Classify Positive Integers to Prove Collatz Conjecture by Mathematical Induction

Zhang Tianshu
Zhanjiang city, Guangdong province, China
Email: chinazhangtianshu@126.com
china.zhangtianshu@tom.com

Abstract

Positive integers which are able to be operated to 1 by set operational rule of the Collatz conjecture and positive integers got via operations by the operational rule versus the set operational rule are one-to-one the same, thus we refer to converse operational routes, apply the mathematical induction, next classify positive integers to prove the Collatz conjecture by substeps according to beforehand prepared two theorems concerned.

AMS subject classification: 11A25, 11P81 and 11Y55

Keywords: mathematical induction; two-way operational rules; classify positive integers; the bunch of integers' chains; operational routes

1. Introduction: The Collatz conjecture is also variously well-known as the Ulam conjecture, Kakutani's problem, the Thwaites conjecture, 3n+1 conjecture, Hasse's algorithm, and the Syracuse problem etc.

Yet it is still both unproved and un-negated a conjecture ever since named after Lothar Collats in 1937.

2. Basic Concepts and criteria

The Collatz conjecture states that take any positive integer n, if n is an even number, then divide n by 2; if n is an odd number, then multiply n by 3 and add 1. Repeat the above process indefinitely, then no matter which positive integer you start with, it will eventually reach a result of 1.

We regard the above-mentioned operational stipulations of the conjecture as the leftward operational rule. Also, regard the operational rule versus the leftward operational rule as the rightward operational rule.

The rightward operational rule stipulates that for any positive integer n, uniformly multiply n by 2. In addition, when n is an even number, if divide the difference of n minus 1 by 3 to get an odd number, then must operate this step, and the operations via this go on.

After begin with any positive integer to operate positive integers got successively by the either operational rule, we regard consecutive positive integers got successively plus arrowheads among them on an unidirection as an operational route.

If positive integer P exists at a certain operational route, then we may term the operational route "an operational route of P" or "the operational route P". Two operational routes of P branch from a positive integer or an integer's expression after pass the operation of P.

Start with 1 to operate positive integers got successively by the rightward operational rule, inevitably will form a bunch of operational routes. We term such a bunch of operational routes "a bunch of integers' chains".

By this token, a whole bunch of integers' chains must consist of infinite many operational routes. Please, see an initial bunch of integers' chains:

```
                              …              …     452→…
                            168↑          680↑   226↑→75→…
                           84↑          340↑→113↑    227→…
                          42↑          170↑          682↑→…
       1                 21↑          85↑          341↑
1→2→4↑→8→16↓→32→64↑→128→256↑→512→1024↑→2048→…
              5→10↓→20→40↓→80→160↓→320→…
                    3↓        13↓       53→106↓→212→…
                     6↓        26↓          35→70↓→140→…
                      12↓        52↓→104→…      23→46↓→15→…
                       24↓        17↓                92→…
                        …          34↓→11→22↓→7→14→28↓→56…
                                    68↓      44→88↓→29…9→18→…
                                     136↓→…      29↓
                                        45→…      58↓→19→…
                                                     …    …
```

First Illustration

Since each positive integer at a bunch of integers' chains comes from an adjacent positive integer before itself, also every such two that root in an even number are an odd number and an even number, thus infer that each of positive integers except for 1 at a bunch of integers' chains is unique.

No matter which positive integer, it exists at the whole bunch of integers' chains so as it is able to be operated to 1 by the leftward operational rule.

Accordingly, if a positive integer exists at a bunch of integers' chains, then the positive integer suits the conjecture certainly too.

Since two kinds of operations reverse for each other, thus positive integers at the whole bunch of integers' chains and positive integers which are able to be operated into 1 by the leftward operational rule are one-to-one correspondence and the same.

Such being the case, we shall refer to the bunch of integers' chains to apply the mathematical induction, afterwards gradually classify positive integers to prove the conjecture by substeps.

Before do the proof, we are necessary to prepare two theorems concerned. The purpose is in order to affirm satisfactory operational results duly, once they appear at operational routes.

Theorem 1* If an integer or an integer's expression suits the conjecture, and that it exists at operational route P, then, P suits the conjecture.

For examples, (1) Let $P=31+3^2\eta$ with $\eta\geq 0$, from $27+2^3\eta\to 82+3\times 2^3\eta\to 41+3\times 2^2\eta\to 124+3^2\times 2^2\eta\to 62+3^2\times 2\eta\to 31+3^2\eta>27+2^3\eta$, so $31+3^2\eta$ suits the conjecture. (2) Let $P=5+2^2\mu$ with $\mu\geq 0$, from $5+2^2\mu\to 16+3\times 2^2\mu\to 8+3\times 2\mu\to 4+3\mu< 5+2^2\mu$, so $5+2^2\mu$ suits the conjecture.

Proof* Suppose C suits the conjecture. At an operational route by the leftward operational rule, if C appears before P, then the operations of C via P reached 1 already, naturally P was operated into 1. If C appears behind P, then the operations of P pass C, afterwards continue along operational route of C to reach 1. In addition, at an operational route by the rightward operational rule, C and P both root in 1, of course, can operate either of them into 1 by the leftward operational rule inversely.

Theorem 2* If an integer or an integer's expression suits the conjecture, and that it exists only at operational route Q, yet operational route P and the operational route Q intersect, then P suits the conjecture likewise.

For example, let $P=95+3^2\times 2^7\varphi$ and $D=71+3^3\times 2^5\varphi$ where $\varphi\geq 0$, from $95+3^2\times 2^7\varphi\to 286+3^3\times 2^7\varphi\to 143+3^3\times 2^6\varphi\to 430+3^4\times 2^6\varphi\to 215+3^4\times 2^5\varphi\to 646+3^5\times 2^5\varphi\to 323+3^5\times 2^4\varphi\to 970+3^6\times 2^4\varphi\to 485+3^6\times 2^3\varphi\to 1456+3^7\times 2^3\varphi$

$\to 728+3^7\times 2^2\varphi \to 364+3^7\times 2\varphi \to 182+3^7\varphi\uparrow \to \ldots$

$\uparrow 121+3^6\times 2\varphi \leftarrow 242+3^6\times 2^2\varphi \leftarrow 484+3^6\times 2^3\varphi \leftarrow 161+3^5\times 2^3\varphi$

$\leftarrow 322+3^5\times 2^4\varphi \leftarrow 107+3^4\times 2^4\varphi \leftarrow 214+3^4\times 2^5\varphi \leftarrow 71+3^3\times 2^5\varphi < 95+3^2\times 2^7\varphi$, we get that $95+3^2\times 2^7\varphi$ suits the conjecture.

Proof* Let D suits the conjecture, and two operational routes intersect at A, then D and A exist at operational route Q, so A suits the conjecture according to the theorem 1. Like that, P and A exist at operational route P, of course, P suits the conjecture according to the theorem 1.

Actually, all positive integers at successively intersecting operational routes suit the conjecture, so long as therein a positive integer is suitable to the conjecture.

3. A Classified Proof by Substeps

We now set to prove the Collatz conjecture by the mathematical induction, then again, classify positive integers to complete the proof by substeps.

1. All positive integers at the initial bunch of integers' chains in the preceding chapter suit the conjecture, and that we are not difficult to find that there are consecutive positive integers≤ 24 therein.

2. After further operate integers at the initial bunch of integers' chains by the rightward operational rule, suppose that there are consecutive positive integers$\leq$ n within all positive integers got practically, where n$\geq$ 24.

In other words, suppose that positive integer n suits the conjecture.

3. After continue to operate foregoing appeared all positive integers by the rightward operational rule, prove that there are consecutive positive integers≤ 2n within all positive integers got ulteriorly.

In other words, prove that positive integer n+1 suits the conjecture.

First, let us divide limits of consecutive positive integers at the number axis into segments according to $2^Y n$ as greatest positive integer per segment, where Y≥0 and n≥24, so as to accord with the proof by the mathematical induction. A simple segmenting illustration is as follows.

1——2^0n———2^1n——————2^2n————————2^3n———→

Second Illustration

Proof * Since there are consecutive positive integers≤ n with n≥24 at a bunch of integers' chains according to the supposition of second step of the mathematical induction, thereout, multiply each and every positive integer≤ n by 2 by the rightward operational rule, then we get all positive even numbers between 2^0n and 2^1n+1 at a bunch of integers' chains extended, irrespective of repeated positive even numbers≤ n.

For odd numbers between 2^0n and 2^1n+1, we can classify them, nothing but, can only abide by a basic norm, i.e. according to the order from small to large to prove them, one by one. That is to say, when we prove M, must first affirm every M-t suits the conjecture, where M and M-t express positive integers or integer's expressions, M >M-t and t=1, 2, 3...

Thus, if n+1 belongs within any kind of odd numbers between 2^0n and 2^1n+1, then it can only be regarded as smallest one therein.

In any case, we first divide all odd numbers between 2^0n and 2^1n+1 into two genera, i.e. 5+4k and 7+4k, where k ≥5.

If n+1 belongs within 5+4k, then from 5+4k→16+12k→8+6k→4+3k < 5+4k, we get that this n+1 suits the conjecture according to the theorem 1.

For 7+4k, we again divide it into three sorts, i.e. 11+12c, 15+12c and 19+12c, where c≥1.

If n+1 belongs within 11+12c, then from 7+8c→22+24c →11+12c >7+8c, we get that this n+1 suits the conjecture according to the theorem 1.

If n+1belongs not within above-mentioned 5+4k and 11+12c, then it can only belong within 15+12c or 19+12c.

So firstly operate 15+12c by the leftward operational rule as follows.

15+12c→46+36c→23+18c→70+54c→35+27c ♣

d=2e+1: 29+27e (**1**)   e=2f: 142+486f→71+243f ♥

♣35+27c↓→c=2d+1: 31+27d↑→d=2e: 94+162e→47+81e↑→e=2f+1:64+81f (**2**)

c=2d: 106+162d→53+81d↓→d=2e+1:67+81e↓→e=2f+1:74+81f (**3**)

d=2e:160+486e♦   e=2f: 202+486f→101+243f ♠

g=2h+1: 200+243h (**4**)   **…**

♥ 71+243f↓→f=2g+1:157+243g↑→g=2h: 472+1458h→236+729h↑→ **…**

f=2g: 214+1458g→107+729g↓→g=2h+1: 418+729h↓→**…**

g=2h: 322+4374h→**… …**

g=2h: 86+243h (**5**)

♠101+243f↓→f=2g+1:172+243g↑→g=2h+1:1246+1458h→**…**

f=2g: 304+1458g→152+729g ↓→**…**

…

…

♦160+486e→80+243e↓→e=2f+1: 970+1458f→485+729f↑→ **…**   **…**

e=2f:40+243f↓→f=2g+1:850+1458g→ 425+729g↑→ **…**

f=2g: 20+243g↓→g=2h: 10+243h (**6**)   **…**

g=2h+1:880+1458h→440+729h↑→ **…**

Annotation:

(1) Each of letters c, d, e, f, g, h …etc at listed above operational routes expresses each of natural numbers plus 0, similarly hereinafter.

(2) Also, there are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦.

We are necessary to define a terminology before analyzing operational results of 15+12c/19+1c. Namely, if an operational result is smaller than a kind of 15+12c/19+12c, and that it first appears at an operational route of 15+12c/19+12c by the leftward operational rule, then we regard the operational result as a first satisfactory result.

Thereupon, we conclude that six kinds of 15+12c monogamously derived from six first satisfactory results at the listed above bunch of operational routes of 15+12c to suit the conjecture, ut infra.

From c=2d+1 and d=2e+1, we get c=2d+1=2(2e+1)+1=4e+3, and 15+12c =15+12(4e+3)=51+48e >29+27e where mark (1), so 15+12c with c=4e+3 suits the conjecture according to the preceding theorem 1.

From c=2d+1, d=2e and e=2f+1, we get c=2d+1=4e+1=4(2f+1) +1=8f+5, and 15+12c=15+12(8f+5) =75+96f > 64+81f where mark (2), so 15+12c with c=8f+5 suits the conjecture according to the preceding theorem 1.

From c=2d, d=2e+1 and e=2f+1, we get c=2d= 4e+2= 4(2f+1) +2=8f+6, and 15+12c=15+12(8f+6) =87+96f >74+81f where mark (3), so 15+12c with c=8f+6 suits the conjecture according to the preceding theorem 1.

From c=2d+1, d=2e, e=2f, f=2g+1 and g=2h+1, we get c=2d+1=4e+1= 8f+1=8(2g+1)+1=16g+9=16(2h+1)+9=32h+25, and 15+12c=15+12(32h +25)=315+384h >200+243h where mark (4), so 15+12c with c=32h+25 suits the conjecture according to the preceding theorem 1.

From c=2d, d=2e+1, e=2f, f=2g+1 and g=2h, we get c=2d=2(2e+1)=4e+2 =8f+2=8(2g+1)+2=16g+10=32h+10, and 15+12c=15+12(32h+10)= 135+

384h > 86+243h where mark (5), so 15+12c with c=32h+10 suits the conjecture according to the preceding theorem 1.

From c=2d, d=2e, e=2f, f=2g and g=2h, we get c=2d =32h, and 15+12c = 15+12(32h) =15+384h > 10+243h where mark (6), so 15+12c with c=32h suits the conjecture according to the preceding theorem 1.

Secondly, operate 19+12c by the leftward operational rule as follows.

19+12c→58+36c→ 29+18c→ 88+54c→ 44+27c ♣

d=2e: 11+27e (**α**) e=2f:37+81f (**β**)
♣ 44+27c↓→c=2d: 22+27d↑→d=2e+1:148+162e→74+81e↑→e=2f+1:466+486f ♥
c=2d+1: 214+162d→107+81d↓→d=2e:322+486e ♠
d=2e+1:94+81e↓→e=2f:47+81f (γ)
e=2f+1:516+486f ♦

g=2h: 129+243h (**δ**) **…**
f=2g+1:258+243g↑→g=2h+1:1504+1458h→752+729h↑→**…**
♥466+486f→233+243f↑→f=2g:700+1458g→350+729g↓→g=2h+1:3238+4374h↓
g=2h: 175+729h↓→**…** **…**
…

g=2h+1:172+243h (**ε**)
f=2g: 101+243g↑→g=2h: 304+1458h→**…**
e=2f+1:202+243f↑→f=2g+1:1336+1458g→**…**
♠322+486e→161+243e↑→e=2f:484+1458f→**…**

♦516+486f→258+243f↓→f=2g+1:1504+1458g→**…**
f=2g: 129+243g↓→g=2h: 388+1458h→**…**
g=2h+1:186+243h (ζ)

Annotation:
(1) Each of letters c, d, e, f, g, h …etc at listed above operational routes expresses each of natural numbers plus 0, similarly hereinafter.
(2) Also, there are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦.

Like that, we conclude that six kinds of 19+12c monogamously derived from six first satisfactory results to suit the conjecture at the listed above bunch of operational routes of 19+12c, ut infra.

From c=2d and d=2e, we get c=2d=4e, and 19+12c=19+12(4e)=19+48e > 11+27e where mark (α), so 19+12c with c=4e suits the conjecture according to the preceding theorem 1.

From c=2d, d=2e+1 and e=2f, we get c=2d = 2(2e+1) = 4e+2 = 8f+2, and 19+12c=19+12(8f+2) = 43+96f >37+81f where mark (β), so 19+12c with c=8f+2 suits the conjecture according to the preceding theorem 1.

From c=2d+1, d=2e+1 and e=2f, we get c=2d+1=4e+3=8f+3, and 19+12c=19+12(8f+3) =55+96f>47+81f where mark (γ), so 19+12c with c=8f+3 suits the conjecture according to the preceding theorem 1.

From c=2d, d=2e+1, e=2f+1, f=2g+1 and g=2h, we get c=2d=2(2e+1)= 4e+2= 4(2f+1)+2 = 8f+6= 8(2g+1)+6 = 16g+14 = 32h+14, and 19+12c = 19+12(32h+14) = 187+384h >129+243h where mark (δ), so 19+12c with c=32h+14 suits the conjecture according to the preceding theorem 1.

From c=2d+1, d=2e, e=2f+1, f=2g and g=2h+1, we get c=2d+1=4e+1= 4(2f+1)+1=8f+5=16g+5=16(2h+1)+5=32h+21, and 19+12c=19+12(32h +21) =271+384h > 172+243h where mark (ε), so 19+12c with c=32h+21 suits the conjecture according to the preceding theorem 1.

From c=2d+1, d=2e+1, e=2f+1, f=2g and g=2h+1, we get c=2d+1=2(2e+ 1)+1=4e+3=4(2f+1)+3=8f+7=16g+7=16(2h+1)+7=32h+23, and 19+12c= 19+12(32h+23)=295+384h > 186+243h where mark (ζ), so 19+12c with c=32h+23 suits the conjecture according to the preceding theorem 1.

Refer to above example illustrations, we summarize out two points below.

Firstly, a kind of 15+12c/19+12c derived from a first satisfactory result and the first satisfactory result coexist at an operational route such as

$15+12c=51+48e=51+3\times2^4e\to154+3^2\times2^4e\to77+3^2\times2^3e\to232+3^3\times2^3e\to$ $116+3^3\times2^2e\to58+3^3\times2e\to29+27e$; also $19+12c=19+48e=19+3\times2^4e\to$ $58+3^2\times2^4e\to29+3^2\times2^3e\to88+3^3\times2^3e\to44+3^3\times2^2e\to22+3^3\times2e\to11+27e$.

Secondly, the greatest exponent of factor 2 of coefficient of variable of each kind of 15+12c/19+12c is exactly the number of times that divided by 2 from a kind of 15+12c/19+12c operate to the first satisfactory result.

Now that each kind of 15+12c/19+12c derived from a first satisfactory result and the first satisfactory result coexist at an operational route, and that the first satisfactory result is smaller than the kind of 15+12c /19+12c.

Therefore, each kind of 15+12c/19+12c derived from a first satisfactory result suits the conjecture according to the preceding theorem 1.

If n+1 belongs within any kind of 15+12c and 19+12c between 2^0n and 2^1n+1 derived from a first satisfactory result, and that it is smallest one therein, then this n+1 is proved to suit the conjecture. Such as, if n+1 belongs within any kind of 15+12c and 19+12c proved above, and that it is smallest one therein, so this n+1 has been proved to suit the conjecture.

Thereinafter, we will further explain that why all first satisfactory result which monogamously derive all kinds of 15+12c/19+12c between 2^0n and 2^1n+1 can earliest appear at operational routes of 15+12c/19+12c?

First, let χ intensively represent variables d, e, f, g, h … etc. within integer's expressions at any bunch of operational routes of 15+12c/ 19+12c by the leftward operational rule, but χ represents not c.

Then, the odevity of part integer's expressions that contain variable χ at any bunch of operational routes of 15+12c/19+12c is still indeterminate.

That is to say, for every such integer's expression, both consider it as an odd number to operate, and consider it as an even number to operate. Thus, let us label such integer's expressions "odd-even expressions".

For any odd-even expression at any bunch of operational routes of 15+12c/19+12c, two kinds of operations synchronize at itself due to the odevity of χ. After regard an odd-even expression as an odd number to operate, get a greater operational result>itself. Yet after regard it as an even number to operate, get a smaller operational result<itself.

Begin with any odd-even expression to operate continuously by the leftward operational rule, every such operational route via consecutive greater operational results is getting longer and longer up to elongate infinitely, and that orderly emerging odd-even expressions therein are getting greater and greater up to infinity.

On the other, for a smaller operational result in synchronism with a greater operational result, if it can be divided successively by 2^{μ} to get an even smaller integer's expression, where μ is the greatest exponent of factor 2 within the smaller operational result, then, when the even smaller integer's expression is smaller than a kind of 15+12c/19+12c, the kind of 15+12c/ 19+12c suits the conjecture according to the theorem 1. Accordingly the operation may stop at here.

If the even smaller integer's expression is greater than any kind of 15+12c/19+12c or the smaller operational result itself is an odd expression, then, in the case we need to operate it continuously.

In other words, at the whole bunch of operational routes of 15+12c/ 19+12c, on the one hand, odd-even expressions are getting both greater and greater, and more and more, along the continuation of operations, up to infinity and infinitely many. Accordingly there are infinitely many operational routes of 15+12c /19+12c.

On the other hand, endlessly stop operations of branches therein, since first satisfactory results always appear at branches therein. Thus, there too are infinitely many operational routes of 15+12c /19+12c which are stopped by, including infinitely many first satisfactory results at them.

By this token, 15+12c/19+12c must be divided into infinite many kinds, just enable that infinitely many first satisfactory results correspond with infinitely many kinds of 15+12c/19+12c monogamously.

Additionally, the variable c of 15+12c /19+12c is able to be endowed with infinitely many natural numbers plus 0, one by one, thus there are infinitely many kinds of 15+12c /19+12c authentically.

Nevertheless, what we need is merely to prove all kinds of 15+12c/ 19+12c between 2^0n and 2^1n+1 according to the requirement of third step of the mathematical induction, yet it isn't all of kinds of 15+12c/ 19+12c.

If let 15+12c=2n+1, figure out c= (n-7)/6; if let 19+12c=2n+1, figure out c=(n-9)/6, so it follows that the number of kinds of 15+12c between 2^0n and 2^1n+1 is smaller than (n-7)/6, and the number of kinds of 19+12c between 2^0n and 2^1n+1 is smaller than (n-9)/6.

From known n≥24 to my way of thinking, if regard n as an infinity, then there are infinitely many integers including all odd numbers of 15+12c and 19+12c between 2^0n and 2^1n+1 to suit the conjecture, although the infinitely more equal not the all, but, once enter into an infinite field, integers inside the infinite field distinguish not big or small, more or less. Therefore, in all senses, we have no occasion to do the proof.

If regard n as a finite-large integer, then 2n+1 is a finite-large odd number, thereupon, each odd number of 15+12c and 19+12c under odd number 2n+1 is a finite-large odd number. Of course, the number of kinds of 15+12c and 19+12c between 2^0n and 2^1n+1 is finite too, i.e. a positive integer which is smaller than (n-7)/6 or (n-9)/6 is a finite number.

At the bunch of operational routes of 15+12c/19+12c by the leftward operational rule, for each operational route therein, either operate it to get a first satisfactory result, or operate it up to an infinity on and on, and that in the latter case, integer's expressions are getting greater and greater along the continuation of operations.

Undoubtedly odd numbers of 15+12c/19+12c between 2^0n and 2^1n+1 are smaller as compared with kindred odd numbers. Or say, the coefficient of χ and the constant term of each kind of 15+12c/19+12c between 2^0n and

2^1n+1 are smaller as compared with unproved kinds of 15+12c /19+12c.

As thus, we can determine all odd numbers of 15+12c/19+12c between 2^0n and 2^1n+1 so long as pass operations for finite times by the rightward operational rule. Or say, we can determine all kinds of 15+12c/19+12c between 2^0n and 2^1n+1 so long as pass operations for finite times by the leftward operational rule to firstly get homologous smaller first satisfactory results, secondly derive them from these first satisfactory results monogamously.

If apply the leftward operational rule to operate, smaller first satisfactory results always appear at the precedence of greater first satisfactory results. When the number of smaller first satisfactory results reaches the number of kinds of 15+12c /19+12c between 2^0n and 2^1n+1 just, we can deduce exactly all kinds of 15+12c /19+12c between 2^0n and 2^1n+1 from these first satisfactory results monogamously.

Hereto, the above-mentioned proof for odd numbers of 15+12c/19+12c between 2^0n and 2^1n+1 should be complete. Yet, in order to impress the proof in minds, we might as well again give two examples in which case suppose $2^Yn+1=193$ where Y=1 plus n=96 or Y=3 plus n=24, to explain: in continued operations, why each odd numbers of 15+12c and 19+12c between 2^0n and 2^1n+1 is always smallest or smaller, and first get them?

First, on the basis of proven 9 kinds of 15+12c and 19+12c concerned, let variable χ be endowed with suitable values to enable each such odd numbers be inside the designated area, as listed below.

χ,	51+48e,	75+96f,	87+96f,	135+384h,	15+384h,	19+48e,	43+96f,	55+96f,	187+384h
0:	51,	75,	87,	135,	15,	19,	43,	55,	187
1:	99,	171,	183,			67,	139,	151	
2:	147,					115,			
3:						163,			

As listed above, from small to large odd numbers of 15+12c and 19+12c under integer 193 have 15, 19, 43, 51, 55, 67, 75, 87, 99, 115, 135, 139, 147, 163, 171, 183 and 187.

Yet, from small to large odd numbers of 15+12c and 19+12c under integer 193 are 15, 19, 27, 31, 39, 43, 51, 55, 63, 67, 75, 79, 87, 91, 99, 103, 111, 115, 123, 127, 135, 139, 147, 151, 159, 163, 171, 175, 183 and 187, therein underlined odd numbers are absentees in the above list.

The absent reason is due to unable to show overlong operational routes, nothing but, we can operate each of them all alone to suit the conjecture, and that point out the belongingness of each of them, ut infra.

From 27→82→41→124→62→31[&]→94→47→142→71→214→107→322 →161→484→242→121→364*→182→91[#]→274→137→412→206→ 103[##]→310→155→466→233→700→350→175[$$]→526→263→790→ 395→1186→593→1780→890→445→1336→668→334**→167→502 →251→754→377→1132→566→283→850→425→1276→638→319 →958→479→1438→719→2158→1079→3238→1619→4858→2429

→7288→3644→1822***→911→2734→1367→4102→2051→6154→3077→9232→4616→2308→1154→577→1732→866→433→1300→650→325→976→488→244→122→61→184→92→46→23 < 27, so 27 suits the conjecture according to the preceding theorem 1. Also, odd number 27 belongs within $27+2^{59}\times 3y$.

Annotation: several signs except for arrowheads at above operational route of 27 will be quoted by latter certain operational routes respectively.

From $31^{\&}$— connect to the operational route of 27 →… →23< 31, so 31 suits the conjecture according to the preceding theorem 1. Also, odd number 31 belongs within $31+2^{56}\times 3x$.

From 39→118→59→178→89→268→134→67→202→101→304→152→76→38<39, so 39 suits the conjecture according to the preceding theorem 1. Also, odd number 39 belongs within $39+2^{8}\times 3k$.

From 63→190→95→286→143→430→215→646→323→970→485→1456→728→364*—connect to the operational route of 27→…→ 61<63, so 63 suits the conjecture according to the preceding theorem 1. Also, odd number 63 belongs within $63+2^{54}\times 3w$.

From 79→238→119→358→179→538→269→808→404→202←67<79, so 79 suits the conjecture according to the preceding theorem 2. Also, odd number 79 belongs within $79+2^{5}\times 3j$. Attention please, the preceding theorem 2 is quoted by us here.

From $91^{\#}$— connect to the operational route of 27→…→ 61 < 91, so 91 suits the conjecture according to the preceding theorem 1. Also, odd number 91 belongs within $91+2^{45}\times3v$.

From 103[##]—connect to the operational route of 27→…→ 61< 103, so 103 suits the conjecture according to the preceding theorem 1. Also, odd number 103 belongs within $103+2^{42}\times3u$.

From 111→334**—connect to the operational route of 27→…→ 61< 111, so 111 suits the conjecture according to the preceding theorem 1. Also, odd number 111 belongs within $111+2^{31}\times3q$.

From 123→370→185→556→278→139→418→209→628→314→157 → 472→236→118 < 123, so 123 suits the conjecture according to the preceding theorem 1. Also, odd number 123 belongs within $123+2^{8}\times3m$.

From 151→454→227→682→341→1024→512 →256 → 128 < 151, so 151 suits the conjecture according to the preceding theorem 1. Also, odd number 151 belongs within $151+2^{5}\times3u$.

From 159→478→239→718→359→1078→539→1618→809→2428→1214→607→1822***—connect to the operational route of 27 →… →122<159, so 159 suits the conjecture according to the preceding theorem 1. Also, odd number 159 belongs within $159+2^{21}\times3s$.

From 175[$$]— connect to the operational route of 27→…→167 < 175, so 175 suits the conjecture according to the preceding theorem 1. Also, odd number 175 belongs within $175+2^{8}\times3v$.

Like that, χ represent too variables j, k, q, s, x, y … etc. within listed above integer's expressions which express the belongingness of proven odd number of 15+12c and 19+12c, i.e. χ=0, 1, 2, 3, 4, 5…

Moreover, we are not difficult to discover that many odd numbers coexist at two bunches of operational routes of 15+12c and 19+12c, such as certain odd numbers at operational route of 27.

If 2n+1≥193, likewise we can prove that odd numbers of 15+12c /19+12c between 2^0n and 2^1n+1 suit the conjecture in the light of the same way.

Erenow, as a matter of fact, we have proved that n+1 as smallest one odd number between 2^0n and 2^1n+1 suits the conjecture, no matter which kind of odd numbers it exists within. Below, we give the cause synthetically.

Since n+1 and an integer which is smaller than n+1 coexist at an operational rule, also n+1 is the smallest odd number between 2^0n and 2^1n, then the integer exists inside range of values from 1 to n evidently, additionally we have supposed that any integer inside the range of values suits the conjecture, thus this n+1 suits the conjecture according to the preceding theorem 1.

To sum up, we have proven that if n+1 belongs within any of genus, sort and kind of odd numbers between 2^0n and 2^1n+1, and that regard it as smallest one therein, then n+1 suits the conjecture uniformly. By now, let us regard the smallest one odd number between 2^0n and 2^1n+1 as first odd number according to the order from small to large. Then we have proven that first odd number between 2^0n and 2^1n+1 suits the conjecture.

In the same old way, we can too prove that second, third, fourth etc. odd number until 2n-1 between 2^0n and 2^1n+1, they all suit the conjecture.

Overall, we have proven that odd numbers of every genus, sort and kind between 2^0n and 2^1n+1 suit the conjecture. Therefore, they all exist at the bunch of integers' chains.

In addition, all even numbers between 2^0n and 2^1n+1 exist at the bunch of integers' chains too, since they all come from positive integers under integer n at the bunch of integers' chains, are operational results got by the rightward operational rule.

Thus, all integers between 2^0n and 2^1n exist at the bunch of integers' chains.

Consequently, all integers between 2^0n and 2^1n suit the conjecture according to the inference of one-to-one correspondence and the same in the chapter 2 of this article.

So far, we have proven that positive integers$\le 2^1n$ suit the conjecture by consecutive positive integers$\le 2^0n$. Like that, we can too prove that positive integers$\le 2^2n$ suit the conjecture by consecutive positive integers$\le 2^1n$ according to the foregoing way of doing, and so on and so forth, up to prove positive integers$\le 2^Yn$ suit the conjecture, where $Y\ge 3$, and $n \ge 24$.

If let Y=X, X+1 and X+2, then on the basis that proven integers between 2^Xn and $2^{X+1}n$ suit the conjecture by positive integers$\le 2^Xn$, we are too able to prove that positive integers between $2^{X+1}n$ and $2^{X+2}n$ suit the conjecture by proven positive integers$\le 2^{X+1}n$ in the same old way.

For greatest positive integer $2^Y n$ per segment, Y begins with 0, next it is endowed with 1, 2, 3, etc natural numbers in proper order, then consecutive positive integers≤$2^Y n$ are getting more and more along with which values of Y are getting greater and greater, and that the later emerged positive integers, the greater. After Y is equal to every natural number plus 0, all positive integers are proven to suit the conjecture. Namely every positive integer is proven to suit the conjecture, as thus, the Collatz conjecture is proven by us thoroughly.

The proof was thus brought to a close. As a consequence, the Collatz conjecture holds water.

References

[1] *Wolfram Mathworld*: http://mathworld.wolfram.com/CollatzProblem.html

[2] Maddux, Cleborne D.; Johnson, D. Lamont (1997). Logo: *A Retrospective*. New York: Haworth Press. P.160. ISBN 0-7890-0374-0.The problem is also known by several other names, including: Ulam's conjecture, the Hailstone problem, the Syracuse problem, Kakutani's problem, Hasse's algorithm, and the Collatz problem.

[3] Leavens, Gary T.; Vermeulen, Mike (December 1992), "3x+1 Search Programs". *Computers & Mathematics with Applications* 24 (11): 79–99, doi: 10.1016/0898-1221(92)90034-F

[4] *a b c* Simons, J.; de Weger, B.; "Theoretical and computational bounds for *m*-cycles of the 3*n* +1 problem", *Acta Arithmetica* (on-line version 1.0, November 18, 2003), 2005.

[5] Steiner, R. P.; "A theorem on the syracuse problem", *Proceedings of the 7th Manitoba Conference on Numerical Mathematics*, pages 553–559, 1977.

[6] Krasikov, Ilia; Lagarias, Jeffrey C. (2003), "Bounds for the 3*x* + 1 problem using difference inequalities". *Acta Arithmetica* 109(3):237–258, doi:10.4064/aa 109-3-4. MR 1980260

[7] Colussi, Livio (9 September 2011). "The convergence classes of Collatz function". *Theoretical Computer Science* 412 (39): 5409–5419, doi: 10.1016/j.tcs. 2011.05.056

[8] Marc Chamberland, A continuous extension of the $3x + 1$ problem to the real line, Dynam. Contin, *Discrete Impuls Systems* 2: 4 (1996), 495–509.

[9] Scollo, Giuseppe (2007), "Looking for Class Records in the 3x+1 Problem by means of the COMETA Grid Infrastructure", *Grid Open Days at the University of Palermo*

[10] Lagarias, Jeffrey C., ed. *(2010). The Ultimate Challenge: the 3x+1 problem,* American Mathematical Society*, ISBN 978-0-8218-4940-8, Zbl 1253.11003.*