Mass interaction principle as a common origin of special relativity and quantum behaviours of massive particles

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Abstract

The author believes there are spacetime particles(STP) which can sense all matter particles ubiquitously. Matter particles will change their states collided by STP . The underlying property of mass is a statistical property emerging from random impact in spacetime. We propose a mass interaction principle (MIP) which states any particle with mass m will involve a random motion without friction, due to random impacts from spacetime. Each impact changes the amount nh (n is any integer) for an action of the particle. Starting from the concept of statistical mass, we propose the fundamental MIP. We conclude that inertial mass has to be a statistical property, which measures the diffusion ability of all matter particles in spacetime. We prove all the essential results of special relativity come from MIP. Speed of light in the vacuum need no longer any special treatment. Instead, speed of STP has more fundamentally physical meaning, which represents the upper limit of information propagational speed in physics. Moreover, we derive the uncertainty relation asserting a fundamental limit to the precision regarding mass and diffusion coefficient. Within this context, wave-particle duality is a novel property emerging from random impact by STP. Further more, an interpretation of Heisenberg's uncertainty principle is suggested, with a stochastic origin of Feynman's path integral formalism. It is shown that we can construct a physical picture distinct from Copenhagen interpretation, and reinvestigate the nature of spacetime and reveal the origin of quantum behaviours from a realistic point of view.

Keywords: *Mass Interaction Principle , Special Relativity, Schrödinger Equation , Quantum Measurement , Entanglement , Uncertainty Relation, Path Integral , Neutrino*

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I. INTRODUCTION

A. Spacetime Fluctuation, STP and Mass Interaction Principle

We believe the energy fluctuations of spacetime are universal, which are defined as STP. Matter particles change their states by all the collisions with STP. The underlying property of mass is a statistical property emerging from random impacts in spacetime. Different particles have different effects of impact by STP, which can be defined as some kind of inertia property of particles. This property corresponds to mass dimension (Following we will prove it happens to be the inertial mass from Schrodinger's equation). Matter particles develop a Brownian motion due to random impacts from spacetime. We strongly suggest that all the probabilistic behaviours of quantum mechanics come from the Brownian motion, which is exactly the origin of quantum nature. However, quantum nature of massless photons inherit from charged matter particle through their electromagnetic interactions. We believe the quantum behaviours of matter particle come from spacetime fluctuation.The energy fluctuation of spacetime is quantised too, among all the particles in spacetime which receive quantised energy from STP. These unique random impacts leads to Brownian-like motions, since exchange of energy between particle and spacetime is not strictly random. Once the time interval of impact are fixed, the exchange of energy has to be quantised, which indeed is the quantum nature of particles. Therefore, all quantum nature of particles is a faithful representation of spacetime quantised fluctuation.

Definition 1. Matter particles will perform random fluctuation motion in spacetime because of stochastic interactions between STP and matter particles, which is not a instantaneous interaction.

For every random impact, the time interval is related to the exchange energy between STP and matter particles. We define $\delta E * \delta t = \delta S$, which is the variation of action of matter particle for every impact. According to the above two fundamental propositions: 1. spacetime fluctuations are universal; 2. spacetime fluctuations are quantised, we propose a mass interaction principle: Any particle with mass m will involve Brownian-like motions without frictions, which is a Markov process, due to random impacts from spacetime. Each impact changes the amount nh (n is any integer)for an action of the particle. The MIP is absolutely essential to all quantum behaviours and mass property of particle, which must be a fundamental principle. We will prove many important principles of modern quantum mechanics can be derived from mass interaction principle. Within this framework, mass interaction principle plays the role of the zeroth interaction, which dictates all quantum behaviours. Moreover, it will be shown that modern quantum field theory is compatible with the mass interaction principle in the sense of quantum statistical partition functions.

B. Inertia Mass is a Statistical Property

Until now, our knowledge of mass, a fundamental concept of physics, comes from Newton's laws of motion especially the first and second laws. The first law states that in an inertial reference frame, an object either remains at rest or continues to move at a constant speed, unless acted upon by a force. However according to the mass interaction principle, free particle has to do Brownian-like motions in spacetime, which is a Markov process. The mass of particle, in order to be sensed by spacetime, has to be collided randomly by STP. Mass cannot be well defined within the interval of two consecutive random collisions. In other words, mass is not a constant property belongs to the particle itself, is a discreet statistical property depends on dynamical collisions of spacetime. We will derive from MIP straightforwardly that mass must be a statistical term which has its own meaning and fluctuations. Moreover, we prove the uncertainty relation asserting a fundamental limit to the precision regarding mass and diffusion coefficient. This implies that both mass and diffusion coefficient of any particle state can not simultaneously be exactly measured. Newton's Second law states that in an inertial reference frame, the vector sum of the forces F on an object is equal to the mass m of that object multiplied by the acceleration of the object. This connects the concept of mass and inertia and in principle defines a fundamental approach to measure the mass of any particle experimentally. However, according to the mass interaction principle, forces on a particle are changed constantly by the random impact of STP. Therefore, we are no longer able to take constant mass for granted. In conclusion, we believe that mass as a statistical property is much more natural within the framework of modern science, which completely overrules Newton's concept of mass based on Mathematical Principles of Natural Philosophy first published in 1687.

C. Mass Interaction Principle and Special Relativity

Due to the stochastic interactions between STP and matter particles, matter particles will perform random fluctuation motions in spacetime. Distinct from classical speed, the fluctuation speed is a relative one with time reversal invariant property. Because this kind of Markov fluctuation is irrelevant with classical motion, it has to remain invariant under transformation of coordinate. This very effect demonstrates two fundamental postulates of special relativity, which are invariant speed of light in vacuum and equivalent inertial reference frame. Importantly, the invariant speed of light is a natural consequence within the framework of mass interaction principle. The postulate of equivalent inertial reference frame is also a natural consequence of irrelevance between classical motion and fluctuation motion. Therefore, we are able to derive all the major effects of special relativity from mass interaction principle. Within our framework, "moving mass", "time dilation" and "length contraction" all have novel physical origins.

D. Realistic Interpretation of Quantum Mechanics

The main idea of Copenhagen interpretation is that the wave function does not have any real existence in addition to the abstract concept. Whether the wave function is an independent entity, the Copenhagen interpretation does not make any statement. In this article we do not deny the internal consistency of Copenhagen interpretation. We admit that Copenhagen′s quantum mechanics is a self-consistent theory. Einstein believed that for a complete physical theory, there must be such a requirement: a complete physical theory should include all of the physical reality, not merely its probable behaviour. From the materialistic point of view, the physical reality should be measured in principles , such as the position q and momentum p of particles. In the Copenhagen interpretation, the particle wave function $\Psi(q,t)$ or the momentum wave function $\Psi(p,t)$ is taken to be the only description of the physical system, which can not be called a complete physical theory. Therefore, in this paper, we propose a mass interaction principle where the coordinate and momentum of particles are objective reality irrespective of observations . With the postulation of mass interaction principle, quantum behaviour will emerge from a statistical description of spacetime random impacts on the experimental scale, including Schrodinger′s equation, Born rule, Heisenberg′s uncertainty principle and Feynman′s path integral formulation. Thus, we believe that non-relativistic quantum mechanics can be constructed under the mass interaction principle. Born rule and Heisenberg′s uncertainty relation are no longer fundamental within our framework.

The paper is organized as follows. The basic hypothesis of mass interaction principle are described in Sec. 2. The derivation of special relativity are given in Sec. 3. Section 4 illustrates the universal diffusion coefficient in spacetime for Brownian-like motions according to mass interaction principle. In Sec. 5, we introduce non-relativistic statistical inertia mass. Section 6 derives a specific universal diffusion coefficient which satisfies Schrodinger's equation. In Sec. 7, we introduce a novel physical origin of quantum spin. In Section 8, an interpretation of Heisenberg′s uncertainty principle is suggested, with a stochastic origin of Feynman′s path integral formalism. It is shown that we can construct a physical picture distinct from Copenhagen interpretation, and reinvestigate the nature of spacetime and reveal the origin of quantum behaviours from the realistic point of view. In Section 9, we reinterpret quantum measurement within the framework of mass interaction principle. The notorious problem of wave packet collapse became well understood. We also explain continuous measurements and quantum entanglements in our framework. Last but no least, we give summaries and outlook of future work.

II. MASS INTERACTION PRINCIPLE

A. Proposing the mass interaction principle

The structure of spacetime, or equivalently the distribution of spactime particles, plays the role to detect the mass of a matter particle moving in spacetime. There are various structures or distributions of STP which meets the criterior as a source of the frictionless Brownian-like motion particle. For example, we can propose two self-consistent structures of spacetime as following.

1. The microscopic spacetime is a discrete system. Matter particles in spacetime could be collided by STP. Though in large scale, spacetime is continuously smooth, the possibility of incontinuity in very small scale allows STP to fluctuate. The randomness of STP' fluctuations will be transmitted to matter particle moving in spacetime.

2. The mass detective particles will be generated from spacetime randomly, because of the engery fluctuation of microscopic spacetime. They can scatter with matter particle randomly, hence change the motion of the matter particle. In specific, mass of the matter particle will emerge as its statistical property.

Apparently there are many possible self-consistent structurs describing the microscopic structures of spacetime. It is beyond the scope of this article to determine which one is valid. We emphrase that the emergent motion of the matter particle is a frictionless Brownian-like motion, ruled by a Markov process. The above two kinds of spacetime mass detective structures will be tested in the following, where we will show the derivation of the spacetime diffusion coefficent.

We propose in each interaction between matter particle and STP, the exchanging action should be nh, with n integer and h the Planck constant. In our MIP framework, there are no instant interactions between matter particle and STP, in other words, the interaction takes time to transfer the energy. If the scattering STP has an extremely low energy such that in Δt, the transfered action is less then h , we conclude that in Δt, the STP cannot collide the particle. Classically, we argue that such a collision is still in process, the particle as well as the STP are in a bound state, not a scattering state. This is similar to a completely inelastic collision in classic mechanics. While in such a process, the conservation of energy and momentum can not be satisfied simultaneously. Because of conservation of energy and momentum, the bound state actually is not a stable state. This observation leads to an important point: there exists a minimal energyE_{min} in Δt so that

$$E_{min}\Delta t = const. \tag{1}$$

In physics , the product of energy and time will have the dimension of action. It is natural to suggest such a constant with action dimension is the Planck constant, so we have

$$E_{min}\Delta t = nh, \quad n \in \mathbb{Z}. \tag{2}$$

At a certain moment, particle can be scattered by many STP with different momenta and energies. In Δt, we assume there are effectively N collisions. The state of the motion will depend on the net effect of the N times collision. This is a principle of superposition. We can use in total N vectors to superposite whole changes of the state of motion, which means if at time t the particle was at position $\vec{X}(t)$, with speed $\vec{V_0}$, then at the moment $t + \Delta t$, its position will be $\vec{x}(t + \Delta t) = \vec{X}(t) + \sum_{i=1}^{N} \Delta X_i$, and speed$\vec{V_0} + \sum_{i=1}^{N} \Delta \vec{V_i}$. This simple analysis tells us in Δt, the ultimate state of motion of the particle can be separated as N different paths. This is the effect of separation of paths. While the weights of these paths, *aka* the probability distribution of universal diffusion, highly rely on the energy distribution of STP. Collisions by STP with different energies end up with different changes of the state of motion.

B. Energy spectrum of STP

To consider the collision between STP and particle, it will be ambiguous if the energy spectrum of STP is not clear at first. In this subsection, we deal with the problem.

Let us consider a cubic with volume L^3, which we call a system. If there are in total N systems in spacetime, we can classify the N systems by states. We label a state by j so that there are N_j systems with energy E_j. The total energy of the ensemble(collection of N systems) is denoted as $\mathcal{E}$, we have

$$N = \sum_j N_j \tag{3}$$

$$\mathcal{E} = \sum_j N_j E_j, \tag{4}$$

for constant $\mathcal{E}$ and N, the possible total number of states in whole spacetime will be $\Omega = \frac{N!}{\prod_j N_j!}$. Physical reality is required by the maximum of Ω. There is a distribution $\{N_j\}$ maximizing Ω, so that

$$\ln\Omega = N\ln N - N - \sum_j N_j \ln N_j + \sum_j N_j \cdots \tag{5}$$

the question is under constraints (3,4), how to maximize $\ln\Omega$. With the method of Lagrangian multiplier,

$$\frac{\partial \ln\Omega}{\partial N_j} - \lambda_1 \frac{\partial \sum_j N_j}{\partial N_j} - \lambda_2 \frac{\partial\left(\sum_j N_j E_j\right)}{\partial N_j} = 0 \tag{6}$$

we can derive

$$\begin{aligned} -\ln N_j - \lambda_1 - \lambda_2 E_j &= 1 \Rightarrow \\ N_j &= e^{-1-\lambda_1-\lambda_2 E_j} \end{aligned} \tag{7}$$

hence the probability of being at state j

$$P_j = \frac{N_j}{N} = \frac{e^{-\lambda_1-\lambda_2 E_j}}{\sum_j e^{-\lambda_1-\lambda_2 E_j}} = \frac{e^{-\lambda_2 E_j}}{\sum_j e^{-\lambda_2 E_j}} \equiv \frac{e^{-\lambda_2 E_j}}{\mathcal{Z}} \tag{8}$$

and the average energy of the ensemble

$$E = \frac{\mathcal{E}}{N} = \sum_j E_j P_j = -\frac{\partial}{\partial\lambda_2}\ln\mathcal{Z} \tag{9}$$

In L^3 , suppose there are $n_{\vec{p}} = 0, 1, 2, \cdots$ STP have momentum $\vec{p}$, for giving distribution $\{n_{\vec{p}}\}$, energy in L^3 is

$$E = \sum_{\{n_{\vec{p}}\}} n_{\vec{p}} E_{\vec{p}} \tag{10}$$

with$E_{\vec{p}} = c|\vec{p}| = cp$. Here STP are massless as proposed. We have

$$\begin{aligned} \mathcal{Z} &= \sum_{\{n_{\vec{p}}\}} e^{-\lambda_2 E} = \prod_{\vec{p}} \left(1 + e^{-c\lambda_2 p} + e^{-2c\lambda_2 p} + \cdots\right) \\ &= \prod_{\vec{p}} \frac{1}{1 - e^{-c\lambda_2 p}} \end{aligned} \tag{11}$$

and the average energy of a system is

$$\begin{aligned} E &= -\frac{\partial}{\partial \lambda_2} \ln \mathcal{Z} = \frac{\partial}{\partial \lambda_2} \sum_{\vec{p}} \ln\left(1 - e^{-c\lambda_2 p}\right) \\ &= \sum_{\vec{p}} \frac{p e^{-c\lambda_2 p}}{1 - e^{-c\lambda_2 p}} = \sum_{\vec{p}} \frac{cp}{e^{c\lambda_2 p} - 1} \end{aligned} \tag{12}$$

when $L \to \infty$, summation becomes integration as follows

$$\sum_{\vec{p}} \to \frac{L^3}{8\pi^3} \int d^3\vec{p}$$

from which we see

$$E = \frac{L^3}{2\pi^2} \int dp \frac{p^3}{e^{c\lambda_2 p} - 1} = \frac{\pi^2 L^3}{30\lambda_2^4} \tag{13}$$

so the density of STP will be

$$\epsilon_{ST} = \frac{\pi^2}{30\lambda_2^4} \tag{14}$$

Recover c and $\hbar$ in above equation, we obtain

$$\epsilon_{ST} = \frac{\pi^2}{30c^3\hbar^3\lambda_2^4}. \tag{15}$$

Now consider the physical meaning of λ_2, which determines the constraint that representing energy distribution of STP. While the multiplier λ_1 which determines the constraint represents the number distribution of STP has no affects on the dynamics of STP. This means we can classify STP arbitrarily, except to satisfy the total energy constraint. From dimensional analysis and MIP, we have

$$\lambda_2 = \frac{g}{\hbar\omega_{ST}} \tag{16}$$

where g is a dimensionless coupling constant, and ω_{ST} is the characteristic frequency of STP. In the limit of extreme relativeity, the colliding of STP can not be seen as perturbations, but strong interaction.

III. MASS INTERACTION PRINCIPLE AND SPECIAL RELATIVITY

In the framework of the principle of mass interaction, the STP do not have self-interaction, so its mass is zero, the speed is constant v_{st}. We now introduce the relative speed invariant hypothesis, which says that the relative speed of STP with respect to the particle in space-time is always the constant v_{st} . The action of STP on a particle is a stochastic dynamic problem. For the particle's coordinate $\vec{x}(t)$, the time derivation $d\vec{x}/dt$, in the strict sense does not exist. In this random process, the instantaneous speed of the particle is a superposition of its classical speed $\vec{v}$ and fluctuation speed $\vec{u}$, that is, $\vec{V} = \vec{v} + \vec{u}$. We know that under time reversion ($T:\ (t, \vec{x}) \to (-t, \vec{x})$), we have

$$T:\ \vec{v} \Rightarrow -\vec{v}, \quad T:\ \vec{u} \Rightarrow \vec{u}$$

This is because the continuous time Markov process, under time reversal, is still a Markov process. The fluctuation speed $\vec{u}$ roots on the collision of STP and the particle. The characteristic of the fluctuation speed is the time reversion invariance. The fluctuation speed has nothing to do with the classical speed under time reversal transformation. So we can always consider the scenario which zero classic speed particle collides with STP. In this scenario, the relative speed of the STP is v_{st} , and will constantly be v_{st} .

Consider the scene where the classic speed is not zero. Since the collision between STP and the particle does not change the classical speed of the particle, it only changes the fluctuation speed. And in the process of the collision betwen STP and the particle, the classic speed is irrelevant. Therefore, at the extreme microscopic level, the speed of STP relative to the particle is constant v_{st} . However, this applies to any moment during the propagation of the particle in spacetime. So at the macro level, the relative speed between STP and the particle is still constant v_{st} . Thus, in all inertial reference systems, we can also conclude that the relative speed between STP and matter particle is v_{st}.

This conclusion can be concisely expressed as the relative particle speed is always v_{st} . Thus, in all inertial systems marked by a classical relative speed, the speed of the space is invariant and is constant v_{st} .

We already know that the mass of the particle actually reflects the collision with STP. The more the collision in the unit time, the greater the statistical mass of the particle. A reasonable inference is that the mass of the particle is proportional to the collision times per

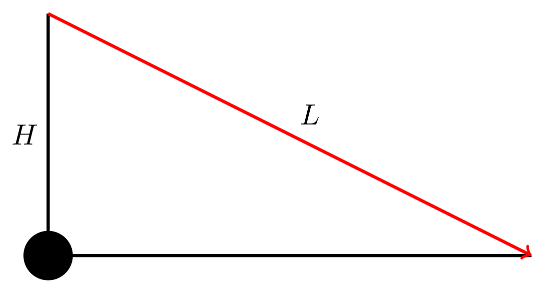


Figure 1: Collision between STP with matter particle.

unit time, that is,

$$m_{ST} = k_{ST}N$$

where N denotes the collision times per unit time , and k_{ST} is a proportional constant.

A. Time Dilation effect

From the relative speed constant assumption, the distribution of STP under the reference system transformation will not change. If the distribution of the STP is uniform and isotropic in the rest frame , then because the speed of STP relative to the particle does not change, in the frame of relative velocity $\vec{v}$. The distribution of STP is still uniform and isotropicl. It should be noted, however, that the time costs of the same collision process in different reference frames are different. This can be explained by the following explanation.

In the rest frame, the STP is at a constant speed v_{st} , moving toward the particle from the distance H . After time t , it will collide with the particle, so the time $t = H/v_{st}$.

However, in the moving frame with constant velocity v, after time t'. The distance between the time and space will be L. And the distance from the particle is$\sqrt{L^2 - H^2}$, Then $t' = L/v_{st}$, the following formula pops

$$\frac{L}{v_{st}} = \frac{\sqrt{L^2 - H^2}}{v}$$

hence

$$t' = \sqrt{t'^2 - t^2}\frac{v_{st}}{v} \Rightarrow T = t'\sqrt{1 - \frac{v^2}{v_{st}^2}}$$

As long as the speed of STP v_{st} equal to the speed of light c , the above equation returns to the relativity of simultaneousity in special relativity. Due to the photon's static mass of 0, the STP and photons have no interaction. On the other hand, due to the isotropy of STP,

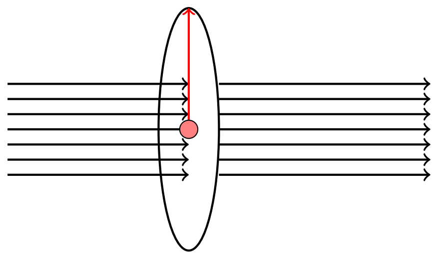


Figure 2: Flux of STP cross a disk.

there can always exist STP moving parallel to the photon, so there are no relative movement between such STP and the photon. Hence the speed of light should be essentially equal to the speed of STP, that is $c = v_{st}$, which is a rigorous conclusion. and is the physical origin of the axiom of invariance of speed of light in special relativity.

B. Relativistic Mass Effect

Now we consider the expression of the particle mass under the frame transformation. Due to the homogeneity and isotropy of STP distribution, we can assume the density of the STP is ρ_0, and in moving frame with constant velocity v, the particle moves along the horizontal direction .

Then in the rest frame, the number of STP passing through the disc on the vertical direction is

$$N_0 = \rho_0 \pi r^2 v_{st} \Delta t$$

The mass of the particle is

$$m_{st} = k_{st} \rho_0 \pi r^2 v_{st} \Delta t$$

In the moving frame with constant velocity v, the number of STP passing through the same disc is

$$N = \rho_0 \pi r^2 v_{st} \Delta t'$$

Since

$$\Delta t' = \Delta t / \sqrt{1 - v^2/v_{st}^2}$$

we have

$$m'_{st} = k_{st} N = k_{st} \rho_0 \pi r^2 v_{st} \Delta t' = m_{st} / \sqrt{1 - v^2/v_{st}^2}$$

When $v_{st} = c$, it is the same expression of relativistic mass as in special relativity.

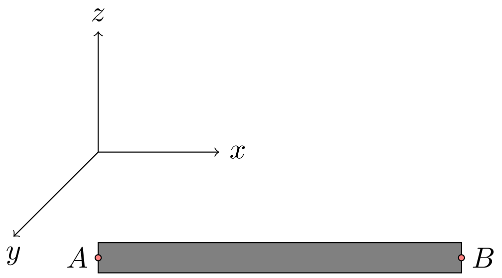


Figure 3: Ruler in rest reference frame.

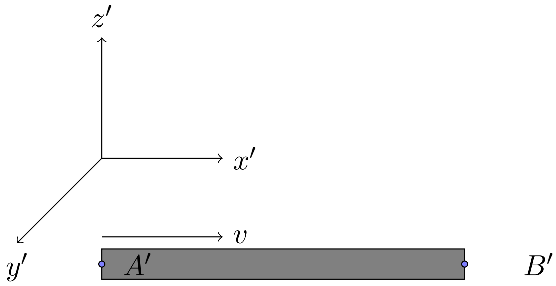


Figure 4: Ruler in moving reference frame.

C. Length Contraction Effect

We consider the relativity of the spatial distance, that is, the measure effect in special relativity. We first consider the rest reference system, the length of the ruler is

$$l_0 = x_B - x_A$$

When the ruler is moving along x- direction in speed of v , as shown in the following figure.

The spacetime coordinates at both ends of the ruler are

$$(x'_A, t'_A), \quad (x'_B, t'_B),$$

as a rigid body, it requires $t'_A = t'_B$. In this coordinate system, the special relativistic transformation is

$$X_A = \frac{x'_A + vt'_A}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad x_B = \frac{x'_B + vt'_B}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence

$$x_B - x_A = \frac{x'_B - x'_A}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and

$$L = x'_B - x'_A = l_0\sqrt{1 - \frac{v^2}{c^2}}$$

In the framework of MIP, we consider the differential distance dx' The MIP requires $\delta(px) = nh = \delta(p'x'),\ n \in \mathbb{Z}$. In all inertial frames, each time the STP acting on matter particle , the changing of action is nh. The basic principle will remain the same regardless of inertial reference frames.

In the motion reference frame, we know that the mass $m' = m_0/\sqrt{1 - v^2/c^2}$, thus inducing $\delta p' = m'\delta v$. In the rest reference frame $\delta p = m\delta v_0$, we can easily seethat to ensure the MIP is independent of reference frame transformation, there must exists the relation

$$dx' = dx\sqrt{1 - v^2/c^2}$$

The length now is the integral of the above formula, and we have

$$l = \int_A^B dx' = l_0\sqrt{1 - v^2/c^2}$$

Thus we have derived the same result as in special relativity. However, its intrinsic meaning is not the same as in special relativity. Since we study within frameword of MIP, in which the STP' relative movement to the particle does not change under refrence frame transformation. Distinct frome macro length contraction effect of special relativity, the differntial distance is also constracted under MIP, which precisely reflects the universal applicability of the MIP.

IV. RANDOM MOTION AND SPACETIME DIFFUSION COEFFICIENT

Let m_{ST} be the statistical mass of the particle . We will prove the spacetime interaction coefficient of a m_{ST} mass particle will be universally given as

$$\Re = \frac{h}{2m_{ST}}. \tag{17}$$

Within the framework of random motion[1], or Wiener process in mathematics [2], this spacetime induced random motion is equivalent to the Markov process, moreover, the spacetime interaction coefficient is nothing but the diffusion coeffient [3]. In this section, we will start our journey from propability theory of random motion[3, 4], and then give a concrete proof that for the random motion induced by MIP, the spacetime interaction coefficient is given exactly by (17). The last two subsections discussed two spacetime model in order to investigate the origin of the spacetime interaction coefficient. From both we obtained the coefficient reading as $\Re = \frac{w\ell}{2}$, in which w is the average speed of the particle and ℓ the mean free distance.

A. Langevin Equation

We argue, the energy distribution of the STP in spacetime, will be in type of Gaussian. For one thing this is ensured by the central limit theorem in probability theory [4]. For another, if the distribution is not Gaussian, the particle in spacetime will have no reason to act randomly. The Gaussion distribution reflects the STP are universal white noises for particles in spacetime. The spacetime background can be seen as a enviroment of white noise, while the particle is moving under the interaction of the enviroment, and its motion is described by a Markov process. The corresponding movement can be determined by the Langevin equation[5]:

$$\frac{dq_i(t)}{dt} = -\frac{1}{2} f_i(\mathbf{q}(t)) + \nu_i(t) \tag{18}$$

where $q_i(t)$ describes the trajectory of the particle, and $f_i(\mathbf{q})$ is a differentiable function, which captures the classical motion of the particle. The ν_i is a white noise, and here means the interaction function induced by STP.For a Markov process, the average contribution of white noise vanishes. However, because of its Gaussian nature, its variation is not zero. We have

$$\langle \nu_i \rangle_\nu = 0, \ \langle \nu_i(t)\nu_j(t') \rangle_\nu = \Omega \delta_{i,j} \delta(t - t') \tag{19}$$

here the $\delta_{i,j}$ in the later equation can be obtained from the spacetime homogeneous property, while $\delta(t-t')$ determined from the Markov property. For a Markov process, only conditions at the very moment determine the dynamics of the system, and all information from future or past are irrelevant. We can write down the basic correlation function by introducing a probability measure $[d\rho(\nu)]$,which is given as

$$[d\rho(\nu)] := \left(\sqrt{\frac{1}{2\pi\Omega\delta(t-t')}} \right)^D [d\nu] \exp\left(-\frac{1}{2\Omega} \int dt \sum_i \nu_i^2 \right) \tag{20}$$

It is easy to see that

$$\int \nu_i(t)[d\rho(\nu)] \; = \; 0 = \langle \nu_i(t) \rangle_\nu \tag{21}$$

$$\int \nu_i(t)\nu_j(t')[d\rho[\nu]] \; = \; \Omega\delta_{i,j}\delta(t-t') = \langle \nu_i(t)\nu_j(t') \rangle_\nu \tag{22}$$

here Ω describes the strength of spacetime interaction on the particle. However, from the definition of measure (20), we can see, ν_i have the unit of m/s, so Ω will have the unit of m^2/s. From previous analysis, each collision leads to a change of an action $\hbar$. h has the unit of angular momentum, $kg \cdot m^2/s$. From this we can define a quantity with mass unit, it is

$$m_{ST} \equiv \frac{h}{\Omega}. \tag{23}$$

The mass m_{ST} has the meaning such that it is the mass collided by STP and is a statistical property. Accordingly, the collision parameter $\Omega = \frac{h}{m_{ST}}$ reflects a physical realistic viewpoint: an object in our real nature, the larger its mass means the smaller its quantum effect.

Langevin equation generates a timedependent probability such that

$$\mathbf{P}[\mathbf{q}, t;\; \mathbf{q_0}, t_0] = \langle \prod_{i=1}^{D} \delta[q_i(t) - q_i] \rangle_\nu,\; t \geq t_0 \tag{24}$$

which means for an operator $\mathcal{O}[\mathbf{q}]$, its average value at time t will be:

$$\langle \mathcal{O}[\mathbf{q}(t)] \rangle_\nu \equiv \int \mathbf{P}[\mathbf{q}, t;\; \mathbf{q_0}, t_0]\mathcal{O}[\mathbf{q}]d\mathbf{q} \tag{25}$$

Using the probability distribution (24), one can immediately verify equation (25). Actually, the distribution (24) can be seen as an evolution process, which says

$$\mathbf{P}[\mathbf{q}, t;\; \mathbf{q_0}, t_0] = \langle q|e^{-H(t-t_0)}|q_0\rangle$$

here the evolution Hamiltonian is the famous Fokk-Planck Hamiltonian, as we will derive its formalism in next subsection.

B. Fokk-Planck Equation

Given the Langevin equation (18), we can derive the corresponding Fokk-Planck equation, as well as the Fokk-Planck Hamiltonian [3].

We consider the time segment from t to $t+\epsilon$, $\epsilon \to 0$, and have the Langevin equation as:

$$q_i(t+\epsilon) - q_i(t) = -\frac{1}{2}\epsilon f_i(\mathbf{q}(t)) + \int\limits_t^{t+\epsilon} \nu_i(\tau)d\tau + O(\epsilon^{3/2}) \tag{26}$$

its related propability distribution is

$$\mathbf{P}[\mathbf{q}, t+\epsilon;\ \mathbf{q}', t] = \langle \delta(\mathbf{q} - \mathbf{q}(t+\epsilon))\rangle_\nu \tag{27}$$

According MIP, everytime the STP collided with the particle, the action of particle will change nh, $n \in \mathbb{Z}$. To obtain the Fokk-Planck equation, we define following discreterization

$$\sqrt{\epsilon}\bar{\nu}_i := \int\limits_t^{t+\epsilon} \nu_i(\tau)d\tau$$

so that the discrete Langevin equation is

$$q_i(t+\epsilon) - q_i(t) = -\frac{1}{2}\epsilon f_i(\mathbf{q}(t)) + \sqrt{\epsilon}\bar{\nu}_i + O(\epsilon^{3/2}) \tag{28}$$

Notice here the time has been discreterized as

$$(t-t')/\epsilon \in \mathbb{Z}^+.$$

Now the Gaussian distribution and the property of Markov progcess determins the average value of discrete white noises ν_i, and we have

$$\langle \bar{\nu_i}\rangle_\nu = 0,\ \langle \bar{\nu_i}(t)\bar{\nu_j}(t')\rangle_\nu = \frac{\hbar}{m_{ST}}\delta_{i,j}\delta_{t,t'} \tag{29}$$

When $\epsilon \to 0$, the Fourier expansion of the probability distribution (27) is

$$\begin{aligned}
\tilde{\mathbf{P}}[\mathbf{p}, t+\epsilon;\ \mathbf{q}', t] &= \int e^{-i\mathbf{p}\cdot\mathbf{q}}\mathbf{P}[\mathbf{q}, t+\epsilon; \mathbf{q}', t]d^D\mathbf{q} \\
&= \langle e^{-i\mathbf{p}\cdot\mathbf{q}(\mathbf{t}+\epsilon)}\rangle_\nu \\
&= \langle e^{-i\mathbf{p}\cdot(\mathbf{q}(\mathbf{t})+\epsilon\frac{\mathbf{dq(t)}}{\mathbf{dt}}+\mathbf{O}(\epsilon^2))}\rangle_\nu \\
&= \langle \exp(-i\mathbf{p}\cdot(\mathbf{q}'(\mathbf{t}) - \epsilon/\mathbf{2f}(\mathbf{q}')))\rangle_\nu \\
&\quad \times \left\langle \exp\left[-i\mathbf{p}\,\mathbf{.} \int\limits_t^{t+\epsilon} \nu(\tau)d\tau\right]\right\rangle_\nu \times \left\langle \exp\left(O(\epsilon^2)\right)\right\rangle_\nu \\
&= \exp\left[-i\mathbf{p}\,\mathbf{.}\,(\mathbf{q}' - \epsilon f(\mathbf{q}')/2)\right] \\
&\quad \times \left\langle \exp\left[-i\mathbf{p}\,\mathbf{.} \int\limits_t^{t+\epsilon} \nu(\tau)d\tau\right]\right\rangle_\nu
\end{aligned} \tag{30}$$

Notice that the last average value can be evaluated out by Gaussian integration, which reads,

$$
\begin{aligned}
&\left(\sqrt{\frac{h}{2\pi}}\right)^{D} \int [d\nu] \exp\left(-\frac{m_{ST}}{2h}\int dt \sum_{i}^{D} \nu_i^2\right) \\
&\quad \times \exp\left[-i\mathbf{p} \cdot \int_{t}^{t+\epsilon} \nu(\tau) d\tau\right] \\
= &\left(\sqrt{\frac{h}{2\pi}}\right)^{D} \int [d\nu] \\
&\quad \times \exp\left(-\frac{m_{ST}}{2h}\int dt \sum_{i} \nu_i^2 - i\mathbf{p} \cdot \int_{t}^{t+\epsilon} \nu(\tau) d\tau\right) \\
= &\left(\sqrt{\frac{h}{2\pi}}\right)^{D} \int [d\nu] \exp\left(-\frac{m_{ST}}{2h}\int dt \sum_{i} \nu_i^2 - i\sqrt{\epsilon}\mathbf{p} \cdot \bar{\nu}\right) \\
&\times \exp\left(+\epsilon\frac{h}{2m_{ST}}\mathbf{p} \cdot \mathbf{p} - \epsilon\frac{h}{2m_{ST}}\mathbf{p} \cdot \mathbf{p}\right) \\
= &\left(\sqrt{\frac{h}{2\pi}}\right)^{D} \int [d^N \left(\nu_i + \frac{ih}{2m_{ST}}\sqrt{\epsilon} p_i\right)] \\
&\times \exp\left(-\frac{m_{ST}}{2h}\int dt \sum_{i=1}^{D} \left(\nu_i + \sqrt{\epsilon}\frac{ih}{2m_{ST}} p_i\right)^2 \right. \\
&\qquad \left. -\epsilon\frac{h}{2m_{ST}}\mathbf{p} \cdot \mathbf{p}\right) \\
= &\exp\left(-\epsilon h \mathbf{p} \cdot \mathbf{p}/2m_{ST}\right)
\end{aligned}
\tag{31}
$$

here we can obtain the Fourier expansion of the probability distribution,

$$
\tilde{\mathbf{P}}[\mathbf{p}, t+\epsilon;\ \mathbf{q}', t] = e^{-\epsilon h/2m_{ST}\mathbf{p}\cdot\mathbf{p} + i\epsilon\mathbf{p}.f(\mathbf{q}')/2 - i\mathbf{p}\cdot\mathbf{q}'} \tag{32}
$$

for $\epsilon \to 0$, expanding (32) will end up with

$$
\tilde{\mathbf{P}}[\mathbf{p}, t+\epsilon;\ \mathbf{q}', t] = e^{-i\mathbf{p}\cdot\mathbf{q}'}(1 - \epsilon H_{FP}(\mathbf{p}, \mathbf{q}') + O(\epsilon^2)).
$$

Here we obtained the Fokk-Planck Hamiltonian

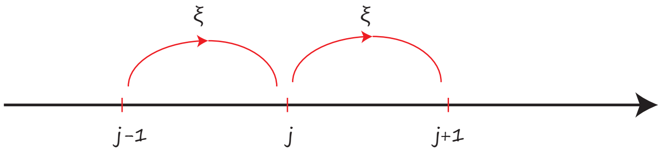
$$
H_{FP}(\mathbf{p},\mathbf{q}) = -\frac{h}{2m_{ST}}\mathbf{p}\cdot\mathbf{p} - i\mathbf{p}\cdot f(\mathbf{q})/2 \tag{33}
$$

From which we can read off the diffusion coefficient induced by collisions between STP and the particle, is exactly $\Re = h/2m_{ST}$. Later we will see in deriving the Schrodinger equation of free particle in spacetime, the spacetime mass $m_{ST} = 2\pi m$ will be identified as the inertial mass, in the framework of non-relativistic quantum mechanics.

C. From Discrete Spacetime to the Spacetime Diffusion Coefficient

Beginning with MIP, we want to investigate the origin of spacetime interaction coefficient. Within the framework of discrete spacetime, spacetime diffusion coefficient $\Re = \frac{h}{2m_{ST}}$ should be derived in terms of parameters of discrete spacetime. Let us consider the simplest discrete model (see Fig.5), where the length union of discrete space is ℓ. $P(j,t)$ is the probability of a particle at lattice site j at time t.

Figure 5: Random jumping model on one dimensional lattice



Because of the discrete nature of the space, all jumpings can only happen between nearest pair of positions. Given the rate of jumping between the nearest neighbour ζ and the isotropy of frictionless space, the evolution of probability should be

$$\partial_t P(j,t) = \zeta(\frac{1}{2}P(j-1,t) + \frac{1}{2}P(j+1,t) - P(j,t)) \tag{34}$$

the first two terms of RHS of (34) describe the fact that jumping forward and backward from neighbors $j-1$ and $j+1$ positions respectively, have the same probability, which is $1/2$, the third term remarks the probability from j position to neighbors. Introducing the fundamental spacing of the lattice ℓ, the eq.(34) goes to

$$\partial_t P(j,t) = \frac{\zeta\ell^2}{2}(\frac{\frac{P(j+1,t)-P(j,t)}{\ell} - \frac{P(j,t)-P(j-1,t)}{\ell}}{\ell}) \tag{35}$$

In the continum limit of spacetime, which says $\ell \to 0$, and $\zeta \to \infty$, but keeping the quantity $\zeta\ell^2$ unchanged, the probability $P(j,t)$ now becomes the probability density $\rho(x,t)$, and the RHS of (34) becomes the definition of second derivative. Thus we have

$$\partial_t \rho(x,t) = \frac{\zeta\ell^2}{2}\partial_x^2 \rho(x,t). \tag{36}$$

It is straightforward to generalise above equation to three dimension case, we have,

$$\partial_t \rho(\vec{r},t) = \frac{\zeta\ell^2}{2}\nabla^2 \rho(\vec{r},t) \tag{37}$$

Comparing with diffusion equation[6]

$$\partial_t \rho(\vec{r},t) = \Re\nabla^2 \rho(\vec{r},t) \tag{38}$$

the microscopic origin of spacetime diffusion coefficient will be

$$\Re = \frac{\zeta \ell^2}{2} \tag{39}$$

Furthermore, we can also discrete time with union $\tau = \frac{\ell}{w}$, where w is the average speed of particle. With $\zeta = \frac{1}{\tau}$, we obtain

$$\Re = \frac{w\ell}{2} \tag{40}$$

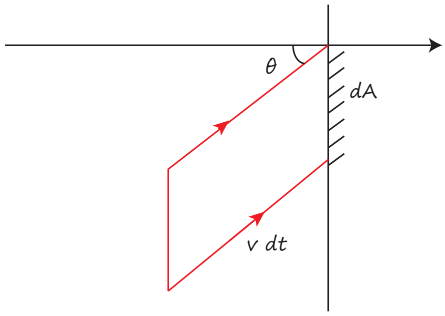
Combining the microscopic structure of discrete spacetime with the MIP, we have

$$\Re = \frac{w\ell}{2} = \frac{h}{2m_{ST}} \tag{41}$$

D. From Spacetime Scattering to the Spacetime Diffusion Coefficient

Particles will be scattered randomly from the STP with the speed of light, which leads to the probability distribution of speed $f(\vec{v})$, the number of partials within $v \to v + dv$ is $f(v)d^3\vec{v}$. Therefore, all the particles cross the section area dA during time dt will be inside the cylinder (see Fig.6).

Figure 6: Probability distribution of spacetime scattering



The volume of this cylinder is

$$V = vdt \cos\theta dA \tag{42}$$

in which the number of particles is

$$N = f(\vec{v})d^3\vec{v}vdt \cos\theta dA \tag{43}$$

Because of the isotropy of space, we have $f(\vec{v}) = f(v)$. From left to right, the number of particle cross the unit area per unit time is

$$\begin{aligned}\Phi &= \int\limits_{v_z>0} \frac{N}{dAdt} \\ &= \int\limits_0^{\frac{\pi}{2}} d\theta \cos\theta \sin\theta \int\limits_0^{2\pi} d\varphi \int\limits_0^{+\infty} f(v)v^3 dv \\ &= \pi \int\limits_0^{+\infty} f(v)v^3 dv \end{aligned} \tag{44}$$

where $v_z > 0$ means $0 < \theta < \frac{\pi}{2}$. The average speed reads

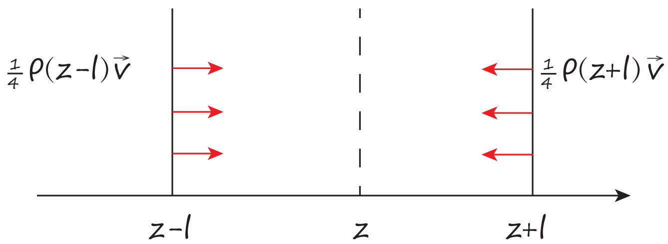
$$w = \frac{\int_0^{+\infty} f(v)vd^3v}{\int_0^{+\infty} f(v)d^3v} = \frac{4\pi}{\rho} \int\limits_0^{+\infty} f(v)v^3 dv \tag{45}$$

where the density of particle number is $\rho = \int_0^{+\infty} f(v)d^3v$. Correspondingly, the number of particle cross the unit area per unit time will be

$$\Phi = \frac{1}{4}\rho w \tag{46}$$

Let mean free path of particles be ℓ, i.e. the average distance traveled by the particle between successive impacts from spacetime. The net flux J_z through the z plane will be (see Fig.3.3)

Figure 7: Mean free distance and scattering flux



$$J_z = \frac{1}{4}\rho(z-\ell)w - \frac{1}{4}\rho(z+\ell)w = -\frac{1}{2}\ell w \partial_z \rho \tag{47}$$

With the equation of continuity

$$\partial_t \rho + \nabla \cdot \vec{J} = 0 \tag{48}$$

and the isotropy of space, we have

$$\partial_t \rho = \frac{1}{2}\ell w \nabla^2 \rho \tag{49}$$

Combining the kinetics of spacetime scattering with quantum nature induced by STP, we obtain

$$\Re = \frac{w\ell}{2} = \frac{h}{2m_{ST}} \tag{50}$$

which is consistent with Eq.41.

V. MASS-DIFFUSION UNCERTAINTY RELATION

A. Classical Theory of Phase Space for Mass and Diffusion Coefficient

We have claimed and proven that particle mass is a statistical property describing the diffusion ability of the particle in spacetime, under continuous interaction of STP. However, MIP itself is describing a special Markov process, which possesses the intrinsic characteristic property of being quantized.

Classically, given the position and momutum of a particle, one can describe how the particle move in spacetime. However, if the mass of a particle is a statistical quantity, the momumtum at a fixed position in spacetime will be in principle ill-defined.

The way out is to introduce the dynamic point of view. We define on each point in spacetime canonical pair $(m_{ST}, \Re)$. They satisfy the classic Poisson relation

$$\{\Re, m_{ST}\}_{P.B.} = 1 \tag{51}$$

in which the Poisson braket reads

$$\{f,\ g\}_{P.B.} := \sum_{i=1}^{D} \left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i} \right)$$

while (q_i, p_i) are a pair of canonical position and momumtum. It is obvious to see if we choose the canonical coordinate be $\Re$, and momentum be m_{ST}, we can immidiately recover the Poisson relation (51).

B. Mass-Diffusion Uncertainty

Under the framework of MIP, for a matter particle, its mass and spacetime diffusion coefficient are not only be classical statistical conjugated quantity to each other, but also satisfy the uncertanty relation

$$\Delta m_{ST} \Delta \Re \geq h/2 \tag{52}$$

The existence of such an uncertainty relation is natural from the viewpoint of MIP. When we measure the mass of a particle, we can neither seperate out the impacts of STP, nor perform measurement during a period of collision between STP and the particle. This rules out the instant measurement for the mass as well as the spacetime diffusion coefficient. A measurement cannot be done without collision of STP. As proposed in MIP, each collision between STP and the particle will change the action of particle at least one unit of h, which means the phase volume defined by mass and spacetime diffusion coefficient will change at least one unit of h. The standard deviation of measurements caused by fluctuation of spacetime background will be

$$\sigma(m_{ST}) = \sqrt{\langle m_{ST}^2 \rangle_\nu - \langle m_{ST} \rangle_\nu \langle m_{ST} \rangle_\nu} \tag{53}$$

$$\sigma(\Re) = \sqrt{\langle \Re^2 \rangle_\nu - \langle \Re \rangle_\nu \langle \Re \rangle_\nu} \tag{54}$$

Suppose the mean value of the mass be $\bar{m} = \bar{m}_{ST}/2\pi$,with 2π the constant for further convenience, and the classic speed be v_0, with $v_0 << c$. We need to calculate the correlation of these two independent vectors , defined as

$$\sigma(m_{ST}, \Re) = \left\langle \left(m_{ST} - \langle m_{ST} \rangle \right) \left(\Re - \langle \Re \rangle \right) \right\rangle \tag{55}$$

The Cauchy-Schwartz inequatity reads

$$\sigma(m_{ST})\sigma(\Re) \geq |\sigma(m_{ST}, \Re)| \tag{56}$$

After a collision of STP, the speed of the particle changed, which comes from the spacetime fluctuation, expressed in Langevin equation with the noise as

$$\Delta V = \nu(t) \tag{57}$$

The fluctuation of STP has significant impacts on both mass and diffusion property of a matter particle. The MIP implies that the fluctuation on the particle depends on collisons of STP. A larger fluctuation means easier to diffuse, hence defines a smaller statistical inertia mass. Because the distribution of STP is a Gaussian distribution we can expand the statistical inertia mass as :

$$m_{ST} = \bar{m}_{ST} + k_1 \nu^{-1} + k_3 \nu^{-3} + \cdots \tag{58}$$

Notice there are no even power of ν in the above expansion, and that is because after taking mean value we need

$$\langle m_{ST} \rangle = \bar{m}_{ST} \tag{59}$$

If there are even powers of ν terms, the above equation will be violated. For the same reason, we can expand the diffusion coefficient as

$$\Re = \bar{\Re} + r_1 \nu + r_3 \nu^3 + \cdots \tag{60}$$

hence

$$\sigma(m_{ST}, \Re) = \langle k_1 r_1 + k_1 r_3 \nu^2 + k_3 r_1 \nu^{-2} + k_3 r_3 + \cdots \rangle \tag{61}$$

Actually, from the consistent convergence condition, we can omit all non-linear terms of these two expansions, with only $k_1 r_1$ term suvives. From dimensional analysis, we see that k_1 has the unit of $kg \cdot s/m$ and r_1 has the unit of m^3/s^2 so that $k_1 r_1$have the same unit of action. Which reflects the fact that collision between STP and the particle will change the action of the particle at least one unit of h. From which we have

$$\sigma(m_{ST}, \Re) = Ch \tag{62}$$

where n is an integer. Due to the previous discussion of path separation and the insensitivity of particle number changing constraint, we can propose that every time the exchanging action between matter particle and STP is exactly a single h . In other words, we think a large change of action,for example $1000h$, actually can be separated out as 1000 times of consequently minimal collision. Each minmimal collision changed the action of the particle a single h changes. In a period of macroscopic time, the collision times will goes to infinity and all the statistical deviations will be consistently the same, using Riemann-Zeta function

$$C = \zeta(0) = -\frac{1}{2} \tag{63}$$

as well as reminding the equation (56). We can obtain the statistical deviation

$$\sigma(m_{ST})\sigma(\Re) \geq |-\frac{1}{2}h| = \frac{h}{2}, \tag{64}$$

which represents the mass-diffusion uncertainty.

C. Particle Statistics in the Framework of MIP

The most important proposition of Copenhagen interperitation of quantum mechanics is the wave-particle duality. This allows one using the superposition rule of plane waves to describe the state of a particle. The kernel of the wave transformation from frequency space to time space will be the factor $\exp(ipx/\hbar)$. Because of the duality, physical quantities of the particle can also be derived from wave, which implies some quantities can be described in phase space as eigenvalues of special operators. However, under the framework of MIP, we need to emphrase again that the wave-like property of the particle is an emergent property due to collision of STP, therefore it is not intrinsic. We can not borrow the quantization hypothesis directly. We consider the action of the particle

$$\begin{aligned} S[\phi(t,x), \partial\phi(t,x), \bar{\nu}(t,x)] \\ = S_0[\phi(t,x), \partial\phi(t,x)] + \sum_{I=1}^{\infty} S_I[\bar{\nu}(t,x)] \end{aligned} \tag{65}$$

where $\phi(t,x)$ describing the classical trajectory of the particle, and S_0 is the related classical action. $S_I[\bar{\nu}(t,x)]$ is the contribution of $I-th$ collision between STP and the particle. It does not depend on the classical trajectory at all, which only depends on the fluctuation of STP. The MIP said this term should contribute integer number of h , that is $S_I = nh$.

The partition function of the particle now is

$$Z = \int [d\phi(t,x)] \exp(-\frac{i}{\hbar} S[\phi(t,x), \partial\phi(t,x), \bar{\nu}(t,x)]) \tag{66}$$

hence

$$\exp\left(-\frac{i}{\hbar} S_I[\bar{\nu}]\right) = \exp\left(-\frac{i}{\hbar} nh\right) = e^{-i2\pi n} = 1 \tag{67}$$

from which we see the introducing of MIP does not change the classical partition function, therefore physical quantity derived from classical action will not be affected.

D. The Mass-diffusion Uncertainty

The uncertainty between mass and spacetime diffusion coefficient implies one can not measure them for the same particle simultaneously. Then a realistic measurement method will be the insimultaneous measurement. However, this method will introduce the different time deviation, defined as

$$\eta := \lim_{\epsilon\to 0} m_{ST}(t+i\epsilon)\Re(t) - \Re(t+i\epsilon)m_{ST}(t) \tag{68}$$

where η describes the different time deviation. Notice here the time interval is imaginary. That is because we have integrated out the self-dynamics of STP and they only play as a background in the case of considering the dynamics of the particle. The collision progress is actually a transmition of energy between STP and the particle, so for the particle, spacetime background served as a themal base and the collision provides an effective temperature. When $\epsilon \to 0$, the RHS of above equation can be defined as the commutation relation between mass and spacetime diffusion coefficient. We rewrite the RHS of above equation as

$$\lim_{\epsilon\to 0}\left(e^{-H\epsilon/\hbar}m_{ST}(t)e^{H\epsilon/\hbar}\Re(t) - e^{-H\epsilon/\hbar}\Re(t)e^{H\epsilon/\hbar}m_{ST}(t)\right) \tag{69}$$

Here the H is defined on the phase space $(m_{ST},\ \Re)$, so one can not simply move toward the left side of $m_{ST}(t)$ or the right side of $\Re(t)$, from Baker-Hausdorff-Campell formula, one extracts the above equation as

$$\begin{aligned}\epsilon\frac{\partial m_{ST}(t)}{\partial t} + \mathcal{O}(\epsilon^2) &= -\frac{\epsilon i}{\hbar}[H, m_{ST}] \\ &\quad -\frac{\epsilon^2}{2\hbar^2}[H,[H,m_{ST}]] + \cdots \end{aligned} \tag{70}$$

$$\begin{aligned}\epsilon\frac{\partial \Re(t)}{\partial t} + \mathcal{O}(\epsilon^2) &= -\frac{\epsilon i}{\hbar}[H, \Re] \\ &\quad -\frac{\epsilon^2}{2\hbar^2}[H,[H,\Re]] + \cdots \end{aligned} \tag{71}$$

It is obvious that Heisenberg equation is satisfied under perturbative collision.

$$i\hbar\frac{\partial m_{ST}(t)}{\partial t} = [H,\ m_{ST}],\ i\hbar\frac{\partial \Re(t)}{\partial t} = [H,\ \Re] \tag{72}$$

For the statistical inertia mass and diffusion coefficient, the evolution factor is determined by $H\epsilon$. The physical meaning is that in time interval $i\epsilon$ the energy transmitted from STP to the particle. This meets the description of MIP. So we can rewrite the action as a compact form of phase space volumn

$$S_I = \hat{\Re}\hat{m}_{ST}$$

The kernel relating the cannonical position space and the cannonical momumtum space reads

$$\hat{m}\exp\left(-\frac{i}{\hbar}S_I[\bar{\nu}]\right) = m_{ST}\exp\left(-\frac{i}{\hbar}S_I[\bar{\nu}]\right), \tag{73}$$

$$\hat{\Re}\exp\left(-\frac{i}{\hbar}S_I[\bar{\nu}]\right) = \Re\exp\left(-\frac{i}{\hbar}S_I[\bar{\nu}]\right) \tag{74}$$

Mathematically, the unique representation of spacetime diffusion coefficient is an differential operator, as

$$\hat{\Re} = i\hbar\frac{\partial}{\partial m_{ST}}, \tag{75}$$

from which the equation (74) will be satisfied immediately, as

$$\begin{aligned}\hat{\Re}\exp\left(-\frac{i}{\hbar}\Re m_{ST}\right) &= i\hbar\frac{\partial}{\partial m_{ST}}\left[\exp\left(-\frac{i}{\hbar}\Re m_{ST}\right)\right] \\ &= \Re\exp\left(-\frac{i}{\hbar}\Re m_{ST}\right)\end{aligned} \tag{76}$$

from which we obtain the critical commutation relation

$$[\hat{\Re}, \hat{m}] = i\hbar \tag{77}$$

E. Position-Momentum Uncertainty Relation

Extending the definition of commutation relation, and recall $m = m_{ST}/2\pi$, we consider the position-momentum commutator

$$\begin{aligned}[x,p] &= \frac{1}{2\pi}x(t+i\epsilon)m_{ST}(t)\frac{\partial x(t)}{\partial t} \\ &\quad -\frac{1}{2\pi}m_{ST}(t+i\epsilon)\frac{\partial x(t+i\epsilon)}{\partial t}x(t) \\ &= i\frac{\epsilon}{2\pi}\left[m_{ST}\left(\frac{\partial x(t)}{\partial t}\right)^2\right. \\ &\quad \left.-\frac{\partial m_{ST}(t)}{\partial t}v(t)x(t) - m_{ST}\frac{\partial^2 x(t)}{\partial t^2}x(t)\right]\end{aligned} \tag{78}$$

Define

$$a_{ST}(t) := \frac{\partial^2 x(t)}{\partial t^2} \tag{79}$$

It is the instantaneous accelaration induced by the collison between STP and the particle. From which we can define the instantaneous "spacetime" force as

$$F_{ST}(t) = ma_{ST}(t) = m\frac{\partial^2 x(t)}{\partial t^2} \tag{80}$$

The statistical average of eq.(78) is

$$[x,p] = m\langle v(t)^2\rangle i\epsilon - \langle\frac{\partial m(t)}{\partial t}v(t)x(t)\rangle i\epsilon - \langle F_{ST}(t)x(t)\rangle i\epsilon \tag{81}$$

The physical meaning of the third term in above equation is clear, it reflects the mean work done by STP acting on the particle. Obviously, this mean work is zero. To understand the first two terms in eq.(81), we should consider the Markov process in detail. From Langevin equation Eq. (18), the classic speed can be expressed as

$$u(t) = -\frac{1}{2}\tilde{u}(x(t)) \tag{82}$$

The fluctuation speed induced by collision between STP and the particle is

$$v(t) = \nu(t) \tag{83}$$

Therefore we could naturally deduce that

$$\begin{aligned} \langle x(t)v(t)\rangle &= \langle -\frac{1}{2}\int f(x(t))dt\nu(t)\rangle_\nu \\ &= \int -\frac{1}{2}\int f(x(t))dt\nu(t)\left(\sqrt{\frac{1}{2\pi\Omega\delta(t-t')}}\right)^D \\ &\times \exp\left(-\frac{1}{2\Omega}\int dt\sum_i \nu_i^2\right)[d\nu] \\ &= 0 \end{aligned} \tag{84}$$

Notice in deriving the second step, we have used the property of Gaussian integral, which leads to the result that the second term of eq. (81) will also have vanishing contribution. Under discretization of the spacetime fluctuation, the mean speed is

$$\int_t^{t+\epsilon} \nu(\tau)d\tau/\epsilon = \bar{\nu}/\sqrt{\epsilon}$$

Therefore

$$\langle \nu^2\rangle = \langle \bar{\nu}^2\rangle/\epsilon = \frac{h}{m_{ST}\epsilon} \tag{85}$$

Substituting this into the first term of Eq. (81),

$$[x,\ p] = i\epsilon m\langle\nu^2\rangle = i\epsilon m\frac{h}{m_{ST}\epsilon} = i\hbar \tag{86}$$

we obtain the fundamental position-momentum uncertainty in quantum mechanics.

F. Energy-Time Uncertainty Relation

A massive particle moves in spacetime, which is represented not only by its momentum and position, but also by its energy and time. According to MIP, every impact of STP will change the action of particle by the amount of nh. As we all known, partition function is determined by energy distribution of the particle not by constraints of particle number.

1. New physics of energy-time uncertainty relation

We need to investigate commutation algebra of energy-time uncertainty relation based on the framework of MIP. Supposing the state function of particle is $\Psi(\vec{x},t)$, then $Et\Psi(\vec{x},t)$ and $tE\Psi(\vec{x},t)$ have totally different physical meanings. From MIP, the motion of particle is a Markov process. It is impossible to define the exact state $\Psi(\vec{x},t)$, since it is a probability function. Fundamentally, time t is a relative quantity, which depends on the zero point of time. However, the absolute zero point of time cannot be well defined. For the same reason, energy E is a relative quantity, which also depends on the zero point of energy. Due to the random impact of STP, the absolute zero point of energy cannot be well defined either. The background noise of STP is stochastic. Under MIP, time and energy both are not exactly well defined. All we can say is their probabilistic distribution. Simultaneously observing time and energy of the particle will lead to the covariance

$$\sigma(E,t) =< (E-\bar{E})(t-\bar{t}) > \tag{87}$$

which is the average value of the change of action between particle and STP. In the interval $\Delta t = t-\bar{t}$, the change of energy is $\Delta E = E-\bar{E}$. The average value of the product is ΔS as

$$\sigma(E,t) =< \Delta S > \tag{88}$$

For any random impact, we have $\Delta S = nh,\ n\in\mathbb{Z}$. The regularisation of ζ function leads to

$$\sigma(E,t) =< \Delta S >= \zeta(0)h = \frac{1}{2}h \tag{89}$$

which is the very uncertainty relation of energy and time. Therefore, the energy and time are statistical quantities, which corresponds to infinite matrixes or a operator. Within this context, energy-time uncertainty relation can be interpreted as non-commutate matrixes.

However, this regularisation scheme cannot reveal the physical origin of energy-time uncertainty relation. We need a more detailed physical interpretation. When no collision between STP and matter particle happens, the particle will move with uniform velocity. After one collision, the particle will acquire an energy ΔE. This energy will feedback to the particle in two different ways. Firstly, the velocity of particle has changed, so its momentum changes correspondingly. Secondly, the mass of particle has changed, so the uncertainty of energy corresponds to the uncertainty of statistical mass. Assuming the status function of the particle depends on time and position as

$$\Psi(\vec{x},t) \in C_\infty(\vec{x},t) \tag{90}$$

here the $C_\infty(\vec{x},t)$ is the set of smooth function in spacetime. As a status function, the normalizable property is very important. If the status function is a decay or increasing function, the normalizability can not be garanteed. In other words, the particle can not be killed or created in spacetime when there are no interactions. Therefore, we can only expect the status function is an oscillation function as

$$\begin{aligned}\Psi(\vec{x},t+\Delta t) &= \Psi(\vec{x},t)\exp(-\frac{i}{\hbar}H\Delta t) \qquad (91)\\ &= \Psi(\vec{x},t) - \Psi(\vec{x},t)\frac{iH\Delta t}{\hbar}\end{aligned}$$

Here H is the Hamiltonian of the particle in spacetime. Accordingly, we have

$$\frac{\Delta\Psi(\vec{x},t)}{\Delta t} = \frac{-i}{\hbar}H\Psi(\vec{x},t) \tag{92}$$

hence we deduce that

$$\begin{aligned}\frac{\Delta(t\Psi(\vec{x},t))}{\Delta t} &= \Psi(\vec{x},t) + t\frac{\Delta(\Psi(\vec{x},t))}{\Delta t}\\ &= \Psi(\vec{x},t) - \frac{i}{\hbar}tH\Psi(\vec{x},t)\\ &= -\frac{i}{\hbar}H(t\Psi(\vec{x},t)) \qquad (93)\end{aligned}$$

In above equation , we used the following substitution

$$\tilde{\Psi}(\vec{x},t) \equiv t\Psi(\vec{x},t),$$

hence

$$(Ht - tH)\Psi(\vec{x},t) = i\hbar\Psi(\vec{x},t) \tag{94}$$

Finally, we obtain the energy-time uncertainty

$$[H,t] = i\hbar \tag{95}$$

as expected.

G. Mass Measurement and Neutrino Oscillation

In previous subsections, we derived the mass-diffusion uncertainty relation. We now discuss a possible important application of this .

In mordern physics, neutrino oscillation is provided as a longstanding puzzle for high energy physics. The current explaination is that neutrinos have a very strange property that they can not be massive and eienstate of flavor symmetry simultaneously. However, in the progress of nuclear reaction, neutrinos are all consdiered as a flavor eigenstate, which means they have definitive flavors. This leads to a strange result that we can not detect the mass of neutrinos.In this article, we will argue that the neutrino oscillation actually reflects the inertia mass is a statistical property. The mass of neutrinos is so small that the statistical deviation is comparable to the mass, so the mass can not be measured accurately. Because of mass-diffusion uncertanty, we can calculate the effect of spacetime diffusion directly, when given the mass differences between various neutrinos. Experimently, physcists now can measure the differences indirectly. The mass square differences are around $2.6\times10^{-3}eV^2$ and $7.58\times10^{-5}eV^2$. This gives a perfect testing arena for the mass-diffusion uncertainty.From the mass-diffusion uncertainty relation,

$$\Delta m\Delta\Re \geq \hbar/2 \tag{96}$$

the diffusion coefficient reads

$$\begin{aligned}\Delta\Re \geq \hbar/2\Delta m &= \frac{6.626\times10^{-34}\times9\times10^{16}}{4\times3.1416\times0.05\times1.6\times10^{-19}} \\ &= 1186.4[m^2/s]\end{aligned} \tag{97}$$

The physical meaning of this calculation is significant,. Every second the neutrino propagates with a growing diffusion cylinder, with the bottom of the cylinder, increasing its area to $1186.4m^2$. If a neutrino goes from sun to earth, its diffusion radius will be about 300 meters.

H. Neutrino Diffusion Experiment

Since sun cannot be seen as point-like source for neutrino ejection, we could design an ideal experiment in labratory, as shown in Fig.7 Electron neutrinos came from reactor and were screened by screening matter, except those moving strictly toward x-direction. According

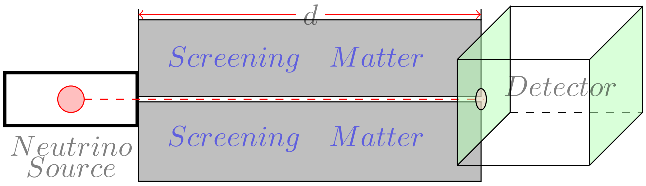


Figure 8: Ideal experiment for neutrino diffusion in lab.

to MIP and due to diffusion of neutrino, after propagating distance d, dectectors at d will detect neutrinos in a disk region with equal probability. The disk area can be calculated as

$$\delta r \simeq \sqrt{\Delta R \frac{d}{c\pi}} \tag{98}$$

If $d \simeq 100km$, $\delta r \simeq 0.3548m$, the disk is macro significant detectable. This phenomenon will provide a strong evidence of MIP.

VI. RANDOM MOTION OF FREE PARTICLE UNDER MIP

A. From MIP to Schrödinger Equation

Random motion of free particle under MIP is a problem of stochastic mechanics, of which the most dramatic difference from Newtonian mechanics is that the derivative $d\vec{x}/dt$ is not well defined[15, 16]. For a stochastic process $\vec{x}(t)$, its speed $\vec{V}$ can be understood as the sum of classical speed $\vec{v}$ and fluctuated speed $\vec{u}$

$$\vec{V} = \vec{v} + \vec{u} \tag{99}$$

With the time reversal transformation $\vec{x} \to \vec{x}, t \to -t$, it's shown that $\tilde{v} = -\vec{v}, \tilde{u} = \vec{u}$. Since a continuous Markov process will still be a Markov process under time reversal, we can have a well defined limit $\vec{u} = 0$ as Newtonian mechanics with

$$\vec{v} = \frac{1}{2}(\vec{V} - \tilde{\vec{V}}) \tag{100}$$

$$\vec{u} = \frac{1}{2}(\vec{V} + \tilde{\vec{V}}) \tag{101}$$

Without the interaction of spacetime, the speed of particle $\vec{v}$ has to be the derivative $\vec{v} = \frac{d\vec{x}}{dt}$. Contrasting from usual Markov process, spacetime random motion is frictionless, otherwise the quantum effect of a particle will decay as time going, which is obviously not the case.

According to the MIP, the coordinate of a free particle is a stochastic process $\vec{x}(t)$, in which the speed $\vec{V}$ can not be expressed in terms of $\frac{d\vec{x}}{dt}$. The speed $\vec{V}$ should be a statistical average corresponding to a distribution $\delta\vec{x} = \vec{x}(t + \frac{1}{\omega}) - \vec{x}(t)$, at the limit of spacetime collision frequency ω going to infinity. In Einstein's theory on Brownian motion, $\delta\vec{x}$ is a Gaussian distribution with zero mean and variance proportional to $\frac{1}{\omega}$[6]. However, Einstein's theory cannot be correct at the limit of spacetime collision frequency ω going to infinity[17, 18]. Therefore, we will construct the operator D as following, which plays the same role as $\frac{d}{dt}$ in Newtonian Mechanics. For any physical function $f(\vec{x}, t)$, we have

$$\begin{aligned}
&\omega(f(\vec{x}(t+\frac{1}{\omega}), t+\frac{1}{\omega}) - f(\vec{x}(t), t)) \\
&= [\partial_t + \sum_i \omega(x_i(t+\frac{1}{\omega}) - x_i(t))\partial_i \\
&\quad + \sum_{ij} \frac{\omega}{2}(x_i(t+\frac{1}{\omega}) - x_i(t))(x_j(t+\frac{1}{\omega}) - x_j(t))\partial_i\partial_j \\
&\quad + \sum_i (x_i(t+\frac{1}{\omega}) - x_i(t))\partial_i\partial_t + \frac{1}{2\omega}\partial_t^2] f(\vec{x}(t), t)
\end{aligned} \tag{102}$$

At the limit of spacetime collision frequency ω going to infinity, in terms of statistical average $< ... >$ for δx, we can define the operator D as

$$\begin{aligned}
Df(x(t), t) &= \\
&\lim_{\omega\to+\infty} \omega < f(\vec{x}(t+\frac{1}{\omega}), t+\frac{1}{\omega}) - f(\vec{x}(t), t) > \\
&= (\partial_t + \sum_i V_i\partial_i + \sum_{ij} \Re_{ij}\partial_i\partial_j) f(\vec{x}(t), t)
\end{aligned} \tag{103}$$

where

$$\vec{V} = \lim_{\omega\to+\infty} \omega < \delta\vec{x} > \tag{104}$$

$$\Re_{ij} = \lim_{\omega\to+\infty} \frac{\omega < \delta x_i \delta x_j >}{2} \tag{105}$$

According to the MIP, the matrix of interaction coefficient is

$$\Re_{ij} = \frac{\hbar}{2m_{ij}} \tag{106}$$

Because of the isotropy of space, the MIP coefficient will be

$$\Re_{ij} = \Re\delta_{ij} \tag{107}$$

which is consistent with Eq.41 and 50. The operator D and its time reversal $\tilde{D}$ are

$$D = \partial_t + \vec{V} \cdot \nabla + \Re\nabla^2 \tag{108}$$

$$\tilde{D} = -\partial_t + \tilde{\vec{V}} \cdot \nabla + \Re\nabla^2 \tag{109}$$

Therefore, the real speed of particle $\vec{V}$ can be written as

$$\vec{V} = D\vec{x} \tag{110}$$

$$\tilde{\vec{V}} = \tilde{D}\vec{x} \tag{111}$$

Correspondingly, its classical speed and fluctuated speed are

$$\vec{v} = D^{-}\vec{x} \tag{112}$$

$$\vec{u} = D^{+}\vec{x} \tag{113}$$

with

$$D^{-} = \frac{1}{2}(D - \tilde{D}) \tag{114}$$

$$D^{+} = \frac{1}{2}(D + \tilde{D}) \tag{115}$$

We define the average acceleration of particles as

$$\vec{a} = D\vec{V} \tag{116}$$

With the invariance of average acceleration under time reversal, the average acceleration of a free particle must be zero, which can be written as

$$D^{-}\vec{v} + D^{+}\vec{u} = 0 \tag{117}$$

$$D^{+}\vec{v} + D^{-}\vec{u} = 0 \tag{118}$$

These conditions are equivalent to the coupled non-linear partial differential equations as following

$$\frac{\partial \vec{u}}{\partial t} = -\Re\nabla^2\vec{v} - \nabla(\vec{u} \cdot \vec{v}) \tag{119}$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v} \cdot \nabla)\vec{v} + (\vec{u} \cdot \nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{120}$$

Random motions of free particles due to the random impacts of STP satisfies the Markov property if one can make predictions for the future of the process based solely on its present state just as well as one could know the process's full history. This is the simplest situation for random motions, the free particle does not involve any external potential. Now, we have an initial value problem, which is to solve $\vec{u}(\vec{x},t)$ and $\vec{v}(\vec{x},t)$ given $\vec{u}(\vec{x},0)=\vec{u}_0(\vec{x})$, and $\vec{v}(\vec{x},0)=\vec{v}_0(\vec{x})$. In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly. Let $\Psi = e^{R+iI}$, where

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{121}$$

$$\nabla I = \frac{1}{2\Re}\vec{v} \tag{122}$$

We can obtain

$$\frac{\partial \Psi}{\partial t} = i\Re\nabla^2\Psi \tag{123}$$

According to the MIP, the universal spacetime diffusion coefficient is the MIP coefficient $\Re = \frac{\hbar}{2m_{ST}}$. Substituting to the last equation, we will get the equation of motion of free particles as

$$i\frac{\partial \Psi}{\partial t} = -\frac{\hbar\nabla^2}{2m_{ST}}\Psi \tag{124}$$

which is the Schr$\ddot{o}$dinger equation essentially. From this emergent Schr$\ddot{o}$dinger equation, we can deduce a series of quantum behaviours. It's important to remark that the spacetime mass m_{ST} in the Schr$\ddot{o}$dinger equation of free particles coincide with the inertial mass m of free particles. Since we only discuss non-relativistic quantum mechanics in the followings, we don't need to distinguish m_{ST} from m any more. From $|\Psi|^2 = e^{2R}$ and $\nabla R = \frac{1}{2\Re}\vec{u}$, we have

$$\vec{u} = \Re\frac{\nabla|\Psi|^2}{|\Psi|^2} \tag{125}$$

which leads to Born rule $\rho = |\Psi|^2$. $\rho(x,t)$ is the probabilistic density of particles in coordinate x at time t. The Born rule is a law of quantum mechanics which gives the probability that a measurement on a quantum system will yield a given result, which became a fundamental ingredient of Copenhagen interpretation. In this paper, we attempt to suggest an interpretation of Born rule according to the MIP, which can provide a realistic point of view for wave function. Emergent from random impacts of spacetime, it's absolutely necessary that wave function is complex. If wave function were a real sine or cosine function[23], according to $\rho = |\Psi|^2$, the probabilistic density of a free particle with definite momentum would oscillate periodically which violates the isotropy of physical space.

B. Physical Meanings of Potential Functions R and I

Substituting $\Psi = e^{R+iI}$ into $\frac{\partial\Psi}{\partial t} = i\Re\nabla^2\Psi$, we equalise the real and imaginary part separately as

$$\partial_t R = -\Re(2\nabla R \cdot \nabla I + \nabla^2 I) \tag{126}$$

$$\partial_t I = \Re[(\nabla R)^2 - (\nabla I)^2 + \nabla^2 R] \tag{127}$$

Combining with previous result $\rho = |\Psi|^2 = e^{2R}$, we have

$$\partial_t \rho = 2\rho \partial_t R \tag{128}$$

$$\nabla \rho = 2\rho \nabla R \tag{129}$$

The differential equation of potential R can be turned into

$$\partial_t \rho = -2\Re\nabla \cdot (\rho \nabla I) \tag{130}$$

With $\nabla I = \frac{1}{2\Re}\vec{v}$, the differential equation of potential R is equivalent to the equation of continuity

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \tag{131}$$

Noticing that the classical momentum of particle is $m\vec{v} = \hbar \nabla I$, we find that the differential equation of potential I goes to

$$\partial_t(\hbar I) + \frac{(\nabla(\hbar I))^2}{2m} - \hbar\Re[(\nabla R)^2 + \nabla^2 R] = 0 \tag{132}$$

Comparing with the Hamilton-Jacobi equation from classical mechanics [24, 25] as

$$\partial_t S + \frac{(\nabla S)^2}{2m} + V(x) = 0 \tag{133}$$

which is particularly useful in identifying conserved quantities for mechanical systems. There are two crucial remarks: Firstly, potential function I is proportional to the Hamilton-Jacobi function S as $S = \hbar I$. Secondly, for a free particle, the influence of spacetime can be summed up to the spacetime potential

$$V_{ST} = -\hbar\Re[(\nabla R)^2 + \nabla^2 R] \tag{134}$$

where the spacetime potential V_{ST} will play the same role of potential V in the Hamilton-Jacobi equation. The spacetime potential V_{ST} vanishes in the classical limit $\hbar = 0$, which is equivalent to $V = 0$ for free particles in classical mechanics. The quantum effect, which corresponding to nonzero $\hbar$, now is the natural result of the existence of the spacetime potential V_{ST}, induced by MIP. In principal, the moving of free particle can be described precisely by the spacetime potential V_{ST} as

$$m\frac{d^2\vec{x}}{dt^2} = -\nabla V_{ST} = \hbar\Re\nabla[(\nabla R)^2 + \nabla^2 R] \tag{135}$$

C. Space-time Random Motion of Charged Particles in Electromagnetic Field

According to the MIP, electromagnetic field only serves as an external potential, which itself is not affected by random impacts of spacetime. In a electromagnetic field $(\vec{E}, \vec{B})$, the charged particle will experience a Lorentz force $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$. Therefore, the average acceleration of charged particles will be

$$\vec{a} = e(\vec{E} + \vec{v} \times \vec{B})/m \tag{136}$$

where m is the inertial mass of charged particle and e is the charge. Based on the spacetime principle, we are able to derive the equation of motion of charged particle in electromagnetic field, which is finally shown to be Schrödinger equation in electromegnatic field, which is

$$i\hbar\partial_t\Psi = \frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\vec{A})^2\Psi + e\phi\Psi \tag{137}$$

where the electromagnetic potential and the electromagnetic field are connected by

$$\vec{B} = \nabla \times \vec{A}, \vec{E} = -\partial_t\vec{A} - \nabla\phi. \tag{138}$$

We do not have average acceleration in absence of electromagnetic field. However, this is not the case when the particle have non-zero electric charge, moving in external electromagnetic field. Identifying the speed in the Lorentz force as the classical speed of random motion of particle in spacetime, we have

$$\partial_t\vec{v} = e(\vec{E} + \vec{v} \times \vec{B})/m - (\vec{v} \cdot \nabla)\vec{v} + (\vec{u} \cdot \nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{139}$$

In the electromagnetic field, the equation of motion of charged particle becomes coupled non-linear partial differential equations as following

$$\frac{\partial\vec{u}}{\partial t} = -\Re\nabla(\nabla \cdot \vec{v}) - \nabla(\vec{u} \cdot \vec{v}) \tag{140}$$

$$\frac{\partial\vec{v}}{\partial t} = e(\vec{E} + \vec{v} \times \vec{B})/m - (\vec{v} \cdot \nabla)\vec{v} + (\vec{u} \cdot \nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{141}$$

In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly. Let $\Psi = e^{R+iI}$ and notice that the canonical momentum of charged particle [26] is $\vec{p} = m\vec{v} + e\vec{A}/c$, we suppose

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{142}$$

$$\nabla I = \frac{1}{2\Re}(\vec{v} + \frac{e\vec{A}}{mc}) \tag{143}$$

In order to prove Eq.(137), we expand the first term of right side of Eq.(137) as

$$\begin{aligned}\frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\vec{A})^2\Psi \;=\; & -\frac{\hbar^2\nabla^2}{2m}\Psi + \frac{e^2A^2}{2mc^2}\Psi \\ & +\frac{i\hbar e}{2mc}(\nabla\cdot\vec{A})\Psi + \frac{i\hbar e}{mc}\vec{A}\cdot(\nabla\Psi)\end{aligned} \tag{144}$$

Substituting $\Psi = e^{R+iI}$, it leads to

$$\begin{aligned}-\frac{\hbar^2}{2m}[\nabla^2 R + i\nabla^2 I + (\nabla R + i\nabla I)^2]\Psi + \frac{e^2A^2}{2mc^2}\Psi \\ +\frac{i\hbar e}{2mc}(\nabla\cdot\vec{A})\Psi + \frac{i\hbar e}{mc}(\vec{A}\cdot(\nabla R + i\nabla I))\Psi\end{aligned} \tag{145}$$

With vector formulas

$$\begin{aligned}\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) \\ +(\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A}\end{aligned} \tag{146}$$

$$\nabla(\nabla\cdot\vec{A}) = \nabla\times(\nabla\times\vec{A}) + \nabla^2\vec{A} \tag{147}$$

and Eq.(142), we will obtain

$$\nabla\times\vec{u} = 0 \tag{148}$$

$$\nabla\times(\vec{v} + \frac{e\vec{A}}{mc}) = 0 \tag{149}$$

Straightforwardly, we have

$$\begin{aligned}i\hbar(\partial_t R + i\partial_t I) = -\frac{\hbar^2}{2m}[\nabla^2 R + i\nabla^2 I \\ +(\nabla R + i\nabla I)^2] + \frac{e^2A^2}{2mc^2} \\ +\frac{i\hbar e}{2mc}(\nabla\cdot\vec{A}) + \frac{i\hbar e}{mc}(\vec{A}\cdot(\nabla R + i\nabla I)) + e\phi\end{aligned} \tag{150}$$

Now, let's prove that the real and imaginary parts are separately equaled as

$$\begin{aligned}\partial_t I = & \frac{\hbar}{2m}(\nabla^2 R + (\nabla R)^2 - (\nabla I)^2) \\ & -\frac{e^2\vec{A}^2}{2mc^2} + \frac{e}{mc}(\vec{A}\cdot(\nabla I)) - \frac{e\phi}{\hbar} \end{aligned} \tag{151}$$

$$\begin{aligned}\partial_t R = & -\frac{\hbar}{2m}(\nabla^2 I + 2(\nabla R)\cdot(\nabla I)) \\ & +\frac{e}{2mc}(\nabla\cdot\vec{A}) + \frac{e}{mc}\vec{A}\cdot(\nabla R) \end{aligned} \tag{152}$$

Taking the gradient from both sides and the definitions $\vec{B} = \nabla\times\vec{A}, \vec{E} = -\partial_t\vec{A} - \nabla\phi$, we have reproduced the Eq.(140). Therefore, we have proved that both sides of Eq.(140) are at most different from a zero gradient function. It's important to notice that the choices of electromagnetic potentials are not completely determined. It allows a gauge transformation [27]

$$\vec{A}' = \vec{A} + \nabla\Lambda \tag{153}$$

$$\phi' = \phi - \partial_t\Lambda \tag{154}$$

For any function $\Lambda(\vec{x}, t)$, the electromagnetic field is invariant. Therefore, the corresponding wave function cannot change essentially, at most changing a local phase factor. Given $\psi' = \psi e^{\frac{ie\Lambda}{\hbar c}}$, Schrödinger equation of charged particle in electromagnetic field is invariant, i.e., $U(1)$ gauge symmetry. By choosing the function $\Lambda(\vec{x}, t)$ properly, we are able to eliminate the redundant zero gradient function. So we have proved Eq.(137) at the end.

D. Stationary Schrödinger Equation from MIP

Let's take a hydrogen atom as an example. Given $\vec{A} = 0$ and $\phi = -\frac{e}{4\pi\epsilon_0 r}$ for a hydrogen atom, the stationary solution of Eq.(137) is

$$E\Psi = \frac{1}{2m}(-i\hbar\nabla)^2\Psi - \frac{e^2}{4\pi\epsilon_0 r}\Psi \tag{155}$$

of which ground state wave function is $\Psi(r,\theta,\varphi) = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$ and $a = 5\times 10^{-11}m$ is the Bohr radius. Corresponding to the classical speed from Eq.(122), it is easy to show that the classical speed of particles must be zero in stationary states. Within the framework of MIP, we should interpret the stationary states from quantum mechanics as a spacetime random motion with zero classical speed. Once we have all the stationary states, we will get the general solution by linear superposition. Therefore, we are going to derive stationary Schrödinger equation from classical speed $\vec{v} = 0$, which can provide a clear physical picture of MIP. Moreover, when $|\vec{v}|$ is large and close to speed of light c, the generalisation of this framework is clear and will be explained in our further work.

The trajectory of random motion of particle can be understood as the superposition of classical path and fluctuated path. During time interval $\triangle t$, there are two contributions to the trajectory as

$$\delta \vec{x} = \vec{u}(\vec{x}, t)\triangle t + \triangle \vec{x} \tag{156}$$

of which distribution satisfies $\varphi(\triangle \vec{x}) = \varphi(-\triangle \vec{x})$ and

$$\int \varphi(\triangle \vec{x}) d(\triangle \vec{x}) = 1$$

. The spacetime coefficient reads

$$\Re = \frac{1}{2\triangle t}\int (\triangle \vec{x})^2 \varphi(\triangle \vec{x}) d(\triangle \vec{x}) \tag{157}$$

The probabilistic density $\rho(x, t)$ evolves as

$$\rho(\vec{x}, t + \triangle t) = \int \rho(x - \delta \vec{x}, t)\varphi(\triangle \vec{x}))d(\triangle \vec{x}) \tag{158}$$

Expanding Taylor series of both sides, we have

$$\partial_t \rho = -\nabla \cdot (\rho \vec{u}) + \Re \nabla^2 \rho \tag{159}$$

which is consistent with Fokker-Planck equation. In any external potential $V(\vec{x})$, there are two contributions to the changing of average speed. One is from random impacts of spacetime, another one is from acceleration provided by external potential. Therefore, the average speed will evolve during time interval $\triangle t$ as

$$\begin{aligned} &\vec{u}(\vec{x}, t + \triangle t) = \\ &\frac{\int (\vec{u}(\vec{x} - \delta \vec{x}, t) - \frac{\triangle t \nabla V(\vec{x} - \delta \vec{x})}{m})\rho(x - \delta \vec{x}, t)\varphi(\triangle \vec{x}))d(\triangle \vec{x})}{\int \rho(x - \delta \vec{x}, t)\varphi(\triangle \vec{x}))d(\triangle \vec{x})} \end{aligned} \tag{160}$$

the denominator of eq. 161 is the normalisation factor of the probability distribution. Expanding Taylor series of both sides, we obtain

$$\frac{d\vec{u}}{dt} = -\nabla V + \Re m(\frac{\nabla^2(\rho \vec{u})}{\rho} - \vec{u}\frac{\nabla^2 \rho}{\rho}) \tag{161}$$

With the condition of stationary state $\partial_t \rho = 0$, it goes to

$$\vec{u} = \Re \frac{\nabla \rho}{\rho} \tag{162}$$

$$\partial_t \vec{u} = 0 \tag{163}$$

It's important to notice that

$$\frac{d\vec{u}}{dt} = \partial_t \vec{u} + (\vec{u} \cdot \nabla)\vec{u} \tag{164}$$

The average speed $\vec{u}$ is not zero in the stationary state, which exactly cancle out its fluctuation speed. Therefore, given the condition of stationary state, we are able to get

$$-2m\Re^2 \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + V(x) = Const. \tag{165}$$

We can prove this constant is exactly the average energy of particle

$$E = \int \rho (\frac{1}{2} m u^2 + V) d^3x \tag{166}$$

Now, we have derived

$$-2m\Re^2 \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + V(x) \; = \; E \tag{167}$$

$$\psi = \sqrt{\rho} e^{-iEt/\hbar} \tag{168}$$

Let $\Re = \frac{\hbar}{2m}$ once again, we arrive at the stationary Schr$\ddot{o}$dinger equation

$$-\frac{\hbar^2 \nabla^2}{2m} \psi + V\psi = E\psi \tag{169}$$

VII. THE ORIGIN OF SPIN IN MIP

In this section, we will investigate the origin of spin based on MIP, in order to derive naturally the uncertain relation among different components of spin and interpret the exotic quantum behaviours observed in the Stern-Gerlach experiments. The concept of spin is a very difficult one, which relates to three other areas in theoretical physics: the spinning rigid body from classical mechanics, the quantisation of angular momentum in quantum mechanics and the spacetime structures in special relativity. It's safe to say at present, we still don't understand the concept of spin thoroughly. Is spin a structure of symmetry or topology? From above sections, we have derived Schrodinger's equation of a charged spinless particle in electromagnetic fields as

$$i\hbar \partial_t \Psi = \frac{1}{2m} (-i\hbar \nabla - \frac{e}{c} \vec{A})^2 \Psi + e\phi \Psi \tag{170}$$

Let's consider the impact of STP on the rotational degree of freedom of particle, which will lead us to the quantisation of angular momentum. At the end, we will derive Pauli's equation of charged particle in electromagnetic fields as

$$i\hbar\partial_t\Psi = \frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\vec{A})^2\Psi + e\phi\Psi - \frac{q\hbar}{2m}\vec{\sigma}\cdot\vec{B}\Psi \tag{171}$$

where $\vec{\sigma}$ is Pauli's matrix, Ψ is a wave function with two components. We have to obtain the coupling term $-\frac{q\hbar}{2m}\vec{\sigma}\cdot\vec{B}$ from MIP. If this coupling term can be induced, we have proved the spin angular momentum of particle is $\hbar/2$ given by the fluctuation of spacetime. Mathematically, wave function $\Psi = e^{R+iI}$ is a complex field, corresponding to two real filed R and I. The gradients of R and I lead to the classical and fluctuated speed of translational motion, which totally has six degrees of freedom (one half from translation, another half from spacetime fluctuation). We need three degrees of freedom to describe rotational motion(e.g., Euler's angles α, β, γ), with another three degrees of freedom to describe fluctuated rotational motion, which all together matches the degrees of freedom of a wave function with two components.

For a free particle, we have $\vec{A} = 0, \varphi = 0$. From the influences of spacetime fluctuation on translational and rotational motions, we would like to derive $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2\nabla^2}{2m}\Psi$, where Ψ is a wave function with two components. Physically, it's crucial to show random rotational motions given by STP has the exact properties of a quantum spin. From above sections on random translational motions, we have $\vec{p} \to -i\hbar\nabla$. Generalising to the random rotational motions, we will obtain the angular momentum as $\vec{s} \to -i\hbar\nabla_{rot}$, where ∇_{rot} is a gradient operator on angles. Specifically, the coordinate of any part λ of a particle is

$$\vec{r^\lambda} = \vec{r} + \vec{\zeta^\lambda} \tag{172}$$

where $\vec{r}$ is the coordinate of centre of mass, $\vec{\zeta}$ is relative coordinate to the centre of mass 'Total angular momentum can be summed up as orbital part and spin part

$$\vec{J} = \sum_\lambda \vec{r^\lambda} \times \vec{p} = \vec{r} \times \vec{p} + \sum_\lambda \vec{\zeta^\lambda} \times \vec{p^\lambda} \tag{173}$$

$$= -i\hbar\vec{r} \times \nabla - i\hbar\nabla_{rot} = \vec{l} + \vec{s} \tag{174}$$

It will induce SU(2) Lie algebra given by spacetime random fluctuations as

$$[s_i, s_j] = i\hbar\epsilon_{ijk}s_k \tag{175}$$

In detail, ∇_{rot} can be conveniently transformed to differential on Euler's angle α, β, γ

$$\begin{aligned}(\nabla_{rot})_x &= \frac{\partial}{\partial \theta_x} \\ &= -cos\alpha ctg\beta\partial_\alpha - sin\alpha\partial_\beta + cos\alpha csc\beta\partial_\gamma\end{aligned} \tag{176}$$

$$\begin{aligned}(\nabla_{rot})_y &= \frac{\partial}{\partial \theta_y} \\ &= -sin\alpha ctg\beta\partial_\alpha + cos\alpha\partial_\beta + sin\alpha csc\beta\partial_\gamma\end{aligned} \tag{177}$$

$$(\nabla_{rot})_z = \frac{\partial}{\partial \theta_z} = \partial_\alpha \tag{178}$$

where $\theta_x, \theta_y, \theta_z$ are angles versus fixed axes x, y, z. Straightforward calculation shows that $[s_i, s_j] = i\hbar\epsilon_{ijk}s_k$. According to Schwartz inequality, we prove the uncertain relation among different components of spin as

$$< (\triangle S_x)^2 >< (\triangle S_y)^2 >\geqslant \frac{\hbar^2}{4} < S_z >^2 \tag{179}$$

At the same time, we obtain the commutation between angle and angular momentum

$$[S_x, \theta_x] = [-i\hbar(\nabla_{rot})_x, \theta_x] = -i\hbar \tag{180}$$

which leads to

$$\triangle S_x \triangle\theta_x \geqslant \frac{\hbar}{2} \tag{181}$$

It should be noticed that, the distinction between SU(2) and SO(3) is important. Ordinary rotations only give to SO(3) orbital angular momentum, which can not correspond to the quantum spin. Within the framework of MIP, for a random process $\vec{x}(t)$, its velocity $\vec{V}$ can be understood as a superposition between classical velocity $\vec{v}$ and fluctuated velocity $\vec{u}$ as

$$\vec{V} = \vec{v} + \vec{u} \tag{182}$$

Under time reversal ($\vec{x} \to \vec{x}, t \to -t$), we have $\tilde{v} = -\vec{v}, \tilde{u} = \vec{u}$, which means a continuous Markov process is still a Markov process under time reversal. Exploring the property of time reversal, we show that angular velocity $\vec{\Omega}$ can be understood as a superposition between classical angular velocity $\vec{\omega}$ and fluctuated angular velocity $\vec{\xi}$ as

$$\vec{\Omega} = \vec{\omega} + \vec{\xi} \tag{183}$$

Furthermore, we construct the corresponding complex momentum and angular momentum as

$$\vec{p} = \vec{p}_v - i\vec{p}_u$$
$$\vec{s} = \vec{s}_\omega - i\vec{s}_\xi$$

Imposing the average acceleration of free particle must be zero, the equation of motion will be

$$D^-\vec{v} - D^+\vec{u} = 0 \quad (184)$$
$$D^+\vec{v} + D^-\vec{u} = 0 \quad (185)$$

Including rotational degrees of freedom, the total derivatives D^- and D^+ are

$$D^- = \frac{1}{2}(D - \tilde{D}) = \partial_t + \vec{v}\cdot\nabla + \vec{\omega}\cdot\nabla_{rot} \quad (186)$$
$$D^+ = \frac{1}{2}(D + \tilde{D}) \quad (187)$$
$$= \vec{u}\cdot\nabla + \Re\nabla^2 + \vec{\xi}\cdot\nabla_{rot} + \Re_{rot}\nabla^2_{rot}$$

Analogy from mass diffusion coefficient $\Re = \frac{\hbar}{2m}$ for translational inertia, there is a mass diffusion coefficient $\Re_{rot} = \frac{\hbar}{2I}$ for rotational inertia, where I is the moment of inertia. Taking an example of rotation around a fixed axes, the fluctuated angle θ satisfies $<\theta^2>= 2\Re_{rot}t$ analogy with fluctuated displacement $<x^2>= 2\Re t$. SU(2) Lie algebra leads to

$$[s^2, s_k] = 0 \quad (188)$$

We can construct common eigenstate Ψ_{mj} of s^2, s_3 as

$$s^2\Psi_{mj} = j(j+1)\hbar^2\Psi_{mj} \quad (189)$$
$$s_3\Psi_{mj} = m\hbar\Psi_{mj} \quad (190)$$

Furthermore, the ladder operator $s_\pm$ will be

$$s_\pm = \frac{1}{\sqrt{2}}(s_1 \pm is_2) \quad (191)$$
$$s_\pm\Psi_{mj} = \sqrt{\frac{1}{2}(j\mp m)(j\pm m+1)}\hbar\Psi_{mj} \quad (192)$$

So we have all eigenstates with half integer spin $s = 1/2, 3/2, 5/2, ...$ In this paper, we assume massive elementary particles have size. According to QED experiments, the radius of electron can not be lager than $10^{-16}cm$. The finite size of electrons implies a nonzero moment of inertia, which are randomly collided by STP to acquire a spin angular momentum. For heavy electrons with higher spins, its excitation energy are much larger than collision energy of STP[32]. Therefore, we only investigate $s = \frac{1}{2}$ state, which satisfies

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{\hbar^2\nabla^2}{2m}\Psi \tag{193}$$

where $\Psi = (\Psi_{+1/2}, \Psi_{-1/2})$ is a wave function with two components'With external magnetic field $\vec{B}$, the particle with $s = \frac{1}{2}$ satisfies

$$i\hbar\partial_t\Psi = -\frac{\hbar^2\nabla^2}{2m}\Psi - \frac{q\hbar}{2m}\vec{\sigma}\cdot\vec{B}\Psi \tag{194}$$

which is Pauli's equation with Pauli's matrixes $\vec{\sigma}$. Within the framework of MIP, mass is a statistical quantity and the line velocity of particle surface is no longer a classical concept. The fluctuated angular velocity given by random impacts of STP is a Markov process, which is time reversal invariant. The Pauli's arguments on faster than light speed is irrelevant in our scenario. When the particle is not randomly collided by STP, the attribute of mass does not exist yet. When the particle is randomly collided by STP, it's mass is not uniformly distributed but statistically fluctuated. According to MIP, every random impact of STP give matter particles an nh action, which one way gives to translational quantum wave, another way to rotational quantum spin.

VIII. QUANTUM MEASUREMENT IN MIP

A. General Principle

There are fundamental distinctions on quantum measurement between MIP and Copenhagen interpretation. Within the framework of MIP, since matter particle is collided randomly by STP, any measurement cannot lead to precise result, which means we cannot make errors as small as possible in principle. Therefore, incommutable observables can not only be measured precisely at the same time, but also cannot be measured precisely separately. Theoretically, all measure values means statistical average, which include intrinsic uncertainty from spacetime besides normal measurement errors. For examples, the momentum uncertainty from MIP is due to the statistical properties of fluctuated mass. As a statistical mass, the minimum fluctuation is $\triangle m_{st}$, which roughly is one part per million of electron mass. The position intrinsic uncertainty $\triangle X_{st}$ from MIP is the mean free distance between two consecutive collision by STP. When the spacetime sensible mass is equivalent to the statistical inertial mass, the equation of motion will be determined by Schrodinger equation. In other words, moving matter particle and propagational wave are unified in spacetime. If we want to measure a matter particle, we need apparatus to interact with particle somehow. However, every such measurement has to interrupt the random motion of particle. Therefore, measurement means the end of a Markov process. When the measurement is finished, a new Markov process will begin. For the moving matter particle, the phases of wave functions before and after measurements is completely irrelevant, which cannot interfere each other. Under this framework, it's unnecessary to introduce hypothesises of wave function collapse or multi universe.

B. EPR Paradox in MIP

In a 1935 paper[33], Einstein with Podolsky and Rosen considered an experiment in which two particles that move along the x-axis with coordinates x_1 and x_2 and momenta p_1 and p_2 were somehow produced in an eigenstate of the observables $X = x_1 - x_2$ and $P = p_1 + p_2$ (these two observables commute $[X, P] = 0$).It's easy to understand that the measurement of the position of particle 1 can interfere with its momentum, so that after the second measurement the momentum of particle 1 no longer has a definite value. However two particles are far apart, how can the second measurement interfere with the momentum of particle 2? And if it does not, then after both measurements particle 2 must have both definite position and momentum, contradicting the quantum uncertainty principle. If it does, there exist some "spooky" interaction between two far apart particle, contradicting the locality principle in the special theory of relativity. The orthodox interpretation of quantum mechanics suppose that the second measurement which gives particle 1 a definite position, prevents particle 2 from having a definite momentum, even though the two particles are far apart. The states of the two particles are so call quantum entanglement.

Let's investigate the experimental process in detailed and estimate every uncertainty relations. Suppose two particles that are originally bound in some sort of unstable molecule at rest fly apart freely in opposite directions, with equal and opposite momenta until their separation becomes macroscopically large. Their separation will evolve as

$$x_1 - x_2 = x_{10} - x_{20} + (p_1 - p_2)t/m \tag{195}$$

where x_{10}, x_{20} are initial positions of two particles. It's noticed that under MIP, every massive particle is collided randomly by STP, the initial separation of two particle cannot be measured precisely. There exists intrinsic uncertainty $\triangle X_{st} = \triangle|x_{10} - x_{20}|$ as the mean free distance between two consecutive collision by STP. According to the uncertainty relation derived from MIP, the momentum difference at least has intrinsic uncertainty as $\triangle P_{st} = \triangle|p_1 - p_2| \geq \frac{\hbar}{\triangle X_{st}}$, because of the commutation $[x_1 - x_2, p_1 - p_2] = 2i\hbar$. Therefore the uncertainty of separation will be

$$\triangle|x_1 - x_2| = \triangle X_{st} + \frac{\hbar t}{\triangle X_{st} m} \tag{196}$$

Its minimum is at $\triangle X_{st} = \sqrt{\frac{\hbar t}{m}}$, leading to

$$\triangle|x_1 - x_2| \geq 2\sqrt{\frac{\hbar t}{m}} \tag{197}$$

Similarly, the total momentum P is not strictly zero under MIP, which includes at least the intrinsic uncertainty due to

$$\triangle P = \triangle m_{st} v \tag{198}$$

where $\triangle m_{st}$is the fluctuation of statistical mass, according to MIP, roughly as one part per million of electron mass. Perform EPR experiment after the second measurement of particle 1, the uncertainty of particle 2 at least will be

$$\triangle p_2 \triangle x_2 = 2\sqrt{\frac{\hbar t}{m}} \triangle m_{st} v \tag{199}$$

More importantly , does the intrinsic uncertainty of particle 2 given by MIP contradict the uncertainty relation given by quantum mechanics? If

$$\triangle p_2 \triangle x_2 \leq \frac{\hbar}{2} \tag{200}$$

it still contradicts uncertainty relation of quantum mechanics, which means that we will observe the quantum entanglement experimentally, because we have to suppose the "spooky" interaction between two far apart particles to satisfy uncertainty relation. Therefore, we obtain the key criterion of quantum entanglement (momentum-position type) as

$$\frac{\triangle m_{st}^2}{m^2} \leq \frac{\lambda_d}{16\pi L} \tag{201}$$

where $\lambda_d = \frac{h}{mv}$ is de Broglie′s wavelength and L is the separation of two particles. So we can conclude that there is a characteristic separation of quantum entanglement as

$$L^* = \frac{\lambda_d}{16\pi}(\frac{m}{\triangle m_{st}})^2 \tag{202}$$

When the separation of two particles is larger than L^*, the inequality of (8) cannot be satisfied which means we are no longer able to determine the existence of quantum entanglement from experimental results. The reason is that the intrinsic uncertainty of particle 2 given by MIP has already satisfy uncertainty relation of quantum mechanics automatically. We cannot deduce the existence of 'spooky' interaction in this scenario. For two electrons moving at the speed of $0.01c$, the corresponding characteristic separation will be $L^* \approx 1m$. For two atoms moving at the speed of $0.01c$, the corresponding characteristic separation will be $L^* \approx 10^6 m$.

IX. FROM MIP TO PATH INTEGRAL

Historically, the basic idea of the path integral formulation can be traced back to Norbert Wiener, who introduced the Wiener integral for solving problems in stochastic process [2]. This idea was extended to the use of the Lagrangian in quantum mechanics by P. A. M. Dirac in his 1933 paper [38].The complete method was developed in 1948 by Richard Feynman [37]. The path integral formulation of quantum mechanics is a description of quantum theory which generalises the action principle of classical mechanics. It replaces the classical notion of a single, unique trajectory for a system with a sum, or functional integral, over an infinity of possible trajectories to compute a quantum amplitude. Although we only investigated non-relativistic quantum mechanics in this paper, it is worthy to remark that the path integral formulation was very important for the development of quantum field theory[**?**]. The advantages of the path integral formulation mostly come from putting space and time on the equal footing, which is convenient to generalise in the relativistic theory. However, the regulator of path integral have caused infamous troubles for the divergence in quantum field theory, which leads to the procedure of renormalization. Within the framework of MIP, all the properties of random motion particle are finite so that we are able to construct a theory without divergence from beginning. In other words, all the quantum behaviours of particles are emergent from the statistical description of stochastic process.

A. Path Integral of Free Particle and Spacetime Interaction Coefficient

There are two kinetic variables with random motion particle in spacetime, which are classical speed $\vec{v}$ and fluctuated speed $\vec{u}$. The corresponding kinetic equations are

$$\frac{\partial \vec{u}}{\partial t} = -\Re\nabla(\nabla\cdot\vec{v}) - \nabla(\vec{u}\cdot\vec{v}) \tag{203}$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v}\cdot\nabla)\vec{v} + (\vec{u}\cdot\nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{204}$$

Setting $\Psi = e^{R+iI}$, we are able to linearise as

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{205}$$

$$\nabla I = \frac{1}{2\Re}\vec{v} \tag{206}$$

which leads to

$$\frac{\partial \Psi}{\partial t} = i\Re\nabla^2\Psi \tag{207}$$

During an infinite small time interval ϵ, the solution can be written in terms of integrals as

$$\Psi(x, t+\epsilon) = \int G(x, y, \epsilon)\Psi(y, t)dy \tag{208}$$

which represents the superposition of all the possible paths from y to x. The critical observation of Feynman is the weight factor $G(x, y, \epsilon)$ will be proportional to $e^{iS(x,y,\epsilon)/\hbar}$, where $S(x, y, \epsilon)$ is the classical action of particle as

$$S(x, y, \epsilon) = \int L(x, y, \epsilon)dt = \int (K - U)dt = (\bar{K} - \bar{U})\epsilon \tag{209}$$

$\bar{K}$ and $\bar{U}$ are average kinetic energy and potential energy separately. In order to show the equivalence between path integral formulation and the spacetime interacting picture, we should derive our basic kinetic equations from the postulation of path integral $G(x, y, \epsilon) = Ae^{iS(x,y,\epsilon)/\hbar}$. For a free particle in spacetime, one has $\bar{U} = 0$,$\bar{L} = \frac{m}{2}(\frac{x-y}{\epsilon})^2$ and $S = \frac{m(x-y)^2}{2\epsilon}$, which leads to

$$\Psi(x, t+\epsilon) = A\int e^{\frac{im(x-y)^2}{2\hbar\epsilon}}\Psi(y, t)dy \tag{210}$$

Setting $y - x = \xi$ and $\alpha = -\frac{im}{2\hbar\epsilon}$, it can be written in terms of

$$\begin{aligned}\Psi(x, t+\epsilon) &= A\int e^{-\alpha\xi^2}\Psi(x+\xi, t)d\xi \\ &= A\int e^{-\alpha\xi^2}(\Psi(x,t) + \xi\frac{\partial\Psi}{\partial x} + \frac{1}{2}\xi^2\frac{\partial^2\Psi}{\partial x^2} + \mathcal{O}(\xi^4))d\xi\end{aligned} \tag{211}$$

With the properties of Gaussian integral

$$\int e^{-\alpha\xi^2}d\xi = \sqrt{\frac{\pi}{\alpha}} \tag{212}$$

$$\int e^{-\alpha\xi^2}\xi d\xi = 0 \tag{213}$$

$$\int e^{-\alpha\xi^2}\xi^2 d\xi = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}} \tag{214}$$

we can obtain

$$\Psi(x, t+\epsilon) = A(\sqrt{\frac{\pi}{\alpha}}\Psi(x,t) + \frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}\frac{\partial^2\Psi}{\partial x^2} + \mathcal{O}(\alpha^{-\frac{5}{2}})) \tag{215}$$

Setting $A = \sqrt{\frac{\alpha}{\pi}}$, we have

$$\Psi(x, t+\epsilon) - \Psi(x,t) = \epsilon\partial_t\Psi(x,t) = \frac{1}{4\alpha}\frac{\partial^2\Psi}{\partial x^2} \tag{216}$$

From this integral, We observed that the most important contribution comes from $y - x = \xi \propto \sqrt{\epsilon}$, where the speed of particle is $\frac{y-x}{\epsilon} \propto \sqrt{\frac{\hbar}{m\epsilon}}$, we see here when $\epsilon \to 0$, the speed divergent in order $\sqrt{1/\epsilon}$. The paths involved are, therefore continuous but possess no derivative, which are of a type familiar from study of stochastic process. With the isotropy of space, we have

$$\partial_t\Psi(\vec{x}, t) = \frac{1}{4\alpha\epsilon}\nabla^2\Psi(\vec{x}, t) \tag{217}$$

Corresponding to the Eq. (207), if one requires the equivalence between path integral formulation and MIP, there must be

$$i\Re = \frac{1}{4\alpha\epsilon} \tag{218}$$

$$\Re = \frac{1}{4i\alpha\epsilon} = \frac{1}{4i(-\frac{im}{2\hbar\epsilon})\epsilon} = \frac{\hbar}{2m} \tag{219}$$

Notice that $\Re$ is only an arbitrary parameter in the Eq.(119). The consistency between path integral and MIP requires $\Re = \frac{\hbar}{2m}$. An arbitrary finite time interval Δt, can be cut into infinitely many pieces of infnitesimal time interval ϵ. And in each ϵ, the collisions leads to many different paths, one can pick one path and consectively another along the time direction, this will end up a path in Δt, sum over all possible paths in Δt gives an integration over path space, which is the celebrated historical summation or path integral. The method here can be straightforwardly generalised to the particle in the external potential as in following section.

B. Path Integral of Particle in an External Potential and Spacetime Interaction Coefficient

In an external potential U, one has $\bar{U} = U(\frac{x+y}{2})$ and $\bar{L} = \frac{m}{2}(\frac{x-y}{\epsilon})^2$, which leads to the action

$$S = \frac{m(x-y)^2}{2\epsilon} - U(\frac{x+y}{2})\epsilon \tag{220}$$

According to the path integral formulation, it must satisfy

$$\begin{aligned} \Psi(x, t+\epsilon) &= A\int e^{\frac{im(x-y)^2}{2\hbar\epsilon} - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar}} \Psi(y,t)dy \\ &= A\int e^{\frac{im(x-y)^2}{2\hbar\epsilon}}(1 - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar})\Psi(y,t)dy \end{aligned} \tag{221}$$

To the lowest order of ϵ, it shows

$$U(\frac{x+y}{2})\epsilon = U(x+\frac{\xi}{2})\epsilon = U(x)\epsilon \tag{222}$$

$$\Psi(x, t+\epsilon) = A\int e^{-\alpha\xi^2}(1 - \frac{iU(x)\epsilon}{\hbar})\Psi(x+\xi, t)d\xi \tag{223}$$

From the properties of Gaussian integral in the previous section, we obtain

$$\Psi(x, t+\epsilon) = A(1 - \frac{iU(x)\epsilon}{\hbar})\sqrt{\frac{\pi}{\alpha}}\Psi(x,t) + A\frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}\frac{\partial^2\Psi}{\partial x^2} \tag{224}$$

Setting $A = \sqrt{\frac{\alpha}{\pi}}$, we have

$$\partial_t\Psi(\vec{x}, t) = \frac{1}{4\alpha\epsilon}\nabla^2\Psi(\vec{x}, t) + \frac{1}{i\hbar}U\Psi(\vec{x}, t) \tag{225}$$

To be consistent with the case of free particle, let's take $\Re = \frac{\hbar}{2m}$ which leads to

$$\partial_t\Psi(\vec{x}, t) = i\Re\nabla^2\Psi(\vec{x}, t) + \frac{1}{i\hbar}U\Psi(\vec{x}, t) \tag{226}$$

framework of modern science, which completely overrules Newton′s Therefore we have derived the equation of motion from MIP.

X. SUMMARIES AND CONCLUSIONS

The main idea of Copenhagen interpretation is that the wave function does not have any real existence in addition to the abstract concept. Whether the wave function is an independent entity, the Copenhagen interpretation does not make any statement. However, in this paper, we have proposed the mass interaction principle as an alternative interpretation which does not involve the abstract concept of wave function, but instead leads us to the realistic point of view of particle and space-time. Within the broader framework of mass interaction principle, we are able to conceive of each individual system as being in a precisely definable state, whose changes with time are determined by definite laws. As long as the present form of Schrodinger's equation is retained in the non-relativistic case, the physical results obtained by the space-time interaction principle are precisely the same as those obtained with usual quantum mechanics. At least, it is entirely possible that we can have a precise and objective description at the quantum level.

Starting from the concept of statistical mass, we propose the fundamental MIP. We conclude that inertial mass has to be a statistical property, which measures the diffusion ability of all matter particles in spacetime. We prove all the essential results of special relativity come from MIP. Speed of light in the vacuum need no longer any special treatment. Instead, speed of STP has more fundamentally physical meaning, which represents the upper limit of information propagational speed in physics. Moreover, we derive the uncertainty relation asserting a fundamental limit to the precision with mass and diffusion coefficient. With the postulation of mass interaction principle, quantum behaviour will emerge from a statistical description of space-time random impacts on the experimental scale, including Schrodinger's equation, Born's rule, Heisenberg's uncertainty principle and Feynman's path integral formulation. Last but not least, applying MIP to quantum measurements, especially to the EPR paradox, we obtain whole new interpretation of quantum entanglement.

In other words, MIP unifies the special theory of relativity and quantum mechanics. Essential results of both theory can be derived naturally from MIP. From realistic point of view, we reinterpret the nature of spacetime, mass and randomness, which reveals the very origins of relativity and quantum behaviours.

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