose first subscript is 2. Box G contains a distance with 2 as a first subscript. We choose the distance $C_{2,10}$ in box G. Connect the chosen distance with the distance in Step 1 to obtain the connected distances $C_{1,2}-C_{2,10}$, shown vertically in the tree diagram and as the first two rows of column R1 of the possible routes.

Step 3: Since the second subscript of the last distance is 10, the first subscript of the next distance should be 10. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2), except that the first subscript of the next distance should be 10. Inspection of the entries in the Royal box indicates that there is no distance whose first subscript is 10. One will go outside the Royal box for the next applicable distance. One chooses $c_{10,9}$ (Note that $C_{10,1}$ is excluded). .Never skip the nearest applicable distance. The excluded subscript numbers , except 1, represent the cities already visited.

Step 4: Since the second subscript of the last distance is 9, the first subscript of the next distance should be 9. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10), except that the first subscript of the next distance should be 9. Inspection of the Royal box shows that there are two distances, namely, $C_{9,8}$ and $C_{9,7}$ with 9 as first subscript. This situation implies that there are two branches from the last distance as in the tree diagram.

Step 5: One will work on distance $C_{9,8}$ followed by $C_{9,7}$.

For $C_{9,8}$	For $C_{9,7}$
Since the second subscript of this distance is 8, the first subscript of the next distance should be 8. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2,10, 9) except that the first subscript of the next distance should be 8, Inspection of the Royal box shows that there are three distances namely, $C_{8,5}$,$C_{8,6}$ and $C_{8,7}$, producing three branches from $C_{9,8}$.	Since the second subscript of this distance is 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2,10, 9) except that the first subscript of the next distance should be 7, Inspection of the Royal box shows that there are two applicable distances namely, $C_{7,6}$ and $C_{7,8}$,producing two tree branches from $C_{9,7}$

Step 6:

For $C_{9,8}$			For $C_{9,7}$	
$C_{8,5}$ Since the second subscript of this distance is 5, the first subscript of the next distance should be 5. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9, 8) except that the first subscript of the next distance should be 5. Inspection of the Royal box shows that there is only one applicable distance namely, $C_{5,6}$ from box A.	$C_{8,6}$ Since the second subscript of this distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 8) except that the first subscript of the next distance should be 6. Inspection of the Royal box shows that there are two applicable distances namely, $C_{6,5}$ and $C_{6,7}$ from boxes A and I respectively.	$C_{8,7}$ Since the second subscript of this distance is 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 8) except that the first subscript of the next distance should be 7. Inspection of the Royal box shows that there is only one applicable distance namely, $C_{7,6}$ from box **I**.	$C_{7,6}$ Since the second subscript of this distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 7) except that the first subscript of the next distance should be 6. Inspection of the Royal box shows that there are two applicable distances namely, $C_{6,8}$ and $C_{6,5}$ from boxes F and A respectively.	$C_{7,8}$ Since the second subscript of this distance is 6, the first subscript of the next distance should be 8. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 7) except that the first subscript of the next distance should be 8. Inspection of the Royal box shows that there are two applicable distances namely, $C_{8,5}$ and $C_{8,6}$ from boxes B and F respectively.

Step 7: One will next determine the next distances for some of the descendants of $C_{9,8}$.

That is for $C_{5,6}$ $C_{6,5}$ $C_{6,7}$, $C_{7,6}$, .

$C_{5,6}$ Since the second subscript of this distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 8, 5) except that the first subscript of the next distance should be 6. Inspection of the Royal box shows that there is only one applicable distance namely, $C_{6,7}$ from box I.	$C_{6,5}$ Since the second subscript of this distance is 5, the first subscript of the next distance should be 5. Note that the next distance should not contain any of the subscripts already used. (No 1, 2,10, 9, 8,6) except that the first subscript of the next distance should be 5. Inspection of the entries in the Royal box indicates that there is no applicable distance whose first subscript is 5. Note that $C_{5,6}$ and $C_{5,8}$ are excluded here. One will go outside the Royal box for the next applicable distance. One chooses $C_{5,4}$ Never skip the nearest applicable distance.	$C_{6,7}$ Since the second subscript of this distance is 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used. (No 1, 2,10, 9,8, 6) except that the first subscript of the next distance should be 7. Inspection of the entries in the Royal box indicates that there is no applicable distance whose first subscript is 7. Note that $C_{7,9}$, $C_{7,8}$ and $C_{7,6}$ are excluded here. One will go outside the Royal box for the next applicable distance. One chooses $C_{7,5}$.	$C_{7,6}$ Since the second subscript of this distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9, 8 7) except that the first subscript of the next distance should be 6. Inspection of the Royal box shows hat there is only one applicable distance namely, $C_{6,5}$ from box A.

By imitating the above steps, one will obtain the following columns

$C_{6,7}$	$C_{5,4}$	$C_{7,5}$	$C_{6,5}$	$C_{5,6}$	$C_{6,5}$	$C_{5,8}$	$C_{8,5}$
$C_{7,4}$	$C_{4,7}$	$C_{5,4}$	$C_{5,4}$	$C_{6,4}$	$C_{5,4}$	$C_{8,4}$	$C_{5,4}$
$C_{4,3}$	$C_{7,3}$	$C_{4,3}$	$C_{4,3}$	$C_{4,3}$	$C_{4,3}$	$C_{4,3}$	$C_{4,3}$
$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$

Step 8: By combining steps 1-7, one obtains the possible routes, R1, R2, R3, R4, R5, R6, R7, R8

R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**
$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**
$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**
$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**
$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**
$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{8,6}$ **6**	$C_{8,7}$ **8**	$C_{7,8}$ **8**	$C_{7,8}$ **8**	$C_{7,6}$ **9**	$C_{7,6}$ **9**
$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{6,7}$ **9**	$C_{7,6}$ **9**	$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{6,5}$ **1**	$C_{6,8}$ **6**
$C_{6,7}$ **9**	$C_{5,4}$ **12**	$C_{7,5}$ **17**	$C_{6,5}$ **1**	$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{5,8}$ **2**	$C_{8,5}$ **2**
$C_{7,4}$ **39**	$C_{4,7}$ **39**	$C_{5,4}$ **12**	$C_{5,4}$ **12**	$C_{6,4}$ **24**	$C_{5,4}$ **12**	$C_{8,4}$ **23**	$C_{5,4}$ **12**
$C_{4,3}$ **10**	$C_{7,3}$ **40**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**
$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**
102	**139**	**95**	**81**	**87**	**79**	**87**	**81**

Shortest Route

From the above table, the shortest route is Route R6 of length 79 units.

$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)C_{7,8}(8)C_{8,6}(6)C_{6,5}(1)C_{5,4}(12)C_{4,3}(10)C_{3,1}(13)= 79$

Important points in the route construction

1. Begin the route construction by choosing from the Royal box (the set of the least ten distances)
2. Branching begins only from distances in the Royal box, and each branch distance must go to a distance in the Royal box.
3. When choosing from outside the Royal box, do not skip the first (nearest) applicable distance.
4. It is important that any possible branch is **not** missed, since such a branch may lead to the shortest route. By hand, draw the tree diagram and check by repeating the drawing

Comparison of the previous approach and the present approach in selecting distances

Previous Approach (A) (viXra:1505.0167)	Present Approach (B)
All the needed distances not among the least ten distances were added to the least ten distances before route construction began, The error here in the shortest route (81 units) found compared to the present approach (79 units) is 2.5%.	The needed distances not among the least ten distances were **not** added to the least ten distances before route construction began. Here, one starts with only the least ten distances and only if a needed distance is not among the set of the least ten distances, would one consider distances greater than those in the set of the least ten distances. The error here is zero or negligible.

A
Data Organization for Previous Approach

A $C_{5,6}$ or $C_{6,5}$ **1**	**G** $C_{2,10}$ or $C_{10,2}$ **7**	**N** $C_{1,3}$ or $C_{3,1}$ **13**	**U** $C_{6,9}$ or $C_{9,6}$ **19**
B $C_{5,8}$ or $C_{8,5}$ **2**	**H** $C_{7,8}$ or $C_{8,7}$ **8**	**P** $C_{9,10}$ or $C_{10,9}$ **14**	**V** $C_{5,9}$ or $C_{9,5}$ **20**
C $C_{1,2}$ or $C_{2,1}$ **3**	**J** $C_{6,7}$ or $C_{7,6}$ **9**	**Q** $C_{8,10}$ or $C_{10,8}$ **15**	**W** $C_{2,3}$ or $C_{3,2}$ **21**
D $C_{8,9}$ or $C_{9,8}$ **4**	**K** $C_{3,4}$ or $C_{4,3}$ **10**	**R** $C_{2,8}$ or $C_{8,2}$ **16**	**X** $C_{1,8}$ or $C_{8,1}$ **22**
E $C_{7,9}$ or $C_{9,7}$ **5**	**L** $C_{1,10}$ or $C_{10,1}$ **11**	**S** $C_{5,7}$ or $C_{7,5}$ **17**	**Y** $C_{4,8}$ or $C_{8,4}$ **23**
F $C_{6,8}$ or $C_{8,6}$ **6**	**M** $C_{4,5}$ or $C_{5,4}$ **12**	**T** $C_{2,5}$ or $C_{5,2}$ **18**	**Z** $C_{4,6}$ or $C_{6,4}$ **24**

B
Data Organization for Present Approach

Royal box

A $C_{5,6}$ or $C_{6,5}$ **1**	$C_{4,5}$ or $C_{5,4}$ **12**	$C_{4,8}$ or $C_{8,4}$ **23**	$C_{6,10}$ or $C_{10,6}$ **34**
B $C_{5,8}$ or $C_{8,5}$ **2**	$C_{1,3}$ or $C_{3,1}$ **13**	$C_{4,6}$ or $C_{6,4}$ **24**	$C_{1,4}$ or $C_{4,1}$ **35**
C $C_{1,2}$ or $C_{2,1}$ **3**	$C_{9,10}$ or $C_{10,9}$ **14**	$C_{2,4}$ or $C_{4,2}$ **25**	$C_{1,9}$ or $C_{9,1}$ **36**
D $C_{8,9}$ or $C_{9,8}$ **4**	$C_{8,10}$ or $C_{10,8}$ **15**	$C_{2,6}$ or $C_{6,2}$ **26**	$C_{5,10}$ or $C_{10,5}$ **37**
E $C_{7,9}$ or $C_{9,7}$ **5**	$C_{2,8}$ or $C_{8,2}$ **16**	$C_{3,5}$ or $C_{5,3}$ **27**	$C_{2,7}$ or $C_{7,2}$ **38**
F $C_{6,8}$ or $C_{8,6}$ **6**	$C_{5,7}$ or $C_{7,5}$ **17**	$C_{7,10}$ or $C_{10,7}$ **28**	$C_{4,7}$ or $C_{7,4}$ **39**
G $C_{2,10}$ or $C_{10,2}$ **7**	$C_{2,5}$ or $C_{5,2}$ **18**	$C_{3,10}$ or $C_{10,3}$ **29**	$C_{3,7}$ or $C_{7,3}$ **40**
H $C_{7,8}$ or $C_{8,7}$ **8**	$C_{6,9}$ or $C_{9,6}$ **19**	$C_{2,9}$ or $C_{9,2}$ **30**	$C_{1,5}$ or $C_{5,1}$ **41**
I $C_{6,7}$ or $C_{7,6}$ **9**	$C_{5,9}$ or $C_{9,5}$ **20**	$C_{3,8}$ or $C_{8,3}$ **31**	$C_{1,6}$ or $C_{6,1}$ **42**
J $C_{3,4}$ or $C_{4,3}$ **10**	$C_{2,3}$ or $C_{3,2}$ **21**	$C_{3,6}$ or $C_{6,3}$ **32**	$C_{4,10}$ or $C_{10,4}$ **43**
$C_{1,10}$ or $C_{10,1}$ **11**	$C_{1,8}$ or $C_{8,1}$ **22**	$C_{1,7}$ or $C_{7,1}$ **33**	$C_{4,9}$ or $C_{9,4}$ **44**
			$C_{3,9}$ or $C_{9,3}$ **45**

Justification of the shortest route.

In the shortest route, R6 (below), seven of the distances are from the Royal box (below) and the other three are the next three distances outside the Royal box (except 11 which is excluded here because of the subscript, 1), namely, 12, 13, and 14, are included in R6. Thus no applicable relatively short distance was skipped or ignored.

R1	R2	R3	R4	R5	R6	R7	R8
$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**
$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**
$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**
$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**
$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{8,6}$ **6**	$C_{8,7}$ **8**	$C_{7,8}$ **8**	$C_{7,8}$ **8**	$C_{7,6}$ **9**	$C_{7,6}$ **9**
$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{6,7}$ **9**	$C_{7,6}$ **9**	$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{6,5}$ **1**	$C_{6,8}$ **6**
$C_{6,7}$ **9**	$C_{5,4}$ **12**	$C_{7,5}$ **17**	$C_{6,5}$ **1**	$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{5,8}$ **2**	$C_{8,5}$ **2**
$C_{7,4}$ **39**	$C_{4,7}$ **39**	$C_{5,4}$ **12**	$C_{5,4}$ **12**	$C_{6,4}$ **24**	$C_{5,4}$ **12**	$C_{8,4}$ **23**	$C_{5,4}$ **12**
$C_{4,3}$ **10**	$C_{7,3}$ **40**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**
$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**
102	**139**	**95**	**81**	**87**	**79**	**87**	**81**

Royal box

A $C_{5,6}$ or $C_{6,5}$ **1**	$C_{4,5}$ or $C_{5,4}$ **12**	$C_{4,8}$ or $C_{8,4}$ **23**	$C_{6,10}$ or $C_{10,6}$ **34**
B $C_{5,8}$ or $C_{8,5}$ **2**	$C_{1,3}$ or $C_{3,1}$ **13**	$C_{4,6}$ or $C_{6,4}$ **24**	$C_{1,4}$ or $C_{4,1}$ **35**
C $C_{1,2}$ or $C_{2,1}$ **3**	$C_{9,10}$ or $C_{10,9}$ **14**	$C_{2,4}$ or $C_{4,2}$ **25**	$C_{1,9}$ or $C_{9,1}$ **36**
D $C_{8,9}$ or $C_{9,8}$ **4**	$C_{8,10}$ or $C_{10,8}$ **15**	$C_{2,6}$ or $C_{6,2}$ **26**	$C_{5,10}$ or $C_{10,5}$ **37**
E $C_{7,9}$ or $C_{9,7}$ **5**	$C_{2,8}$ or $C_{8,2}$ **16**	$C_{3,5}$ or $C_{5,3}$ **27**	$C_{2,7}$ or $C_{7,2}$ **38**
F $C_{6,8}$ or $C_{8,6}$ **6**	$C_{5,7}$ or $C_{7,5}$ **17**	$C_{7,10}$ or $C_{10,7}$ **28**	$C_{4,7}$ or $C_{7,4}$ **39**
G $C_{2,10}$ or $C_{10,2}$ **7**	$C_{2,5}$ or $C_{5,2}$ **18**	$C_{3,10}$ or $C_{10,3}$ **29**	$C_{3,7}$ or $C_{7,3}$ **40**
H $C_{7,8}$ or $C_{8,7}$ **8**	$C_{6,9}$ or $C_{9,6}$ **19**	$C_{2,9}$ or $C_{9,2}$ **30**	$C_{1,5}$ or $C_{5,1}$ **41**
I $C_{6,7}$ or $C_{7,6}$ **9**	$C_{5,9}$ or $C_{9,5}$ **20**	$C_{3,8}$ or $C_{8,3}$ **31**	$C_{1,6}$ or $C_{6,1}$ **42**
J $C_{3,4}$ or $C_{4,3}$ **10**	$C_{2,3}$ or $C_{3,2}$ **21**	$C_{3,6}$ or $C_{6,3}$ **32**	$C_{4,10}$ or $C_{10,4}$ **43**
$C_{1,10}$ or $C_{10,1}$ **11**	$C_{1,8}$ or $C_{8,1}$ **22**	$C_{1,7}$ or $C_{7,1}$ **33**	$C_{4,9}$ or $C_{9,4}$ **44**
			$C_{3,9}$ or $C_{9,3}$ **45**

Discussion and Conclusion

The length of the shortest route was found to be 79 units; but the sequence of cities of the shortest route is not unique. One sequence of the cities of the shortest route is given by $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)C_{7,8}(8)C_{8,6}(6)C_{6,5}(1)C_{5,4}(12)C_{4,3}(10)C_{3,1}(13)= 79$. If the direction of travel of this route is reversed, one obtains the route given by
$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,8}(6)C_{8,7}(8)C_{7,9}(5)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = 79$
The future in the approach for solving the traveling salesman problem lies in the approach (data ordering and route construction)) whereby one concentrates on the smallest distances, and by judicious selection, constructs the shortest route. Such an approach reduces the redundant use of brute force. For the nine cities visit, using brute-force, one would have to consider about 362,880 possibilities. Each possibility would be a column of nine distances. One of these 362,880 columns would be the shortest route to visit the nine cities without returning to City 1. In the approach used in this paper, only eight columns were constructed.

The error in the shortest route of length 79 units determined is zero or negligible.

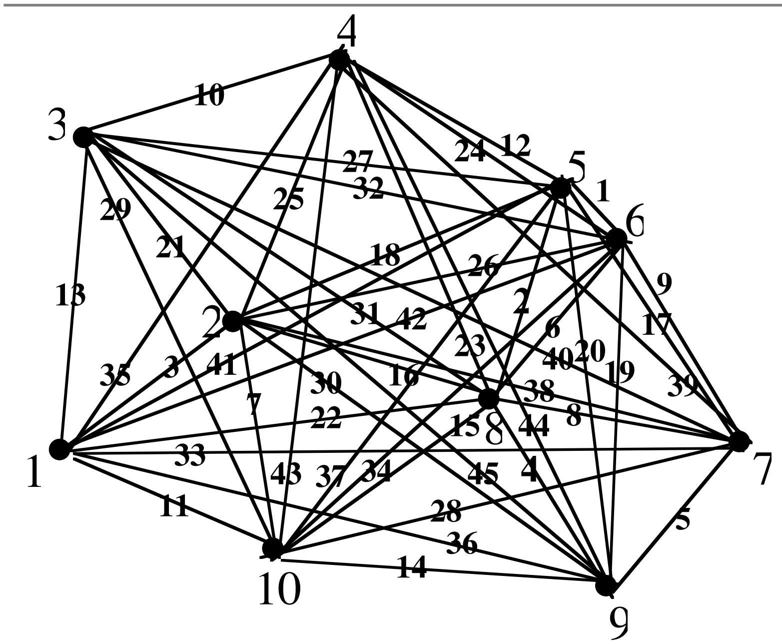
Application of the approach in determining the shortest route

The approach used in this paper can be applied in workforce project management and hiring as well as in a country's workforce needs and immigration quota determination.

Future and Next Task

Write a computer code to implement the solution process in this paper.

Now, by moving the cursor (using the mouse), enjoy the following travel:
$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)C_{7,8}(8)C_{8,6}(6)C_{6,5}(1)C_{5,4}(12)C_{4,3}(10)C_{3,1}(13)= 79$ is equivalent to
$C_{1,2}(3) + C_{2,10}(7) + C_{10,9}(14) + C_{9,7}(5) + C_{7,8}(8) + C_{8,6}(6) + C_{6,5}(1) + C_{5,4}(12) + C_{4,3}(10) + C_{3,1}(13)= 79$



Travel Route

From City 1 to City 2;
from City 2 to City 10;
from City 10 to City 9;
from City 9 to City 7;
from City 7 to City 8;
from City 8 to City 6;
from City 6 to City 5;
from City 5 to City 4;
.from City 4 to City 3;
and finally, from City 3 to City 1.

Adonten