Radiation Pressure Inside a Collapsing Shell

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Abstract

Birkhoff's theorem says that the spacetime inside a spherically symmetric collapsing shell is Minkowskian. Given this fact, signals exchanged between the shell and a central observer as well as between observers on the shell itself are examined, and it is shown that these signals will become infinitely blueshifted as the shell approaches its Schwarzschild radius. It is also shown that the space inside the shell will contract to zero volume in the frame of a shell observer as the shell approaches its Schwarzschild radius.

Observations from a Central Observer

Let us consider an observer at rest at the center of a collapsing spherically symmetric collapsing shell. According to Birkhoff's theorem, the spacetime inside the shell, where the central observer resides, will be flat. An observer in freefall on the collapsing shell does so with speed (in natural units measured by her clock) [1]:

$$\frac{dr}{d\tau} = \sqrt{\frac{2GM}{r}} \tag{1}$$

Therefore, the freefall observer will see observers at rest at *r* moving past her at the speed given in Equation 1. Since the central observer is also at rest relative to observers at rest at any *r*, Equation 1 will also give the relative velocity between the freefall observer and the central observer when the shell is at *r*. Since the spacetime between the freefall observer and central observer is flat, they will each see the other's clock dilated by the Special Relativity Relationship:

$$d\tau = dt\sqrt{1 - V^2} = dt\sqrt{1 - \frac{2GM}{r}} \tag{2}$$

In each of the observer's reference frame, the other observer will appear to be moving toward them with the speed in Equation 1. This means that signals they exchange will be increasingly blueshifted as the shell falls, and will be infinitely blueshifted as $r \to 2GM$.

We can use the velocity addition formula of Special Relativity to calculate the velocity that a freefalling observer will measure for an observer on the opposite side of the shell relative to her:

$$V = -\frac{2\sqrt{\frac{2GM}{r}}}{1+\frac{2GM}{r}} = -\frac{2\sqrt{2GMr}}{r+2GM} \tag{3}$$

Therefore, we can conclude that a collapsing shell of matter will see radiation within the shell emitted by particles on the shell infinitely blueshifted as it approaches its Schwarzschild radius. Since radiation pressure is proportional to the frequency of the radiation, this blueshift would create an infinite radiation pressure and temperature opposing the collapse.

Length contraction inside the shell

As discussed, the spacetime inside the spherically symmetric shell is flat Minkowski spacetime. So let us consider a meter stick inside the shell that stretches from the center of the shell out to a distance *2GM* (the shell is at a radius greater than *2GM* so the entire stick is in flat spacetime). Because the meter stick will appear to be moving in the frame of the freefalling observer, its length in her frame would be:

$$L = 2GM\sqrt{1 - \frac{2GM}{r}} \tag{4}$$

We see from Equation 4 that as the freefalling observer approaches $r = 2GM$ the length of the meter stick in her frame will contract to zero length. Thus, in her frame the center of the collapsing shell and $r = 2GM$ are at the same point as she approaches $r = 2GM$. This implies that in her frame, there will be no space beyond $r = 2GM$.

Conclusion

It was shown that the radiation pressure inside a collapsing spherical shell would increase to infinity as the shell approaches its Schwarzschild radius. This pressure would provide a force opposing gravitational collapse all the way to the event horizon. Furthermore, in the frame of the freefalling observer, the volume from $r = 0$ to $r = 2GM$ will be length contracted to zero in her frame and therefore there will be no room to collapse beyond $r = 2GM$. These two consequences suggest that collapse to a black hole is not physically possible.

References

[1] Raine, D., Thomas, E.: Black Holes: A Student Text. Imperial College Press, (2015).