Investigation into the Cosmological Properties of a 'Shell Universe'

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Abstract

The black hole Schwarzschild solution is examined in the context of a cosmological model where the Universe is a 3D shell surrounded by a vacuum of time. The anisotropy of the metric is discussed and it is shown by analyzing light signals on a Kruskal-Szekeres coordinate chart that since the anisotropy lies along the time dimension, we will still observe isotropic space in the internal metric. It is shown that the model predicts an accelerated expansion that agrees with current observations of the expansion history of our Universe, namely that the initial expansion is infinitely fast, and then the expansion slows for some time followed by an accelerated expansion. Using measured data for the Hubble constant and transition redshift, the limits for the coordinate age of the Universe and total lifetime of the Universe are calculated. Using these values, distance modulus (calculated from the model) is plotted against redshift and compared to cosmological data. The angular portion of the metric is interpreted, and it is discussed in terms of the celestial spheres of the CMB and Big Bang. In particular, it is shown that the internal Schwarzschild metric is able to describe the spherical nature of the Big Bang (the Big Bang must be a sphere behind the CMB since it occurred in the finite past and is fully contracted in the spatial dimension).

Introduction

Consider a collection of particles distributed as a 2D spherical in space such that any local interactions between the particles is negligible and it is surrounded externally and internally by vacuum. If this shell is made to uniformly expand or left to collapse, the particles will follow worldlines defined by the Schwarzschild metric. In this paper, we will examine a similar scenario where the spherically symmetric 'shell' is the entire Universe at a given time. Observation has shown that the Universe is:

- Spherically symmetric.
- Homogenous in space
- Inhomogeneous across time.

We will also make one further assumption in this paper:

- The Universe only ever occupies a single instant of Cosmic time[1] and moves from one moment of cosmic time to the next where the time measured by observers between cosmic times depends on their respective motions. In

[1] In the classical approximation. Quantum uncertainty would blur that instant.

other words, the 3D spatial distribution of energy in the Universe is physically moving through the time dimension from the past into the future, and energy only exists in the present. So if one were to view the Universe on a spacetime diagram, they would only see the Universe at one value of time with the rest of the diagram empty. A worldline in this scenario is like the dot of a laser pointer following a prescribed path as opposed to a drawn out line fixed in the spacetime.

This further assumption implies that the spherically symmetric Universe is 'surrounded' by vacuum in the time dimension, analogously to how the aforementioned 2D shell was surrounded by a vacuum of space. Since the only spherically symmetric vacuum solution in General Relativity is the Schwarzschild metric, this assumption implies that the metric of the Universe is the black hole metric, which will be referred to as the 'internal Schwarzschild metric' in this paper. If we neglect the angular part of this metric, we find that the metric can be put in a form identical to the FRW metric, with the only exception being that the scale factor is not dependent on the matter distribution but is rather an energy-independent function of time. But where this metric is truly distinct from the FRW metric is in the angular term. For the internal Schwarzschild metric, the radius of the angular component is just time, as opposed to the FRW metric where the radius is distance times the scale factor (note that the anisotropy of the Schwarzschild metric will be addressed in the 'Metric Anisotropy' section of this paper).

Consider the celestial spheres around an observer in the Universe. When we look out to distant events, we can use the redshift from these events to determine their distance from us. Events with the same distance from us can be thought of as residing on a celestial sphere, such that all these events are separated from us by the same magnitude of space and time. We can classify these spheres into three types:

1. Dynamic Spheres – These are the spheres that galaxies reside on. Objects on these spheres maintain a constant coordinate distance from us and move forward in time. We are able to move toward or away from objects on these spheres by moving through space. If we fix our sights on a particular galaxy, the light we see from that galaxy is being emitted later in time as we ourselves move through time.

2. Static Spheres – These are spheres fixed in time. The Cosmic Microwave Background is the most obvious example of these spheres. Light from the CMB sphere is always emitted from the same cosmological time, but as we ourselves move through time, we see light from that time emitted from farther and farther away from us in space, giving the impression that the CMB sphere is growing. We cannot move toward or away any objects on this sphere because it is frozen in time. Both metrics are able to capture this behaviour, but they do so in different ways.

3. The Dark Sphere – The Dark Sphere is where the internal Schwarzschild metric succeeds and the FRW metric fails. The Dark Sphere is the Big Bang and lies beyond the CMB. It is in principle unobservable for two reasons. First, the CMB is opaque so that any light from the Big Bang cannot penetrate it. Second, even if

the CMB was not locking our view, any light from that sphere would be infinitely redshifted in the frame of all future observers since the scale factor on that sphere is zero. But though we cannot see the Dark Sphere, it must be there if the model of the Universe is consistent. Because the Big Bang happened a finite time in our past, it must in some sense be located a finite distance from us in the sky somewhere behind the CMB. Furthermore, it must be a sphere because there is no preferred direction in the Universe (if the cosmological principle holds). The Big Bang is sometimes pictured as an infinitely dense point containing all space. It is also sometimes described as an infinitely expansive 3D space with infinite density. The reality, however must be something in between. The Big Bang must be a 2D spherical surface with infinite density and finite area. The density is infinite because the scale factor is zero, and it must have a finite spherical surface area for the reasons given above. The FRW metric breaks down here and cannot account for the fact that based on our observations of the Universe, the Big Bang must have those properties. It is also unclear in the FRW metric what distance should be assigned to that sphere. The CMB appears to grow over time because we get light emitted at that time from greater and greater distances. But space is fully contracted at the Big Bang, so the sphere would not be able to appear to grow. How can the CMB sphere grow if the Big Bang sphere cannot? The internal Schwarzschild metric, handles these conditions effortlessly thanks to the structure of the angular component of the metric.

These spheres are shown in terms of the internal Schwarzschild metric in Figure 1. Figure 1 shows the Schwarzschild coordinates of the internal metric plotted on the Kruskal-Szekeres coordinate plane. In these coordinates, space is the 't' coordinate emanating from the center of the diagram (Big Bang) and time is the 'r' coordinate depicted as hyperbolas (time is flowing forward as r goes toward zero). The upper right quadrant of this diagram represents a single fixed direction ($\theta = const, \phi = const$). So each bold line representing a sphere would be a point on each sphere over time. Note that light on this diagram travels on 45-degree lines.

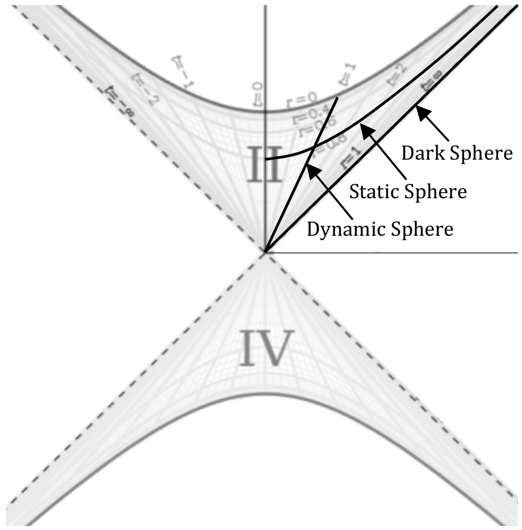

Figure 1 – Celestial Sphere Types on Kruskal-Szekeres Coordinate Chart[2]

In this paper, we will first examine the space time of the internal metric from the perspective of the inertial observer and compare results to experimental data. We will then examine the angular part of the metric more closely. Finally, we will examine at the classical theory of Black Holes in this context.

The Schwarzschild Metric

The Schwarzschild metric is the simplest solution to Einstein's field equations. It is a vacuum solution for the spacetime around a spherically-symmetric distribution of energy. The general form of the metric can be expressed as:

$$d\tau^2 = \frac{r}{u-r}dr^2 - \frac{u-r}{r}dt^2 - r^2 d\Omega^2 \quad (1)$$

Depending on the ratio $\frac{u}{r}$, we get three distinct descriptions of spacetime:

1. $u = 0$: This gives us the flat Minkowski metric of Special Relativity.

2. $\frac{u}{r} < 1$: This describes the external metric for the spacetime surrounding a spherically-symmetric energy distribution occupying a finite amount of space for an infinite amount of time
3. $\frac{u}{r} \geq 1$: This describes the internal metric for the spacetime surrounding a spherically-symmetric energy distribution occupying an infinite amount of space for a finite amount of time

[2] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

The internal metric is typically considered an extension of the external metric, giving rise to the theory of Black Holes where an observer can cross from the external to the internal metric by freefalling toward an infinitely dense mass. In this paper, we will be examining the internal metric completely independently of the external metric such that we do not need to consider the pre-existence of a gravitationally collapsed mass from the external metric in order for the spacetime of the internal metric to exist. We simply hypothesize that the internal metric is the metric of the Universe and so will not view it in the context of Black Holes (though this context will be addressed later in the paper).

An important observation is that the internal metric describes a vacuum solution to the field equations. But the Universe is clearly filled with energy, so how can this solution apply? In order to satisfy the requirements of the metric, the Universe must be "*a spherically-symmetric energy distribution occupying an infinite amount of space for a finite amount of time*". For this metric to be a cosmological description, it must be that Universe only truly exists in the present and in a very real sense moves into the future. The surrounding vacuum is the future, and the Universe is freefalling through time toward the temporal center of the metric.

Time being the radial dimension of the metric combined with the fact that the solution is a vacuum solution gives a mathematical justification for our intuitive notions of past, present, and future. The anisotropy along the radial direction gives us an arrow of time that distinguishes the 'past' and 'future' analogous to the way the external solution gives us an absolute distinction between 'up' and 'down'. And the vacuum as described above gives us a boundary between them, that boundary being the 'present' time.

Metric Anisotropy and the Angular Term

According to the Cosmological Principle, which is supported by observation, the Universe is homogenous and isotropic. But the Schwarzschild metric is anisotropic, so how can this metric be a true description of our Universe? It is important to note that the anisotropy of the internal metric lies exclusively along the time dimension. On the top half of Figure 1, light travels on 45 degree lines upwards to an observer (for our example, we'll put the observer at $t = 0$) at some time r. The diagram is showing the spacetime for a fixed θ and ϕ. The observer at r will see 2 light signals intercepting it: one coming from the positive t direction and one coming from the negative t direction. Thus, if the observer is oriented in a particular direction, the light coming to her face is the signal from the positive t direction, while the light coming to the back of her head is coming from the negative t direction. But both of these directions face outward from the center of the sphere (they face the past). Therefore, the Universe in front of her will look just like the Universe behind her. The anisotropy of the metric does not manifest itself as an anisotropy in space, but rather of time. Since we did not specify θ and ϕ, we can conclude that this is true for every direction. The metric is anisotropic in past, present, and future, but one cannot see this anisotropy in space; we cannot choose to look toward or away from the center of the Universe because the center is a point in time, not space. We are always looking away from the center no matter what direction we face – there is no preferred direction, no observable anisotropy. Contrast the above argument with the external metric where on a diagram similar to Figure 1, it shows that one light signal always comes from the direction of the center of the metric while the other signal comes from the opposite direction. So in that metric one can look toward or away from the center at will and the external spacetime is therefore observably anisotropic.

But, the angular portion of the metric does collapse while the spatial dimension expands, so how do we account for the observation that the Universe looks more dense in the past? The brightness of a given object that we observe will be proportional to its position in time, *r,* relative to us since the radius of the sphere is time. According to Equation 1 along a null geodesic, $\frac{dt}{dr} = \frac{r}{u-r}$, implying that the variation in space for a given variation in observed brightness will increase exponentially as we look at galaxies closer in time to the Big Bang.

Consider two equal-area patches of sky. In the first patch we observe galaxies from the near past while in the other patch we observe galaxies from the distant past. After measuring the brightness of the galaxies in each patch, we determine that the variation in brightness (Δr) is approximately equal in both patches. In this circumstance, we might see a more dense collection of galaxies in the second patch even though the galaxies in the first patch should have less angular separation due to the collapsing angular term in the metric. This is due to the fact that for the same variation in brightness, we see a larger variation in space in the second patch compared to the first. In other words, the amount of space being 'projected' onto the second patch is much greater that the amount of space being projected onto the first because the second patch is farther in the past where the spatial dimension is contracted. Therefore, we would still expect to observe a more dense collection of galaxies in the second patch than we would on the first. The increase in density is a kind of relativistic optical illusion.

Finally, since the radius of the metric is time, arc lengths become a function of distances in time rather than space. Imagine an observer at some time *r* in Figure 1 such that (as has been stated) the Universe in front of her looks the same as that behind her. If she is centered in a ring, the point on the ring in front of her will be the same distance in space and time away from her as the point directly behind her. Since, according to the metric, arc lengths are measured using time as the radius, the circumference of the ring will be the 2π times the distance from the observer to the ring in time, rather than space. So the area of the celestial spheres of the past appear to be growing because we are moving farther away from them in time and thus the radii relative to us are increasing. The radius of the CMB itself will appear to grow to a maximum radius of $\sim u$ (the CMB will have a slightly smaller radius since it occurred slightly after $r = u$) as we approach $r = 0$.

Freefall Through Time

Let us take the center of our galaxy as the origin of an inertial reference frame. We can draw a line through the center of the reference frame that extends infinitely in both directions radially outward. This line will correspond to fixed angular coordinates (θ, ϕ). There are infinitely many such lines, but since we have an isotropic, spherically symmetric

Universe, we only need to analyze this model along one of these lines, and the result will be the same for any line.

The radial distance in this frame is kind of a compound dimension. It is a distance in space as well as a distance in time. The farther away a galaxy is from us, the farther back in time the light we currently receive from it was emitted. Fortunately the $\frac{u}{r} \geq 1$ spacetime of the Schwarzschild solution plotted in Kruskal-Szekeres coordinates provides us with a method to understand this radial direction. Figure 1 showed the $\frac{u}{r} \geq 1$ solution on a Kruskal-Szekeres coordinate chart where, in this model, the hyperbolas of constant *r* represent spacelike slices of constant cosmological time and the rays of *t* represent spatial distances. We will not be considering differences in angles until a later section in the paper, so we only need to consider the two halves of Figure 1. We will focus on the upper half where the half represents an observer pointed in a particular direction and the positive *t*'s represent the coordinate distance from the observer in that particular direction while the negative *t*'s represent coordinate distance in the opposite direction.

We must first determine the paths of inertial observers in the spacetime. For this we need the geodesic equations for the internal Schwarzschild metric [1] given in Equation 1. In these equations u represents a time constant that in the external metric would be the Schwarzschild radius (in Figure 1, the value of u is 1). The following equations are the geodesic equations for *t* and *r* ($r \leq u$):

$$\frac{d^2t}{d\tau^2} = \frac{u}{r(u-r)}\frac{dr}{d\tau}\frac{dt}{d\tau} \tag{2}$$

$$\frac{d^2r}{d\tau^2} = \frac{u}{2r^2}\left[\frac{u-r}{r}\left(\frac{dt}{d\tau}\right)^2 - \frac{r}{u-r}\left(\frac{dr}{d\tau}\right)^2\right] - (u-r)\left(\frac{d\Omega}{d\tau}\right)^2 \tag{3}$$

In Equations 1, 2, and 3, we use units where $c = 1$ and equations 2 and 3 assume no angular motion. Looking at points $0 < r < u$, then by inspection of Equation 2 it is clear that an inertial observer at rest at *t* will remain at rest at *t* ($\frac{d^2t}{d\tau^2} = 0$ if $\frac{dt}{d\tau} = 0$). Also, we see that if an observer is moving inertially with some initial $\frac{dt}{d\tau}$, then if $\frac{dr}{d\tau} < 0$, the coordinate speed of the observer will be reduced over time (the coordinates are expanding beneath her) and if $\frac{dr}{d\tau} > 0$, the coordinate speed will be increased over time (the coordinates are collapsing beneath her).

Let us therefore examine Equation 3 for an observer with no angular motion. Combining Equations 1 and 3, equation 3 becomes:

$$\frac{d^2r}{d\tau^2} = -\frac{u}{2r^2}\left[1 + \left(\frac{d\Omega}{d\tau}\right)^2\right] - (u-r)\left(\frac{d\Omega}{d\tau}\right)^2 \tag{4}$$

For $\frac{d\Omega}{d\tau} = 0$, notice that the observer's acceleration through cosmological time is similar to the form of Newton's law of gravity, where *r* (a time coordinate) varies from *u* to 0 (If the

Schwarzschild constant was $2GM$, as it would be in the external solution, Equation 4 would be Newton's gravity). Also, anyone moving inertially starting with non-zero $\frac{dt}{d\tau}$ will experience the same acceleration through time as someone with zero $\frac{dt}{d\tau}$ since dt does not appear in Equation 4.

So we will first use Figure 1 to describe the freefall of the galaxies through the cosmological time dimension where galaxies (or galaxy clusters) follow lines of constant t (and any such observer can choose $t = 0$ as their coordinate). The 'Big Bang' will have occurred in Figure 1 along the line $r = 1$. We know this because the above analysis showed that space expands if $\frac{dr}{d\tau}$ is negative, so for our current cosmological time, our worldlines must be moving toward $r = 0$.

The Scale Factor

Expressions for the proper time interval along lines of constant t and Ω and the proper distance interval along hyperbolas of constant r and Ω from Equation 1 are:

$$\frac{dr}{d\tau} = \pm\sqrt{\frac{u-r}{r}} = \pm a \qquad (5)$$

$$\frac{ds}{dt} = \pm\sqrt{\frac{u-r}{r}} = \pm a \qquad (6)$$

Where a is the scale factor. First we should notice that neither Equation 5 nor 6 depend on the t coordinate. This is good because the t coordinate marks the position of other galaxies relative to ours. Since all galaxies are freefalling in time inertially, the particular position of any one galaxy should not matter. The proper velocity and proper distance only depends on the cosmological time r.

What is notable here is that in Schwarzschild coordinates, the scale factor is equal to the velocity through the time dimension for an observer at rest $\left(\frac{dt}{d\tau} = \frac{d\Omega}{d\tau} = 0\right)$. When $r = u$, Equations 5 and 6 are both 0. At this point (the Big Bang), it is our proper velocity in time that is zero. So at that instant, we are no longer moving through time and therefore all points in space are coincident (the observer can reach every point in space without moving through time, all paths are light-like). So this why the scale factor goes to zero there and why the lines of t in Figure 1 converge at that point; it is an instant where our velocity through cosmological time goes to zero as our speed through cosmological time changes from positive to negative (we can see that if we draw a worldline through the center point, $\frac{dr}{d\tau}$ will change signs as it passes the $r = u$ point). In fact, for any choice of time coordinate, that point will be a stationary point in those coordinates.

At $r = 0$, both equations 5 and 6 are infinite. So when the worldlines enter or exit one of the $r = 0$ hyperbolas, they do so at infinite proper speed *through the time dimension*. If something is travelling through space at the speed of light, the proper distance between points in space is zero. In this case, since we have infinite proper velocity in the time dimension, the proper distance between points in space will be infinite, because you would traverse an infinite amount of time in order to move through an infinitesimal amount of space. What we see then is that at $r = 0$ space will be infinitely expanded and thus the scale factor is infinite. A plot of the scale factor vs. r (with $u = 1$) is given in Figure 2 below:

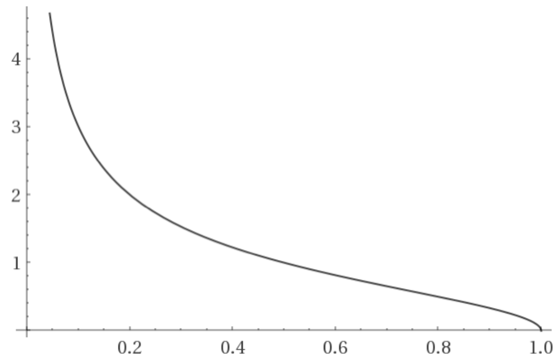

Figure 2 – Scale Factor vs. r for $u = 1$

Cosmological Parameters

In order to compare this model to cosmological data, we must solve for u and find our current position in time (r_0) in the model. Reference [3] gives us a 95% confidence interval for the measured transition redshift at $z_t = 0.426^{+0.27}_{-0.089}$. We can use the fact that $\sqrt{\frac{u-r}{r}}$ is the scale factor and get the expression for cosmological redshift caused by the expansion [1] (note that this Equation was derived from the FRW metric in the reference, but the internal metric, when setting $d\Omega = 0$, can be put in the same form as the FRW metric with a coordinate change, so the equation below is still valid for the internal metric):

$$z = \sqrt{\frac{r_{emit}}{(u-r_{emit})}}\sqrt{\frac{u-r}{r}} - 1 \tag{7}$$

We can see in Figure 2 that there is an inflection point that corresponds to the transition redshift in the model. To find this inflection point, we need to derive the Hubble parameter and deceleration parameter equations using the scale factor. The Hubble parameter is given by:

$$H = \frac{\dot{a}}{a} = \frac{d}{dr}\left(\sqrt{\frac{u-r}{r}}\right)\sqrt{\frac{r}{u-r}} = \frac{u}{2r(u-r)} \tag{8}$$

And the deceleration parameter is given by:

$$q = \frac{\ddot{a}a}{\dot{a}^2} = \frac{4r}{u} - 3 \tag{9}$$

The transition redshift occurs when $q = 0$, giving us $\left(\frac{r}{u}\right)_t = 0.75$. With this and Equation 7, we can find $\left(\frac{u}{r}\right)_0$:

$$z_t = 0.426^{+0.27}_{-0.089} = \sqrt{\frac{1}{\frac{1}{0.75}-1}}\sqrt{\left(\frac{u}{r}\right)_0 - 1} - 1 \rightarrow \left(\frac{u}{r}\right)_0 = \left(\frac{1.426^{+0.27}_{-0.089}}{\sqrt{3}}\right)^2 + 1 \quad (10)$$

Giving:

$$\left(\frac{u}{r}\right)_0 = 1.678^{+0.281}_{-0.082} \quad (11)$$

The current Hubble constant, as measured by the Planck mission was found to be $H_0 = 67.8 \pm 0.9\ (\mathrm{km/s})/\mathrm{Mpc}$ and from the Hubble Space telescope $H_0 = 73.48 \pm 1.66\ (\mathrm{km/s})/\mathrm{Mpc}$. With these and Equation 11, we can solve for limiting values of *u* and r_0 (after converting the units of H_0 so that *u* is measured in Gly):

$$H_0 = \left(\frac{u}{r}\right)_0 \left[\frac{1}{2u\left(1-\left(\frac{r}{u}\right)_0\right)}\right] \rightarrow u = \left(\frac{u}{r}\right)_0 \left[\frac{1}{2H_0\left(1-\left(\frac{r}{u}\right)_0\right)}\right] \quad (12)$$

Note that in Equation 12, H_0 is in units of $(Gy)^{-1}$. Before presenting the results, let us derive the expression for *t* vs. *r* along a null geodesic where the geodesic ends at the current time r_0. We can do this by setting $d\tau = rd\Omega = 0$ in Equation 1 and integrating:

$$t = \int_{r_0}^{r} \frac{r}{u-r} dr = u\ln\left(\frac{u-r_0}{u-r}\right) + (r_0 - r) \quad (13)$$

Table 1 below gives the values of u, r_0, a_0, q_0, r_t (coordinate time at transition redshift), H_t (Hubble constant at the transition redshift), and t_t (coordinate distance of transition redshift) given the measured bounds of z_t and H_0. All times are in Gy, distances are in Gly, and *H* are in $(\mathrm{km/s})/\mathrm{Mpc}$.

z_t	H_0	u	r_0	$u - r_0$	a_0	H_t	r_t	t_t	$r_t - r_0$	q_0
0.337	68.7	30.4	19.0	11.4	0.77	85.8	22.8	8.5	3.8	-0.5
0.337	66.9	31.2	19.5	11.7	0.77	83.6	23.4	8.8	3.9	-0.5
0.337	75.14	27.8	17.4	10.4	0.77	94.3	20.9	7.9	3.5	-0.5
0.337	71.82	29.1	18.2	10.9	0.77	89.5	21.8	8.1	3.6	-0.5
0.696	68.7	28.5	14.3	14.2	1.00	91.8	21.4	12.7	7.1	-1.0
0.696	66.9	29.3	14.7	14.6	1.00	89.3	22.0	13.0	7.3	-1.0
0.696	75.14	26.0	13.0	13.0	1.00	100.4	19.5	11.5	6.5	-1.0
0.696	71.82	27.3	13.7	13.6	1.00	95.8	20.5	12.1	6.8	-1.0

Table 1: Limiting Cosmological Parameter Values Based on z_t and H_0 Measurements

Note that these values cannot be calculated for the CMB because of lack of precision in z_t and H_0 measurements (The CMB is too close to $r = u$ to get meaningful values given the imprecise measurements). Table 2 has the proper times from the Big Bang to the transition redshift and current time for stationary, inertial observers ($dt = rd\Omega = 0$) by integrating

Equation 1 (there is not enough precision in the measurements to calculate this for the CMB). The column τ_{tot} gives the time from $r = u$ to $r = 0$. The expression for τ_{tot} turns out to be quite simple[3]:

$$\tau_{tot} = \frac{\pi}{2} u \tag{14}$$

The column τ_{remain} gives the time between $r = r_0$ and $r = 0$.

z_t	H_0	τ_0	τ_t	τ_{tot}	τ_{remain}
0.337	68.7	34.6	29.1	47.8	13.2
0.337	66.9	35.7	30.1	49.2	13.5
0.337	75.14	31.7	26.5	43.7	12.0
0.337	71.82	33.3	27.9	45.7	12.4
0.696	68.7	36.3	27.2	44.8	8.5
0.696	66.9	37.4	28.0	46.0	8.6
0.696	75.14	33.3	25.1	41.0	7.7
0.696	71.82	34.8	26.2	42.9	8.1

Table 2: Limiting Proper Times Based on z_t and H_0 Measurements (Time is in Gy)

Note that while the coordinate times for the current age of the Universe $(u - r_0)$ are close to current estimates (for high z_t), the proper time τ_0 is actually much larger. This is because in the early Universe, observers are moving slower through the time dimension and therefore they accrue more proper time per unit coordinate time early on. But the speed through the time dimension increases over time such that even though we are presently only about halfway through the "coordinate life" of the Universe (according to Table 1), the amount of proper time remaining is actually much less than the amount of proper time that has already passed (according to Table 2).

Next we would like to use the u and r_0 values found to create an envelope on a Hubble diagram to compare to measured supernova data. First we need to find r as a function of redshift. We can do this by solving for r_{emit} in Equation 7 where $a_0 \equiv \sqrt{\frac{u-r}{r}}$, the present value of the scale factor:

$$r = u \frac{z^2+2z+1}{a_0^2+z^2+2z+1} \tag{15}$$

Next we substitute Equation 15 into Equation 13 to get coordinate distance in terms of redshift:

$$t = u \left[\ln \left(\frac{r_0 (a_0^2+z^2+2z+1)}{u} \right) - \frac{z^2+2z+1}{a_0^2+z^2+2z+1} \right] + r_0 \tag{16}$$

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[3] Thinking of τ_{tot} as a 'Universal Period' allows us to define a Universal constant $U = \frac{\pi}{2} u$ for time and space. Equation 14 is the maximum amount of time that can be measured between the Big Bang and $r = 0$. So if we set $U = \frac{\pi}{2} u = c = 1$ then we are working in units where space and time have the same units and all measurable times will be between 0 and 1. When working in these units, the constant in the interior Schwarzschild metric will be $u = \frac{2}{\pi}$.

Finally, we convert Equation 16 to the distance modulus, μ, which is defined as:

$$\mu = 5\log_{10}\left(\frac{t}{10}\right) \tag{17}$$

Where t in Equation 17 is in units of parsecs. A plot of distance modulus vs. redshift is shown in Figure 3 below plotted over data obtained from the Supernova Cosmology Project [6]. Curves calculated from all combinations of u and r_0 in Table 1 are plotted, giving an envelope for the model's prediction of the true Hubble diagram.

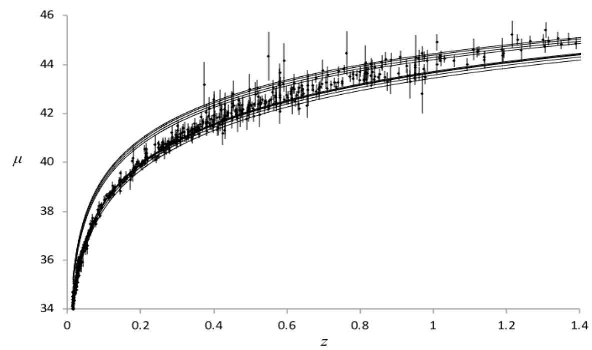

Figure 3 – Distance Modulus vs. Redshift Plotted with Supernova Measurements

Note that the lower curves correspond to the $z_t = 0.696$ data, suggesting that, if this model is correct, the true transition redshift is closer to 0.696 than 0.337.

In [7], the authors analyze a large sample of quasar data to obtain distance moduli at higher redshifts than is possible with supernova data. Although not definitive, the results of this analysis suggests that the "Dark Energy" density may be increasing with time, which does not fit with the ΛCDM model. However, the accelerated expansion predicted by the Schwarzschild solution *is* consistent with this type of expansion. Figure 4 shows the same predicted envelope from Figure 3 for the Hubble diagram plotted out to higher redshifts with the quasar data from [7] also shown with error bars. The black diamonds in the figure are the 18 high-luminosity XMM-Newton quasar points described in [7].

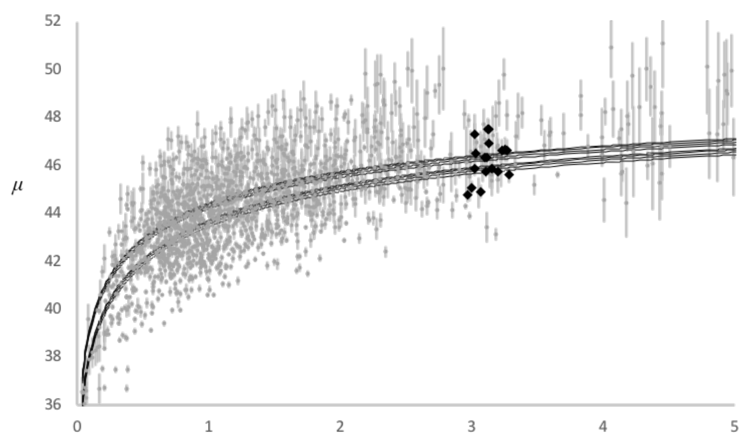

Figure 4 – Distance Modulus vs. Redshift Plotted with Quasar Measurements

The ‘Big Bounce’?

A plot of τ vs. *r* from the uppermost to lowermost hyperbola in Figure 1 is given in Figure 10 below. It illustrates well the relationship to typical spatial projectile motion (for $u = 1$).

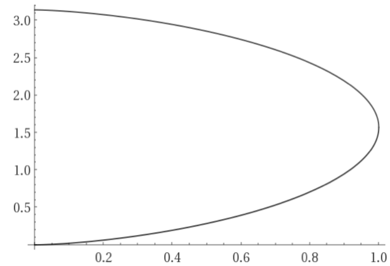

Figure 10 - τ vs. *r*

Consider a perfectly rigid and elastic ball in simple Newtonian mechanics. If we throw it straight up in the air with initial velocity $\frac{dx}{d\tau}$, the velocity will continuously decrease until at some height $\frac{dx}{d\tau} = 0$, at which point the ball will reverse direction and fall with increasingly negative $\frac{dx}{d\tau}$ until it returns to the ground. When it hits the ground (which we will assume has infinite inertia), since the ball is perfectly rigid and elastic, it will experience an infinite acceleration that will bounce it back toward its maximum height and this cycle will continue ad infinitum. So, there are two turnaround points for the ball. One point is maximum height, where the ball does not experience any special acceleration; it just stops moving through space as it turns around. The second point is a hard acceleration that the ball can really feel a (infinite) force changing its direction.

Likewise, we can see that the cosmology in question is a similar situation except that the Universe is the ball and the acceleration is through time rather than space. Looking at Figure 1, we can imagine an inertial observer moving from the top of the diagram to the bottom along a line of constant *t*. At the center of the diagram, the Big Bang, the motion of the inertial observer relative to the time coordinate shifts from positive to negative $\left(\frac{dr}{d\tau} = 0\right)$. This is analogous to the ball reaching its maximum height. Then, as it reaches $r = 0$, we find that $\frac{d\tau}{dr} = 0$ which we might interpret as the worldline instantly turning back on itself, analogous to the infinite acceleration of the rigid ball when it hits the ground. But since the Universe is quantum in nature and there would be some uncertainty in the coordinate time corresponding to the present Universe, this turnaround may not be as sharp and abrupt as the classical theory predicts. The ‘quantum blur’ of the Universe might smooth out that singularity, giving the Universe what would be analogous to giving the ball some stiffness, such that when it bounces, it does so in a smooth way.

Relationship to the External Solution

Let us consider a meter stick at rest at the center of a collapsing spherically symmetric collapsing shell in space. The meter stick inside the shell stretches from the center of the shell out to a distance *2GM* (the shell is at a radius greater than *2GM* so the entire stick is in flat space). An observer in freefall on the collapsing shell does so with speed (in natural units measured by her clock) [5]:

$$\frac{dr}{d\tau} = -\sqrt{\frac{2GM}{r}} \tag{29}$$

Therefore, the freefall observer will see observers at rest at *r* moving past her at the speed given in Equation 29. Since the meter stick is also at rest relative to observers at rest at any *r*, Equation 29 will also give the relative velocity between the freefall observer and the meter stick when the shell is at *r*. Since the spacetime between the freefall observer and central observer is flat, they will each see the other's clock dilated by the Special Relativity Relationship:

$$d\tau = dt\sqrt{1 - V^2} = dt\sqrt{1 - \frac{2GM}{r}} \tag{30}$$

Because the meter stick will appear to be moving in the frame of the freefalling observer, its length in her frame would be:

$$L = 2GM\sqrt{1 - \frac{2GM}{r}} \tag{31}$$

We see from Equation 31 that as the freefalling observer approaches $r = 2GM$ the length of the meter stick in her frame will contract to zero length. So observers in freefall will see the space beyond $r = 2GM$ fully contracted as they approach $r = 2GM$. Furthermore, the clock of an observer at the center of the shell will be slowed as the shell collapses (the clock of an observer at the center ticks at a rate equal to an observer at rest at the location of the shell) such that if she exchanges light signals with the shell as it collapses, the time she measures for the light to return will shrink to zero as the shell reaches the Schwarzschild radius. Thus, she also effectively sees the space within the shell shrink to zero as the shell approaches the Schwarzschild radius.

But the freefalling observer of the external solution will never fall into a 'black hole'. It would take an infinite amount of time in the frame of an observer at infinity for the freefalling observer to reach the event horizon. But the Universe itself will reach $r = 0$ in a finite amount of time in the frame of the infinite observer and therefore the freefalling observer will only reach the $r = 2GM$ location when the entire Universe has reached $r = 0$. Thus, she will never actually reach any event horizon, she will reach $r = 0$ when the entire Universe has reached $r = 0$.

Conclusion

By adding the assumption that the mass/energy of the Universe only exists as a shell surrounded by a vacuum of time (i.e. that the Universe does not pre-exist at all times, but moves through time) to the Cosmological Principle, the following can be concluded:

- The metric of the Universe is described by the black hole metric where the radii and arc lengths are measured in time rather than space
- The Big Bang is an infinitely dense sphere with finite 2D area
- The metric predicts that the expansion of the Universe will undergo a period of deceleration followed by a period of acceleration without the need for a Cosmological Constant.
- It was shown that the inhomogeneity of the metric does not result in the observation of an inhomogeneous Universe since the radius of the Universe is the time dimension and all light comes to us from the past direction. The result of this is that there is no 'preferred direction' since all light we observe comes from the same direction in time.
- By using data for the current value of the Hubble constant and estimates for the transition redshift, bounds on the values for the constant u in the metric and the current time r_0 were calculated and the Hubble diagram for the metric was compared to supernova and quasar data. It was found that if the true transition redshift is close to 0.69, then the model fits well with the observed data.
- It is conjectured that as the Universe reaches the center of the metric (a finite time in the future), it will bounce and worldlines will reverse on themselves evolving back toward the Big Bang. It was also speculated that quantum uncertainty might smooth out the bounce, acting as some finite stiffness for the Universe.

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