Method of Localisation and Controlled Ejection of Swarms of Likely Charged Particles

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Abstract

This work considers Coulomb forces acting on a charged point particle located between the two coaxial, likely charged rings laying at some distance between one another. Conditions under which the charged particle, while being affected by Coulomb repulsion forces of charges of the rings, localises in the space between the rings and moves along a closed trajectory around the axis of charged rings, not leaving the space between them, were determined. Also, conditions at which a controlled ejection of the charged particle from its localisation zone (or the capability to control the kinetic energy and direction of ejection of the particle) is possible, were determined. Based on the obtained results, we conclude that the considered method of localisation and controlled ejection of charged particles is applicable both in experiments on the nuclear synthesis in swarms of localised positively charged particles and for the formation of beams of likely charged particles with the given velocity of motion relatively to the charged rings.

Keywords: Coulomb forces, electric charge, nuclear synthesis, particle beam.

Figure 1 illustrates an interaction between a particle and two rings having like electric charges (charges of the particle and the rings are of the same sign). By ring we mean a solid torus (a bagel-shaped manifold). The following definitions are used at the figure:
m is an electrically charged particle;
Pa is a Pa electrically charged ring;
Pb is a Pb electrically charged ring;
Fa is a surface on which unit vectors of forces acting on the particle from the side of the Pa ring are laying;
Fb is a surface on which unit vectors of forces acting on the particle from the side of the Pb ring are laying.

Let us assume that the particle is a point particle and forces acting from the side of rings on the point particle are coming from the circles formed by centres of infinite set of cross-sections of the rings. The rings have the same circular cross-sections (Figure 2). The planes at which the circles of rings are laying are parallel. The axes of the $\vec{X}$ and $\vec{Y}$ coordinate system in which the interaction is reviewed are laying in a plane which is parallel to those of circles of the rings and is located in the middle between the planes

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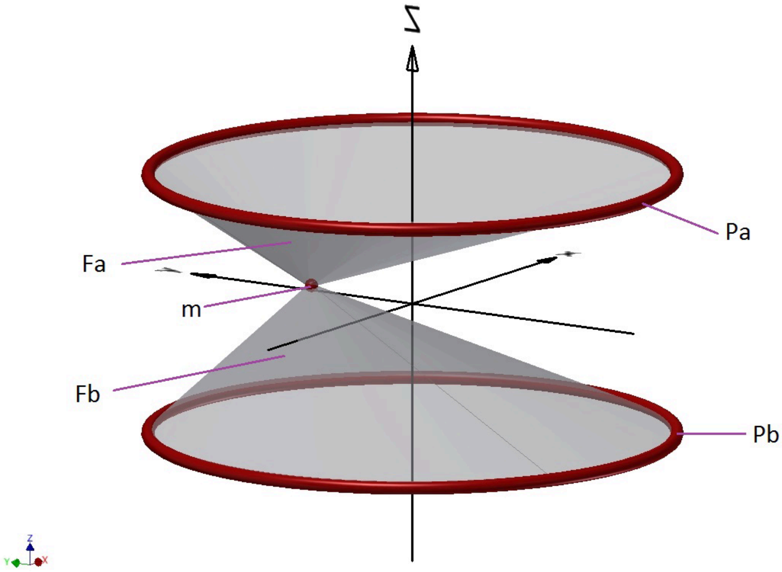

Figure 1: Interaction of a charged particle and two charged rings.

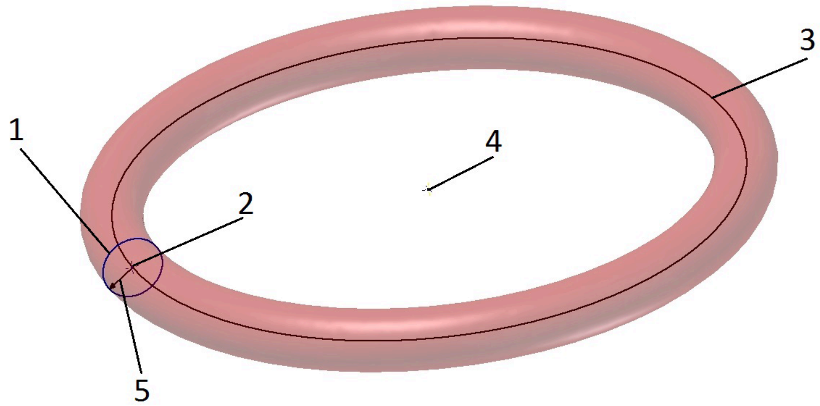

Figure 2: 1 is a circular cross-section of the ring; 2 is a centre of the circular cross-section of the ring; 3 is a circle formed by centres of infinite set of cross-sections of the ring (the circle of the ring); 4 is the centre of the circle of the ring; 5 is a radius of the cross-section of the ring.

of circles of the rings. Projections of centres of circles on the $\vec{X}, \vec{Y}$ plane coincide. The positive axis $\vec{Z}$ is coming from the projection point of centres of circles of the rings to the $\vec{X}, \vec{Y}$ plane and directed toward the Pa ring. Coordinates of the centre of the circle of the Pa ring are $(0, 0, h)$ (the coordinate definition order is (x, y, z)). Coordinates of the centre of the circle of the Pb ring are $(0, 0, -h)$. Coordinates of the particle location are (x, y, z).

In order to determine the equation of motion of the charged particle under the action of Coulomb forces of charges of the rings, we will use the following variables and constants:
m is a mass of the particle;
$\vec{R} = \vec{x} + \vec{y} + \vec{z}$ is the radius vector of the particle position;
$\vec{r} = \vec{x} + \vec{y}$ is a vector parallel to the $\vec{X}, \vec{Y}$ plane, linking the $\vec{Z}$ axis and the particle position point;
q is an electric charge of the particle;
Q_a is an electric charge distributed uniformly in the Pa ring;
Q_b is an electric charge distributed uniformly in the Pb ring;
$\vec{R}_{a0}$ is a vector coming from the centre of the circle of the Pa ring to the point on this circle;
$\vec{R}_{b0}$ is a vector coming from the centre of the circle of the Pb ring to the point on this circle;
$\vec{R}_{ma}$ is a vector issued from the point of the circle of the Pa ring to which the $\vec{R}_{a0}$ vector comes to, to the point of location of the particle;
$\vec{R}_{mb}$ is a vector issued from the point of the circle of the Pb ring to which the $\vec{R}_{b0}$ vector comes to, to the point of location of the particle;
γ is an angle between the vectors $\vec{r}$ and $\vec{R}_{a0}$ equal to the angle between the vectors $\vec{r}$ and $\vec{R}_{b0}$;
ω is a magnitude of an angular rotation velocity of the $\vec{r}$ vector.

In order to consider forces acting on the particle from the side of two rings as those that are coming from the circles formed by centres of infinite set of cross-sections of the rings, the following conditions should be satisfied: $R_{ma} >> r_s$ and $R_{mb} >> r_s$ where r_s is the radius of cross-sections of the rings (Figure 2).

Let us write down the equation of motion of the particle as an equality of the force acting on the particle to the sum of Coulomb forces applied to the particle from the side of all electric charges of the rings:

$$m\frac{d^2\vec{R}}{dt^2} = \frac{qQ_a}{2\pi}\int_0^{2\pi}\frac{\vec{R}_{ma}}{R_{ma}^3}d\gamma + \frac{qQ_b}{2\pi}\int_0^{2\pi}\frac{\vec{R}_{mb}}{R_{mb}^3}d\gamma, \tag{1}$$

where:

$$\vec{R}_{ma} = \vec{r} + (z-h)\,\hat{z} - \vec{R}_{a0}, \tag{2}$$

$$\vec{R}_{mb} = \vec{r} + (z+h)\,\hat{z} - \vec{R}_{b0}, \tag{3}$$

$$R_{ma}^2 = R_{a0}^2 + (z-h)^2 + r^2 - 2rR_{a0}\cos(\gamma), \tag{4}$$

$$R_{mb}^2 = R_{b0}^2 + (z+h)^2 + r^2 - 2rR_{b0}\cos(\gamma). \tag{5}$$

In the (1), the integration is performed with respect to the γ variable, with constants R_{a0}, R_{b0}, $\vec{R}$, h, and variables $\vec{R}_{ma}$ and $\vec{R}_{mb}$.

Let us determine the projection of the force (1) to the $\vec{r}$ vector:

$$m\left(\frac{d^2\vec{R}}{dt^2}\cdot\vec{r}\right)=\frac{qQ_a}{2\pi}\int_0^{2\pi}\frac{\left(\vec{R}_{ma}\cdot\vec{r}\right)}{R_{ma}^3}d\gamma+\frac{qQ_b}{2\pi}\int_0^{2\pi}\frac{\left(\vec{R}_{mb}\cdot\vec{r}\right)}{R_{mb}^3}d\gamma. \qquad (6)$$

Let us determine the projection of the force (1) to the $\vec{z}$ vector:

$$m\left(\frac{d^2\vec{R}}{dt^2}\cdot\vec{z}\right)=\frac{qQ_a}{2\pi}\int_0^{2\pi}\frac{\left(\vec{R}_{ma}\cdot\vec{z}\right)}{R_{ma}^3}d\gamma+\frac{qQ_b}{2\pi}\int_0^{2\pi}\frac{\left(\vec{R}_{mb}\cdot\vec{z}\right)}{R_{mb}^3}d\gamma. \qquad (7)$$

After the transformation of equations (6) and (7), taking into account (2) - (5), we obtain:

$$m\frac{d^2r}{dt^2}-m\omega^2r=\frac{qQ_a}{2\pi}\int_0^{2\pi}\frac{(r-R_{a0}\cos(\gamma))}{\left(R_{a0}^2+(z-h)^2+r^2-2rR_{a0}\cos(\gamma)\right)^{3/2}}d\gamma+$$

$$+\frac{qQ_b}{2\pi}\int_0^{2\pi}\frac{(r-R_{b0}\cos(\gamma))}{\left(R_{b0}^2+(z+h)^2+r^2-2rR_{b0}\cos(\gamma)\right)^{3/2}}d\gamma, \qquad (8)$$

$$m\frac{d^2z}{dt^2}=\frac{qQ_a}{2\pi}\int_0^{2\pi}\frac{(z-h)}{\left(R_{a0}^2+(z-h)^2+r^2-2rR_{a0}\cos(\gamma)\right)^{3/2}}d\gamma+$$

$$+\frac{qQ_b}{2\pi}\int_0^{2\pi}\frac{(z+h)}{\left(R_{b0}^2+(z+h)^2+r^2-2rR_{b0}\cos(\gamma)\right)^{3/2}}d\gamma. \qquad (9)$$

We introduce and determine the following functions:

$$r_a^2=R_{a0}^2+(z-h)^2+r^2,\quad r_b^2=R_{b0}^2+(z+h)^2+r^2, \qquad (10)$$

$$s_a=\frac{r}{r_a},\quad s_b=\frac{r}{r_b},\quad k_a=\frac{R_{a0}}{r_a},\quad k_b=\frac{R_{b0}}{r_b},\quad l_a=\frac{z-h}{r_a},\quad l_b=\frac{z+h}{r_b}. \qquad (11)$$

Using (10) and (11), we transform (8) and (9):

$$m\frac{d^2r}{dt^2}-m\omega^2r=\frac{q}{2\pi}\int_0^{2\pi}\left(\frac{Q_a}{r_a^2}\frac{(s_a-k_a\cos(\gamma))}{(1-2s_ak_a\cos(\gamma))^{3/2}}+\frac{Q_b}{r_b^2}\frac{(s_b-k_b\cos(\gamma))}{(1-2s_bk_b\cos(\gamma))^{3/2}}\right)d\gamma, \qquad (12)$$

$$m\frac{d^2z}{dt^2}=\frac{q}{2\pi}\int_0^{2\pi}\left(\frac{Q_a}{r_a^2}\frac{l_a}{(1-2s_ak_a\cos(\gamma))^{3/2}}+\frac{Q_b}{r_b^2}\frac{l_b}{(1-2s_bk_b\cos(\gamma))^{3/2}}\right)d\gamma. \qquad (13)$$

As the following conditions are satisfied:

$$h\neq 0,\quad z\neq\pm h, \qquad (14)$$

we will have from (10) and (11):

$$2s_ak_a=\frac{2rR_{a0}}{R_{a0}^2+(z-h)^2+r^2}<1,\quad 2s_bk_b=\frac{2rR_{b0}}{R_{b0}^2+(z+h)^2+r^2}<1. \qquad (15)$$

Therefore, for (12) and (13), under the conditions of (14), we can use the expansion into a Maclaurin series:

$$\frac{1}{(1-a)^{3/2}} = \sum_{n=0}^{\infty} \frac{(2n+1)!a^n}{2^{2n}(n!)^2}, \quad a < 1. \tag{16}$$

Applying the (16), we will obtain from the (12):

$$m\frac{d^2r}{dt^2} - m\omega^2 r = \frac{q}{2\pi}\sum_{n=0}^{\infty}\frac{(2n+1)!}{2^n(n!)^2}\left(\frac{Q_a}{r_a^2}s_a^{n+1}k_a^n + \frac{Q_b}{r_b^2}s_b^{n+1}k_b^n\right)\int_0^{2\pi}\cos^n(\gamma)d\gamma -$$

$$-\frac{q}{2\pi}\sum_{n=0}^{\infty}\frac{(2n+1)!}{2^n(n!)^2}\left(\frac{Q_a}{r_a^2}s_a^{n}k_a^{n+1} + \frac{Q_b}{r_b^2}s_b^{n}k_b^{n+1}\right)\int_0^{2\pi}\cos^{n+1}(\gamma)d\gamma. \tag{17}$$

Using the values of definite integrals:

$$\int_0^{2\pi}\cos^{2n}(\gamma)d\gamma = 2\pi\frac{(2n)!}{2^{2n}(n!)^2}, \quad \int_0^{2\pi}\cos^{2n+1}(\gamma)d\gamma = 0, \quad n = 0, 1, ..., \infty, \tag{18}$$

we integrate and transform the right part of the (17):

$$m\frac{d^2r}{dt^2} - m\omega^2 r = q\sum_{n=0}^{\infty}\frac{(4n+1)!}{2^{4n}(2n)!(n!)^2}\left(\frac{Q_a}{r_a^2}s_a^{2n+1}k_a^{2n} + \frac{Q_b}{r_b^2}s_b^{2n+1}k_b^{2n}\right) -$$

$$-q\sum_{n=1}^{\infty}\frac{(4n)!}{2^{4n}(2n-1)!(n!)^2}\left(\frac{Q_a}{r_a^2}s_a^{2n-1}k_a^{2n} + \frac{Q_b}{r_b^2}s_b^{2n-1}k_b^{2n}\right). \tag{19}$$

Then, applying (16) and (18), we will have from the (13):

$$m\frac{d^2z}{dt^2} = q\sum_{n=0}^{\infty}\frac{(4n+1)!}{2^{4n}(2n)!(n!)^2}\left(\frac{Q_a l_a}{r_a^2}s_a^{2n}k_a^{2n} + \frac{Q_b l_b}{r_b^2}s_b^{2n}k_b^{2n}\right). \tag{20}$$

Under the following conditions:

$$Q_a = Q_b = Q_0, \quad R_{a0} = R_{b0} = R_0, \quad F_0 = \frac{qQ_0}{R_0^2}, \tag{21}$$

we rewrite (19) and (20) as follows:

$$\frac{m}{F_0}\left(\frac{d^2r}{dt^2} - \omega^2 r\right) = \sum_{n=0}^{\infty}\frac{(4n+1)!R_0^2}{2^{4n}(2n)!(n!)^2}\left(\frac{s_a^{2n+1}k_a^{2n}}{r_a^2} + \frac{s_b^{2n+1}k_b^{2n}}{r_b^2}\right) -$$

$$-\sum_{n=1}^{\infty}\frac{(4n)!R_0^2}{2^{4n}(2n-1)!(n!)^2}\left(\frac{s_a^{2n-1}k_a^{2n}}{r_a^2} + \frac{s_b^{2n-1}k_b^{2n}}{r_b^2}\right), \tag{22}$$

$$\frac{m}{F_0}\frac{d^2z}{dt^2} = \sum_{n=0}^{\infty}\frac{(4n+1)!R_0^2}{2^{4n}(2n)!(n!)^2}\left(\frac{l_a s_a^{2n}k_a^{2n}}{r_a^2} + \frac{l_b s_b^{2n}k_b^{2n}}{r_b^2}\right). \tag{23}$$

Let us introduce and determine dimensionless functions:

$$\breve{r} = \frac{r}{R_0}, \quad \breve{z} = \frac{z}{R_0}, \quad \breve{h} = \frac{h}{R_0}, \tag{24}$$

$$\breve{r}_a^2 = \frac{r_a^2}{R_0^2} = 1 + \left(\breve{z} - \breve{h}\right)^2 + \breve{r}^2, \quad \breve{r}_b^2 = \frac{r_b^2}{R_0^2} = 1 + \left(\breve{z} + \breve{h}\right)^2 + \breve{r}^2, \tag{25}$$

$$\breve{F}_r = \frac{m}{F_0}\left(\frac{d^2r}{dt^2} - \omega^2 r\right), \quad \breve{F}_z = \frac{m}{F_0}\frac{d^2z}{dt^2}. \tag{26}$$

Then (22) and (23) will look as follows:

$$\breve{F}_r = \sum_{n=0}^{\infty} \frac{(4n+1)!\breve{r}^{2n+1}}{2^{4n}(2n)!(n!)^2}\left(\frac{1}{\breve{r}_a^{4n+3}} + \frac{1}{\breve{r}_b^{4n+3}}\right) - \sum_{n=1}^{\infty} \frac{(4n)!\breve{r}^{2n-1}}{2^{4n}(2n-1)!(n!)^2}\left(\frac{1}{\breve{r}_a^{4n+1}} + \frac{1}{\breve{r}_b^{4n+1}}\right). \tag{27}$$

$$\breve{F}_z = \sum_{n=0}^{\infty} \frac{(4n+1)!\breve{r}^{2n}}{2^{4n}(2n)!(n!)^2}\left(\frac{\breve{z}-\breve{h}}{\breve{r}_a^{4n+3}} + \frac{\breve{z}+\breve{h}}{\breve{r}_b^{4n+3}}\right). \tag{28}$$

As follows from (25), (26) and (28), if $z = 0$, then $d^2z/dt^2 = 0$ as well. Thus, at $z = 0$ and $dz/dt = 0$ the particle will move within the $\vec{X}, \vec{Y}$ plane. Let us determine the projection of the force (1) on a vector which is normal to the $\vec{r}$ vector also:

$$m\left(\frac{d^2\vec{R}}{dt^2} \times \vec{r}\right) = \frac{qQ_a}{2\pi}\int_0^{2\pi} \frac{\left(\vec{R}_{ma} \times \vec{r}\right)}{R_{ma}^3} d\gamma + \frac{qQ_b}{2\pi}\int_0^{2\pi} \frac{\left(\vec{R}_{mb} \times \vec{r}\right)}{R_{mb}^3} d\gamma, \tag{29}$$

and let us consider the (29) under the conditions of (21), at $z = 0$ and at $dz/dt = 0$:

$$\frac{m}{r}\frac{d\left(r^2\omega\right)}{dt} = -\frac{qQ_0}{\pi}\int_0^{2\pi} \frac{R_0 \sin(\gamma)}{\left(R_0^2 + h^2 + r^2 - 2rR_0\cos(\gamma)\right)^{3/2}} d\gamma. \tag{30}$$

Taking into account that:

$$\int_0^{2\pi} \frac{R_0 \sin(\gamma))}{\left(R_0^2 + h^2 + r^2 - 2rR_0\cos(\gamma)\right)^{3/2}} d\gamma = 0, \tag{31}$$

integrating the (30), we will get:

$$z = 0, \quad \frac{dz}{dt} = 0, \quad mr^2\omega = Const. \tag{32}$$

Graphing the dependence of dimensionless functions $\breve{F}_r$ and $\breve{F}_z$ on the values of $\breve{r}$, at various values of $\breve{z}$ and $\breve{h}$, using the equations (27), (28), (32), and graphing the functions similar to dimensionless those $\breve{F}_r$ and $\breve{F}_z$ obtained from the equations (19) and (20) at $Q_a \neq Q_b$ allows for the following conclusions:

1. Under the condition of $\breve{h} < 2^{-1/2}$, there are the values of variables $(r, z, dr/dt, dz/dt, \omega)$ that determine initial conditions of particle motion at which the particle can be localised in the space between the charged rings.

2. There are conditions of variation of the distance between the rings and conditions of variation of charges of the rings, both overall and sectoral (i.e., charges of certain segments of rings), at which localised particles will be ejected from the localisation zone and will accelerate under the action of Coulomb repulsion forces of the rings along certain directions and to certain kinetic energies.

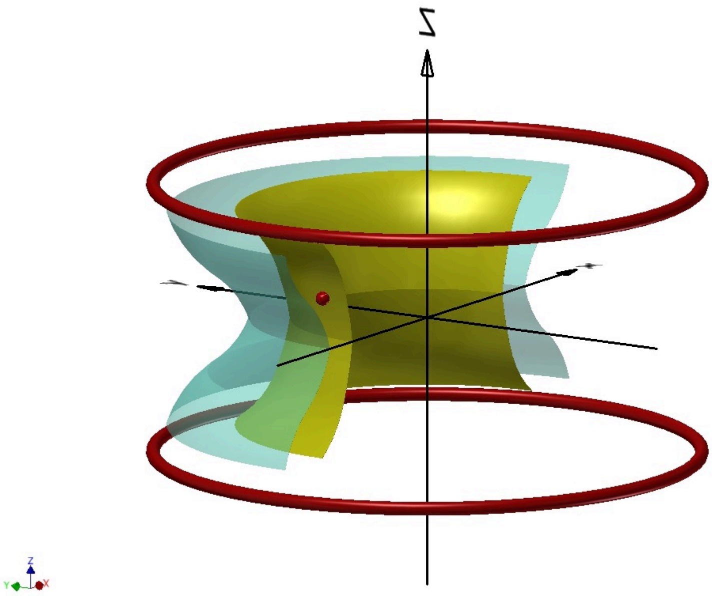

Figure 3: Zones of localisation and ejection of a charged particle during its interaction with the two charged rings.

Figure 3 demonstrates two surfaces (marked transparent blue and yellow) that split the space between the rings into three areas. The surfaces halved by the $\vec{X}, \vec{Z}$ plane are shown. In the area between the surfaces two forces determined by the equations of (27) and (28) act on the particle under the condition of $\breve{h} = 0.5$: the force parallel to the $\vec{Z}$ axis is directed toward the $\vec{X}, \vec{Y}$ plane while the force parallel to the $\vec{r}$ vector is directed to the $\vec{Z}$ axis. The magnitude of force parallel to the $\vec{r}$ vector and acting on the particle at the transparent blue surface equals to zero. The magnitude of force parallel to the $\vec{Z}$ axis and acting on the particle at the yellow surface equals to zero. In the area beyond the transparent blue surface the force parallel to the $\vec{r}$ vector and acting on the particle is directed away from the $\vec{Z}$ axis. In the area beyond the yellow surface the force parallel to the $\vec{Z}$ axis and acting on the particle is directed away from the $\vec{X}, \vec{Y}$ plane. Therefore, in the area between the surfaces there will exist some certain set of trajectories not contacting to the surfaces; while moving along them, the particle will be localised. This set includes a circular trajectory in the $\vec{X}, \vec{Y}$ plane at the following conditions:

$$\breve{h} = 0.5, \quad z = 0, \quad \frac{dz}{dt} = 0, \quad \omega_0^2 = \frac{F_0}{mR_0}, \quad \breve{\omega}^2 = \frac{\omega^2}{\omega_0^2}, \quad \breve{r} \approx 0.556446..., \tag{33}$$

$$\breve{\omega}^2 \approx \sum_{n=1}^{\infty} \frac{2\,(4n)!}{2^{4n}\,(2n-1)!\,(n!)^2} \frac{\breve{r}^{2n-2}}{\left(1+\breve{h}^2+\breve{r}^2\right)^{2n+1/2}} - \sum_{n=0}^{\infty} \frac{2\,(4n+1)!}{2^{4n}\,(2n)!\,(n!)^2} \frac{\breve{r}^{2n}}{\left(1+\breve{h}^2+\breve{r}^2\right)^{2n+3/2}}.$$

If the particle enters the area beyond the yellow surface (see Figure 3) while the distance between the rings is changing or the charges of the rings are changing, it is repelled from the zone between the rings with the force directed parallel to the $\vec{Z}$ axis away from the $\vec{X}, \vec{Y}$ plane. If the particle enters the area beyond the transparent blue surface (see Figure 3) while the distance between the rings is changing or the charges of the rings are changing, it is repelled from the zone between the rings with the force directed parallel to the $\vec{r}$ vector away from the $\vec{Z}$ axis.

As calculations show, the localisation of particles in the inner space between the rings is possible only for particles that have strictly determined initial conditions of their motion relatively to the charged rings. Particles unit vectors of which velocities are laying within the $\vec{X},\vec{Y}$ plane and having definite initial values of moments of momenta relatively to the $\vec{Z}$ axis and definite initial values of their radial velocities relatively to the $\vec{Z}$ axis will overcome the repulsion of rings and will concentrate along certain circular trajectories between the rings. These initial conditions of particles' motion relatively to the rings are determined as follows:

Based on the law of conservation of the sum of kinetic and potential energies of the particle during its motion within the $\vec{X},\vec{Y}$ plane ($z = 0$, $dz/dt = 0$), we will obtain:

$$\frac{m}{2}\left(\frac{dr}{dt}\right)^2 = E_0 - \frac{J_0^2}{2mr^2} - U\left(r, h\right), \tag{34}$$

$$E_0 = \frac{m}{2}\left(\frac{dr_0}{dt}\right)^2 + \frac{J_0^2}{2mr_0^2} + U\left(r_0, h\right), \quad J_0 = mr_0^2\omega_0, \tag{35}$$

where:
r_0 is a magnitude of a radius vector of initial position of the particle in the coordinate system where the interaction is considered,
dr_0/dt is an initial radial velocity of the particle,
ω_0 is a magnitude of an initial angular velocity of the particle.

Using the definition of a potential [1], applying (10) and (11), we find the potential energy of the particle in the system of two charged rings:

$$U\left(r, z, h\right) = \frac{q}{2\pi}\int_0^{2\pi}\left(\frac{Q_a}{r_a}\frac{1}{\left(1 - 2s_a k_a \cos\left(\gamma\right)\right)^{1/2}} + \frac{Q_b}{r_b}\frac{1}{\left(1 - 2s_b k_b \cos\left(\gamma\right)\right)^{1/2}}\right)d\gamma. \tag{36}$$

Under the conditions:

$$Q_a = Q_b = Q_0, \quad R_{a0} = R_{b0} = R_0, \quad z = 0, \tag{37}$$

$$s = \left(1 + \breve{h}^2 + \breve{r}^2\right)^{1/2}, \quad k = \breve{r}/s^2, \tag{38}$$

we will have from the (36):

$$U\left(\breve{r}, \breve{h}\right) = \frac{qQ_0}{\pi R_0 s}\int_0^{2\pi}\left(\frac{1}{\left(1 - 2k\cos\left(\gamma\right)\right)^{1/2}}\right)d\gamma, \tag{39}$$

From the values of functions (38) we will get:

$$\breve{h} \neq 0, \quad 2k < 1. \tag{40}$$

Therefore, under the conditions of (40), we can represent the integrated function in the (39) as an infinite series:

$$U\left(\breve{r}, \breve{h}\right) = \frac{qQ_0}{\pi R_0 s}\sum_{n=0}^{\infty}\frac{(2n)!k^n}{2^n\left(n!\right)^2}\int_0^{2\pi}\cos^n\left(\gamma\right)d\gamma. \tag{41}$$

Using the (18), we integrate the (41):

$$U\left(\breve{r},\breve{h}\right)=\frac{2qQ_0}{R_0 s}\sum_{n=0}^{\infty}\frac{(4n)!k^{2n}}{2^{4n}\,(2n)!\,(n!)^2}. \tag{42}$$

We substitute s and k with their determinations (38) and finally obtain:

$$U\left(\breve{r},\breve{h}\right)=\frac{2qQ_0}{R_0}\sum_{n=0}^{\infty}\frac{(4n)!}{2^{4n}\,(2n)!\,(n!)^2}\frac{\breve{r}^{2n}}{\left(1+\breve{h}^2+\breve{r}^2\right)^{2n+1/2}}. \tag{43}$$

Then we will determine four dimensionless functions:

$$\breve{A}=\frac{mR_0^3}{4qQ_0}\left(\frac{d\breve{r}}{dt}\right)^2,\quad \breve{B}=\frac{E_0R_0}{2qQ_0},\quad \breve{C}=\frac{J_0^2}{4mqQ_0R_0},$$

$$\breve{U}=\sum_{n=0}^{\infty}\frac{(4n)!}{2^{4n}\,(2n)!\,(n!)^2}\frac{\breve{r}^{2n}}{\left(1+\breve{h}^2+\breve{r}^2\right)^{2n+1/2}}. \tag{44}$$

Using (44), we rewrite the (34):

$$\breve{A}=\breve{B}-\frac{\breve{C}}{\breve{r}^2}-\breve{U}. \tag{45}$$

Let us find subsequently the first, the second and the third partial derivatives of $\breve{A}$ with respect to $\breve{r}$ and let us determine the three functions of $\breve{r}$ and $\breve{h}$:

$$\frac{\partial\breve{A}}{\partial\breve{r}}=\frac{2\breve{C}}{\breve{r}^3}-\frac{\partial\breve{U}}{\partial\breve{r}}. \tag{46}$$

$$\frac{\partial^2\breve{A}}{\partial\breve{r}^2}=-\frac{6\breve{C}}{\breve{r}^4}-\frac{\partial^2\breve{U}}{\partial\breve{r}^2}. \tag{47}$$

$$\frac{\partial^3\breve{A}}{\partial\breve{r}^3}=\frac{24\breve{C}}{\breve{r}^5}-\frac{\partial^3\breve{U}}{\partial\breve{r}^3}. \tag{48}$$

From the functions (45) - (47) we form a system of three algebraic equations relatively to the unknowns $\breve{B}$, $\breve{C}$, $\breve{r}$ depending on the value of $\breve{h}$:

$$1.\quad \breve{B}=\frac{\breve{C}}{\breve{r}^2}+\breve{U},\quad 2.\quad \breve{C}=\frac{\breve{r}^3}{2}\frac{\partial\breve{U}}{\partial\breve{r}},\quad 3.\quad \frac{3}{\breve{r}}\frac{\partial\breve{U}}{\partial\breve{r}}=-\frac{\partial^2\breve{U}}{\partial\breve{r}^2}. \tag{49}$$

From (48) and (49) we obtain an inequality:

$$\frac{12}{\breve{r}^2}\frac{\partial\breve{U}}{\partial\breve{r}}\neq\frac{\partial^3\breve{U}}{\partial\breve{r}^3}, \tag{50}$$

for the purpose of determination of the $\breve{h}$ values at which both the system of equations (49) and the inequality (50) are true. The system of equations (49) and the inequality (50) determine conditions under which the $\breve{A}$ function (45) has an inflection point at which the value of the function equals to zero. As also follows from these conditions, the function (46) has an extremum at the inflection point of the function (45); at the point of this extremum the function (46) also equals to zero. The function (45) is a dimensionless function of the squared radial velocity of the particle. The function (46) is a dimensionless function of radial acceleration of the particle. Therefore, the radial velocity and the radial acceleration of the particle at the inflection point of the $\breve{A}$ function (45) will equal to zero. The particle having the initial conditions of its motion as determined from (49) and (50), with the negative value of radial velocity will overcome the repulsion of the rings, and the trajectory of its motion will gradually transform to circular with certain constant values of $\breve{r}$ and $\breve{\omega}$. Particles which initial conditions of motion do not comply the conditions of localisation will be ejected from the system of rings to the infinity.

The dynamics of the particle moving from the infinity toward the rings and which trajectory transforms to the circular as determined by the conditions of (33) is demonstrated at Figure 4. The graphs of the following three functions are drawn there:
- a red curve determines the magnitude of the dimensionless radial velocity of the particle depending on the distance to the origin of the coordinate system (the $\breve{A}^{1/2}$ function obtained from the (45));
- a blue curve determines the dimensionless radial acceleration of the particle depending on the distance to the origin of the coordinate system (the $\partial\breve{A}/\partial\breve{r}$ function (46));
- a yellow curve determines the direction of the dimensionless force acting on the particle parallel to the $\vec{Z}$ axis (the negative values of the function mean that forces are pressing the particle to the $\vec{X}, \vec{Y}$ plane) depending on the distance to the origin of the coordinate system, with $\breve{z} = 0.01$ (the $\breve{F}_z$ function (28)).

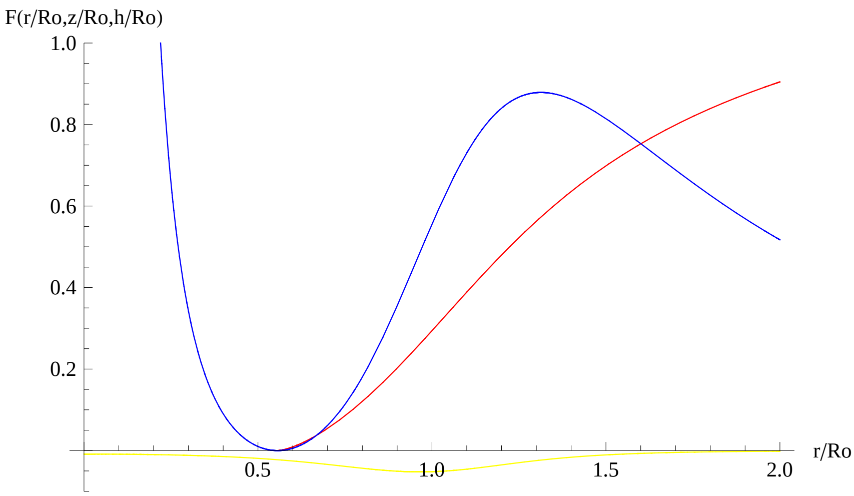

Figure 4: The dynamics of localisation of a charged particle during its interaction with the two charged rings.

The results of the study regarding trajectories of particles' motion performed by using the numeric methods demonstrate that there is a theoretical possibility of creating the conditions for the nuclear synthesis in swarms of positively charged particles localised using the method as determined hereinabove, and that there is a theoretical possibility of formation of beams of likely charged particles with the given kinetic energy and with the given direction of their motion relatively to the system of charged rings.

If conclusions formulated in «Modified Coulomb Forces and the Point Particles States Theory» [2] concerning the existence of the proton and the electron condensates are correct, then the system of the two charged rings will help create the conditions for the formation and localisation of volumes of both proton and electron condensates. Localised volumes of condensates could possibly be used either as a mean for jet-propelled motion or as a tool for the destruction of space objects (asteroids and comets) potentially threatening Earth or for changing their motion trajectories.

References

[1] *Jackson, J.D.* Classical Electrodynamics // John Wiley & Sons, Inc., New York, 1962

[2] *Tukaev, I.N.* Modified Coulomb Forces and the Point Particles States Theory// http://vixra.org/pdf/1707.0270v1.pdf