Deepening Questions about Electron Deep Orbits of the Hydrogen Atom

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Abstract. In previous works, we analyzed and countered arguments against the deep orbits, as discussed in published solutions. Moreover, we revealed the essential role of Special Relativity as source of electron deep orbits (EDOs). We also showed, from a well-known analytic method of solution of the Dirac equation, that the obtained EDOs have a positive energy. When including the magnetic interactions near the nucleus, we observed a breakthrough in how to satisfy the Heisenberg Uncertainty Relation (HUR) for electrons confined near the nucleus, in a radial zone of only a few fm. Here we chose a different method, by directly facing the HUR for such confined electrons, from which we deduce the coefficient γ of these highly relativistic electrons. Then we show the effective Coulomb potential due to a relativistic correction, can maintain the electrons in containment. Next we resume and deepen our study of the effects of EM interactions near the nucleus. We first obtain computation results: though approximate, we can effectively expect high-energy resonances near the nucleus. These results should be confirmed by using QFT-based methods.

1. Introduction

The concept of nuclear, or near-nuclear, electrons goes back nearly 100 years (Appendix A). After being acceptable for a decade, it was rejected for several reasons and buried. The deep-orbit-electron aspect periodically resurfaced for various reasons, but was always rejected in subsequent publications. With the advent of cold fusion and the possibility of experimentally testing for these levels (Appendix B), the arguments against such orbits have been actively countered in this context (Appendix A). In earlier papers [1][2][3[4], we have analyzed works on electron deep orbits (EDOs) [5][6][7] obtained by means of relativistic quantum equations (Dirac and Klein-Gordon) and answered the principal criticisms found in the literature and some indicated by colleagues. In particular, we have verified, extended, and improved the results of [7], obtained with a modified nuclear potential to take into account the finite dimension of the proton.

In our most recent paper [8], we answered a recent criticism by proving that the sign of the energy of EDO's solutions of the Dirac equation is positive. We also recalled the essential role of Special Relativity in the existence of the EDO's. This latter point was already indicated in [4] and particularly analyzed in [9]. Nevertheless, the studied methods used to find EDO's have imperfections that make them disputable on some important questions, e.g. the satisfaction of the Heisenberg uncertainly relation (HUR). Therefore, we began to address the EDO question from another angle, by starting a study on the role of magnetic interactions.

First, we analyzed several works on this subject from Barut [10][11][12][13] as well as the subsequent works on the so-called "Barut-Vigier model" [14][15][16][17][18] on the Hydrogen atom. These latter papers were developed in a non-relativistic context, unlike those of Barut himself. Finally, while computing magnetic interactions near the nucleus, we obtained a first positive answer about the essential question of the HUR satisfaction. Then, we addressed another important question: the stability of an electron resonance near the nucleus. For doing this, we first used a well-known [19] [20] classical approximation: to look for a local minimum of energy (LME) near the nucleus, while respecting the HUR. However, the results were not conclusive, insofar as the computed LME appeared deep inside the proton, i.e. where the relations used for computing it are no longer correct.

Here, we completely change strategy:

- We start from the HUR, to determine a size order of the momentum p for an electron confined in a region of mean radius $\langle r \rangle$ around the nucleus.
- From p we can directly deduce an approximate expression for the relativistic coefficient γ as a function of r.

- As γ is very high for the expected region of the EDO's, we take into account the relativistic corrections to the Coulomb potential, yielding an effective dynamical electric potential energy V_{eff} [1][21][22].This potential is strong enough to confine highly-relativistic electrons to very deep orbits. Moreover, this new method allows progress into further questions, in particular for the computation of the LME. Now, we obtain interesting results: though these computations are approximate, they allow us to expect stable resonances for EDO's.

The HUR, which began as a big problem, leads us not only to find its proper solution but also to progress in the solution of the next questions. This study confirms the central role of the Relativity. Special Relativity is not only the source of EDO's, but as already felt previously, it also appears that relativistic methods may be necessary to prove the existence of EDO's.

2. Relativistic confinement energies and the relativistic coefficient γ

We take up and deepen elements discussed in section 6 of [8], by starting from the Heisenberg Uncertainty Relation, (HUR) to be satisfied by an electron confined to a very small volume around the nucleus. In particular, we previously saw that electrons on deep orbits are *strongly* relativistic. Under these conditions, instead of computing the well-known relativistic coefficient γ for such electrons indirectly, as previously, we *directly deduce an approximate minimum value for* γ for an electron confined in a region corresponding to an average radius $<r>$ around the proton.

To make such a computation using HUR, it is usual [19][23] to consider that the dispersion ("uncertainty") on the norm of the momentum $|\mathbf{p}|$ satisfies $\Delta|\mathbf{p}|\ \Delta r \geq \hbar/2$; to accept $\Delta|\mathbf{p}|$ as an average estimate of the momentum; and to attribute $<r>$ to Δr. So, we write $p \geq \hbar/2r$, where p stands for $|\mathbf{p}|$ and r for $<r>$, in order to simplify notation. Then, we consider the relativistic expression of momentum: $p = \gamma mv$, where m is the mass of the electron and v its velocity. Thus, we have to satisfy the relation $\gamma mv \geq \hbar/2r$, i.e. $\gamma v \geq \hbar/2mr$.We put $s = \hbar/mr$, a quantity of physical dimension "speed", which gives the inequality $\gamma v \geq s/2$, and thus $(\gamma v)^2 = (cv)^2/(c^2 - v^2) \geq s^2/4$.
After some simple algebraic transformations[1], we obtain $\gamma^2 \geq 1 + \hbar^2/4(mcr)^2 = 1 + (\underline{\lambda}_c)^2/4r^2$, where $\underline{\lambda}_c$ is the "reduced" Compton wavelength of the electron $\hbar/mc$. As $\underline{\lambda}_c \sim 386$ F and for the EDOs r is of order a few F, one has $(\underline{\lambda}_c)^2/4r^2 >> 1$, so one can write the following inequality deduced from the HUR:

$$\gamma \geq \underline{\lambda}_c/2r \qquad (1a)$$

We note that the introduction of the Compton wavelength allows us to simplify the relation. Of course this expression (and the involved approximations) is valid only under the condition above on r, i.e. $r^2 << (\underline{\lambda}_c)^2/4$.
In the previous references [19][23] the coefficient "1/2" is removed to give an order of size for the momentum p: $p \sim \hbar/r$. Under this condition, one can show the following relation:

$$\gamma \sim \underline{\lambda}_c/r \qquad (1b)$$

It is rather remarkable to obtain such a relation involving the Compton wavelength, even if the principle used for the computations is coarse.
To give a size order of γ for EDOs: if computing γ with (1b) and for $r = 2$F, we can expect a relativistic coefficient of order ~ 193, i.e. close to 200 (!), and if we consider the inequality relation $\gamma \geq \underline{\lambda}_c/2r$, we have γ_{min} near 100

3. Consequences on the effective Coulomb potential energy V_{eff}

In [8], while seeking to resolve important physical questions for EDOs, such as the satisfaction of the HUR and the existence of a resonance near the nucleus thanks to a local minimum of the energy, we have principally considered magnetic interactions, because we expect them to yield high potential

[1] From $(cv)^2/(c^2 - v^2) \geq s^2/4$. We deduce $(cv)^2 \geq [(cs)^2 - (vs)^2]/4$, $v^2(c^2 + s^2/4) \geq (cs)^2/4$, $(v/c)^2 \geq (s^2/4)/(c^2 + s^2/4)$. With $S = (s^2/4)/(c^2 + s^2/4)$, $\gamma^2 = 1/1 - (v/c)^2 \geq 1/1 - S = (c^2 + s^2/4)/c^2 = 1 + s^2/4c^2$

energy. But here, because of the high level of the relativistic coefficient γ, it is interesting to consider the effects of the relativistic correction of the static Coulomb potential, indicated in [21][22], under the resulting form of an effective dynamical potential noted $V_{\mathbf{eff}}$, and already considered in [4]. In this latter reference, the coefficient γ was computed on a very different basis, and the energy shifts were not negligible but moderate.

We recall the general form of V_{eff}, below (Eq. 2), comes from the development of relativistic quantum equations (Dirac and Klein-Gordon) with the expression of the relativistic energy of a particle in a central field for a Coulomb potential energy V:

$$V_{eff} = V\,(E/mc^2) - V^2/2mc^2. \quad (2)$$

Nevertheless, in the case of the atomic electrons in light elements, one has $E \sim mc^2$ and $V << mc^2$, so the corrective term is always neglected and one has $V_{eff} = V$.

In the case of a relativistic electron, one can show $V_{\mathbf{eff}}$ takes the following form:

$$V_{eff} = \gamma V + V^2/2mc^2\,, \quad (3)$$

With the expected expression of γ as function of r indicated above, the expression of V_{eff} reads:

$$V_{eff} \sim -(\hbar/mcr)(\alpha c\hbar/r) + (\alpha c\hbar/cr)^2/2m = -(\alpha\hbar^2/mr^2)(1-\alpha/2) \sim -\alpha\hbar^2/mr^2 \quad (4a)$$

If we take the inequality $\gamma \geq \underline{\lambda}_c/2r$ (as in equation 1a), one obtains:

$$|V_{eff}| \geq \alpha\hbar^2/2mr^2 \quad (4b)$$

With this last relation, and for any radius $r \leq \underline{\lambda}_c/2 \sim 193$ F, we can show the three following results for the relativistic potential:

-1. V_{eff} is always attractive

-2. $|V_{eff}| \geq |V|$. So one has a strengthening of the static Coulomb potential

-3. V_{eff} has a behavior in K/r^2 when r decreases (and thus $|V|$ increases), with $K \sim 9\text{x}10^{-41}$ in SI units. Previously [4], these results were obtained only for quasi-circular orbits, but with no condition on the radius.

To have an idea of the size order of V_{eff} near the nucleus, by computing it for r = 2F, we obtain the following approximate values:

V_{eff} of order -140 MeV, whereas the kinetic energy KE = $(\gamma - 1)\,mc^2 \sim$ 98 MeV.

With such a high value, V_{eff} can confine an electron in this region. We showed previously that Special Relativity is the source of the EDO's. Here we have shown that the HUR, which seemed an impediment for the EDO's, provides its proper resolution thanks to Relativity.

4. The question of stability of the EDOs. Seeking a local minimum of the energy

4.1. Principles

We use a well-known approximation to estimate the possibility of a stable resonance near the nucleus before applying more complex and complete tools to solve the problem. We proceed so because the area near the nucleus is a region where several strong interactions must be considered under different conditions. Nevertheless, we expect to determine which interactions have greatest importance in the generation of resonance. For doing this, we consider the relativistic expression of energy, in which the norm of momentum $|\mathbf{p}|$ is replaced by $\hbar/r$, in order to respect the HUR and to obtain the following term noted E_H ("H" for Heisenberg):

$$E_H = \sqrt{\frac{\hbar^2 c^2}{r^2} + m^2 c^4} \quad (5)$$

At E_H, we add a term V representing a potential energy, where V is a function of the radius. Thus we obtain the total energy E, represented by the following relation:

$$E = E_H + V \quad (6)$$

Then, we look for a local minimum of energy E for various combinations of potentials included in the term V and we determine the radius of this local minimum. In fact, a potential is "interesting" for the

resonance, i.e. to be kept for the rest of the study, not only by considering the energy levels, but also if the (average) radius in the approximation for the local minimum is acceptable, i.e. near and maybe outside the nucleus (here it is a proton).

Of course, V includes the electric Coulomb potential, but we recall that, earlier in this study [8], we considered potential energies of magnetic interactions, after analyzing the works referring to the "Vigier-Barut Model" [14][15][16][17][18]. These interactions include Spin-Orbit (*SO*) and Spin-Spin (*SS*), taken in their attractive form, as well as a repulsive term in $1/r^4$ coming from the square of the nuclear vector potential **A**. More precisely, this term comes from an expression $(\mathbf{P} \pm e\mathbf{A})^2$, associated with the minimal coupling between one charged particle and an "exterior" EM field, as e.g. in a Pauli equation. **P** is the kinetic momentum of the particle, e its electric charge, m its mass, and **A** the vector potential of the EM field. $\mathbf{P} \pm e\mathbf{A}$ is sometimes called the "canonical" or the "dynamical" momentum. The vector potential produced by a magnetic moment **m** can be expressed by using: $\mathbf{A}(r) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times \hat{\mathbf{r}}}{|\mathbf{r}|^2}$. The complete energy term associated with $\mathbf{A}^2$ has the form $e^2\mathbf{A}^2/2m$ and is considered [24][25] to be expressing a "diamagnetic" energy with a behavior in $1/r^4$. Though it is usually negligible at atomic levels, it has a considerable importance here, not only because it is strongly increasing near the nucleus but also, as it is repulsive, it avoids a "fall at the center." This fall is almost inevitable in "macrocospic" (non-quantum) computation, if keeping only attractive interactions, and would give a result with questionable physical meaning.

There are actually two similar diamagnetic terms to consider, as we can see when considering the Hamiltonian of a two-body system electron + proton, as in [16] [18]:

-1. One is caused by the interaction of the electric charge of the electron with the intrinsic magnetic moment of the proton spin.
-2. The other is caused by the symmetric interaction: between the electric charge of the proton and the intrinsic magnetic moment of the electron spin.

In spite of this apparent symmetry, the strength of both terms is very different because of the great difference between the values squared of the electron and proton magnetic moments. In fact, the term #2 is equal to ~ 240 times the #1 and "absorbs" completely this term. Finally, its coefficient C is computed by the following expression:

$$C \sim (\mu_0/4\pi)\,[e^4\hbar^2/(4\, m_e^2 m_p)] \sim 1.3\ 10^{-71} \text{ in SI units} \qquad (7)$$

Further repulsive potentials include the well-known "centrifugal term" in $1/r^2$, which is available only if the angular momentum L, as determined by its quantum number l, is not null. On another hand, the *SO* interaction is also available only if $l \neq 0$. The lesser known Spin-Orbit interaction, S_pO, involving again the orbital momentum of the electron, but with the nuclear spin instead of the electron spin, also needs $l \neq 0$. Nevertheless, the magnetic moment of the proton μ_p is of order 10^3 (in fact ~660) smaller in absolute value than that of the electron, μ_e. So, the S_pO interaction, in either its attractive or repulsive version, is completely "absorbed" by the *SO* interaction and can be neglected. Note finally that the "Heisenberg" term E_H (5), having a positive sign, is also repulsive, but with a $1/r$ behavior for r very small and tending towards 0. Under these conditions, it can be surpassed by attractive terms in $1/r^3$, such as the magnetic potentials *SO* and *SS*, when r –> 0.

To put some order in all these terms possibly involved in the computations, we list them below while specifying some associated conditions, possible eliminations by "absorption", and their computation expressions. (Further refinements are mentioned in section 4.2.2.ii.)

1. E_H: relativistic energy taking into account the HUR for an electron confined at a radius r.

Nature: repulsive. Expression defined in (5). Always Taken Into Account (ATIA)

2. The Coulomb potential. Nature: attractive.

In the previous study, we considered only the static potential energy $V_{Cb} = \alpha c\hbar/r$, as the relativistic coefficient γ was computed afterwards, after inserting further attractive potential energies coming from magnetic interactions. But now, as we have a high value of γ, derived beforehand from the HUR, we can directly take the effective potential V_{eff} (3), (4b). ATIA

3. *SO* interaction: available only if $l \neq 0$. Nature: we choose the *attractive* version, derived from rules on composition of angular momenta: here, $j = l - ½$. We use the expression of energy given in [8], by taking $l = 1$ and thus $j = ½$. In order to establish this calculation expression, we suppose that one can use the same quantum rules for deep orbits as for atomic orbitals, based on the quantum $\hbar$, to evaluate L•S. Initially, in not considering a possible relativistic correction for E_{SO} (see the next subsection), we had $<E_{SO}> \sim (-1.7\text{x}10^{-53}/ < r^3>)$ Joules $\sim (- 1.1\text{x}10^{-34}/ < r^3>)$ eV.

4. *SS* interaction: always "available." Nature: first, we choose the *attractive* version, corresponding to a singlet state. The approximate expression [8] is derived by extrapolation from the atomic case: the total energy shift between the singlet state (attractive, s=0) and the triplet state (repulsive) is $A \sim 5.87\text{x}10^{-6}$ eV [26] for the fundamental state, i.e. for $r \sim 53\text{x}10^{-12}$ (Bohr radius), and the energy shift is equal to (-3/4) A. We deduced $<E_{SS}> \sim (-10^{-55} / < r^3>)$ Joules $\sim (- 0.64\text{x}10^{-36} / < r^3>)$ eV. With a $1/r^3$ dependence, as for the *SO* interaction, *SS* is *taken into account only if the former is not*, since $|E_{SS}| << |E_{SO}|$ unless $l = 0$.

5. V_{centr} : "Centrifugal term." Nature: repulsive. Expression: $V_{centr} = l(l+1)\hbar^2/2mr^2$. One supposes simply l=1, which gives $V_{centr} = \hbar^2/mr^2$. As for E_{SO}, V_{centr} has value $\neq 0$ only for $l > 0$. V_{centr} even dominates V_{eff}, which is also in $1/r^2$ when r becomes very small: V_{eff} has order $10^{-40}/r^2$, whereas V_{centr} has order $\sim 10^{-38}/r^2$. Despite its very weak coefficient (of order 10^{-52}) only E_{SO} (for point particles), as it is in $1/r^3$, can surpass V_{centr}.

6. V_4: "Diamagnetic" term coming from the square of the vector potentials $\mathbf{A}^2$. Nature: repulsive. Approximate expression in SI units: $V_4 = 1.3\text{x}10^{-71}/r^4$, as indicated above in section 4.1. ATIA.

To summarize, we look for a local minimum of energy $E = E_H + V$, with $V = CbPot + V_4 + OptPot$, where

- *CbPot* was the static potential V_{Cb} in previous computations, and is now V_{eff} in the new ones.
- *OptPot* is a combination (possibly empty) of "optional" potential energy terms #3, #4 and #5.

4.2. Analysis of the computation results for a local minimum of energy near the nucleus

4.2.1. Quick recall of the previous computations

- Trial with $OptPot = E_{SO}$, and thus $E = E_H + V_{Cb} + E_{SO} + V_4 \sim E_H - \alpha c\hbar/r - 1.7\text{x}10^{-53}/r^3 + 1.3\text{x}10^{-71}/r^4$.

Local Minimum of Energy (LME) is at ~ 0.001 F. Of course this result has no physical meaning for electron orbitals. Moreover, if we compute E at r = 2F, we find $E \sim E_{SO} \sim$ -13 GeV. Such a "hadronic" value seems unreasonable unless we are considering very high-energy particle resonances. We had also noted that even with adding the (repulsive) centrifugal term V_{centr} to *OptPot*, one obtains almost the same excessive energy result.

Moreover, as $\gamma >> 1$, one has to take into account a *relativistic expression* of the *SO* interaction including γ . The *SO* interaction energy is proportional to the "difference" between the Larmor and the Thomas precessions. The Larmor precession ω_L depends on the effective magnetic field seen by the electron as it orbits the charged nucleus. The Thomas precession ω_T is a relativistic geometric effect on the electron spin axis, in the opposite direction of the Larmor precession. The relativistic Larmor precession is $\omega_L' = \gamma\omega_L$, where ω_L is the usual precession at low speed, i.e. when $\gamma = (1- v^2/c^2)^{(-1/2)} \sim 1$. The full relativistic expression of the Thomas precession corresponds to $\omega_T = - [\gamma^2/(\gamma+1)]\omega_L$ (see e.g. [27]) = $-[\gamma/(\gamma+1)]\omega_L'$. The total precession rate is thus equal to the following expression:

$$\omega_{tot} = \omega_L' + \omega_T = [\gamma - \gamma^2/(\gamma+1)]\omega_L = [\gamma/(\gamma+1)]\omega_L \quad (8)$$

Because of the high value of γ for EDO electrons (estimated to be $\gamma > 100$ near the nucleus), we have $\omega_{tot} \sim \omega_L$. Thus, the total *SO* interaction energy for the highly-relativistic EDO electron is functionally twice that for the well-known usual precession $\omega_L/2$ at atomic levels, which increases with electron proximity to the nucleus. Surprisingly, other than within this factor of 2, relativity has no effect on the net precession rates.

This change would increase, in the same ratio, the value of *SO* computed in the trial above, which was already unrealistically high. Then we decide, for the time and as working hypothesis for the sequel, not to take into account a *SO* interaction. Of course, this amounts to putting $l = 0$.

• Trial with $OptPot = E_{SS}$, i.e. $E = E_H + V_{Cb} + E_{SS} + V_4 \sim E_H - \alpha c\hbar/r - 10^{-55}/r^3 + 1.3\text{x}10^{-71}/r^4$. Of course, we must suppose $l = 0$, which implies $E_{SO} = 0$ and $V_{centr} = 0$. The LME is reached at $r \sim 0.17$ F, i.e. again inside the proton, albeit less deeply than with E_{S0}. Moreover, at $r = 2$ F, we had $E_{SS} \sim -81$ MeV, to compare (while assuming a quasi-circular orbit) with a kinetic energy order ~76 MeV, and $\gamma \sim 150$.

To summarize, we conclude that, without major adjustments, neither above trial can determine a realistic LME.

4.2.2. Present results of computations

Now, the static potential is replaced by the effective V_{eff} due to relativistic dynamical corrections, the sum with any "optional" attractive potential energy gives an even greater attractive strength than determined previously. It is thus useless to try a combination with *OptPot* including E_{SO} or even $E_{SO} + V_{centr}$. For the same reason, with *OptPot* including the attractive E_{SS}, we obtain again a LME inside the nucleus, at $r \sim 0.16$ F, very near the previous result with E_{SS}.

(i). Results of a potential including a repulsive *SS* interaction.

We can test a simulation with the *repulsive* version of the *SS* interaction, corresponding to a triplet state (s=1). At r = Bohr radius, the energy shift is positive and equal to (1/4) A. One deduces the repulsive version of the interaction, noted E_{SSR}: $E_{SSR} = |E_{SS}|/3 \sim 3.4\text{x}10^{-56}/r^3$ J $\sim 2.2\text{x}10^{-37}/r^3$ eV.

• Trial with $E = E_H + V_{eff} + E_{SSR} + V_4 \sim E_H + V_{eff} + 3.4\text{x}10^{-56}/r^3 + 1.3\text{x}10^{-71}/r^4$, and $l = 0$

This time, Fig. 1, the LME is reached at $r \sim 1.07$ F, with $E \sim -61$ MeV. So the LME is outside the proton. We have also the following results: $\gamma \sim 365$, kinetic energy KE ~ 185 MeV, total potential energy PE ~ -250 MeV. Moreover, the potential wall due to the HUR is of order ~ 20 MeV, at $r \sim 5$F. The curve of E is plotted on the *Fig.*1, with the radius in the interval [0.9 F, 4 F].

One finds also a LME (not included in Fig. 1) corresponding to the Bohr radius, but the energy E does not agree with the classical value. To simplify the reason, V_{eff} is computed here with an expression of γ as a function of r, which is relevant only for very small value of r, i.e. $r \leq \lambda\!\!\!^{-}_c/2 \sim 193$ F, whereas V_{eff} for a non-relativistic electron is practically equal to the static Cb potential. Of course, this result about a local minimum of energy, with a potential well outside the nucleus, is just a coarse approximation; but it gives size orders and an "interesting" combination of potentials capable of obtaining a realistic resonance for EDO, without yet using quantum equations.

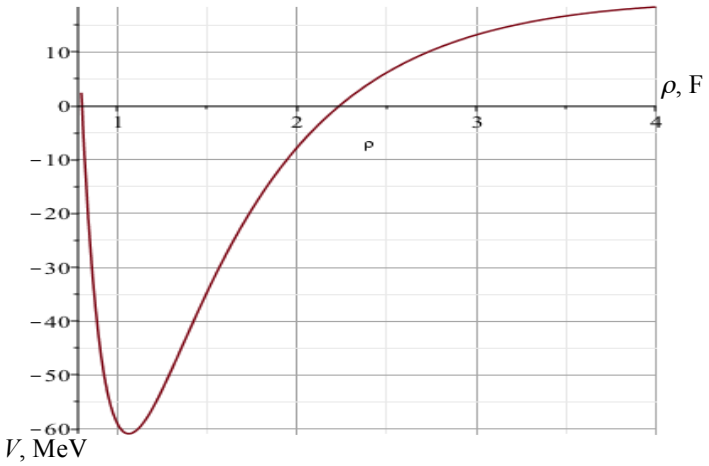

Fig 1. Plot of E in MeV as a function of ρ in F, for $E_{SO} = 0$ and a repulsive E_{SSR}

Note that for an electron to be inside the nucleus would not be scandalous per se, since one knows the electron of an orbital *s* has a finite probability to be inside the nucleus. In a more-extreme case, if considering a muonic atom of lead, where the muon plays the role of an electron, one knows [28] the muon is more inside the nucleus than outside, in a region where the potential, according to a classical approximation, is parabolic. But, similar to this last example, the expressions of the potential energies are completely different inside the nucleus from the ones outside. Thus the computations of local minima such as those indicated in the section 1 above, with LME inside, are simply erroneous, since

they are computed with incorrect expressions and thus must be modified in a yet-to-be-determined manner.

• Now, we can try computations with a modified electro-static potential inside the nucleus, by using the expression $V_i(r) = -\left[\frac{3}{2} - \frac{1}{2}\left(\frac{r^2}{R_0^2}\right)\right]\frac{e^2}{R_0}$ [7], for $r < R_0$, the charge radius of the proton taken equal to ~ 0.84 F. By using the usual expression for $r \geq R_0$, we determine a Cb potential, V_{mod}, modified for any radius r. Next, by replacing V by V_{mod} in the expression (3) of V_{eff}, we define an "effective Cb potential" with relativistic correction, V_{effmod}. While taking account of the repulsive *SS* interaction i.e. with $E = E_H + V_{effmod} + E_{\text{SSR}} + V_4$, it is interesting to note a slight shift of the LME, which passes from ~1 F to 1.1F.

(ii). Taking into account a relative weakening of strong EM interactions near the nucleus.
On one hand, some EM interactions become very strong when the radius decreases, because of behavior in inverse powers of r, mostly in powers -2,-3 and -4. Nevertheless, one also has to take into account effects that can weaken the Coulomb potential before even arriving at the proton. In particular, we consider some radiative corrections derived from QED, such as the electron self-energy and the vacuum polarization, which generates a cloud of virtual pairs of electrons and positrons confined in a localized region around the electron, smearing its charge. In fact, self-energy decreases the binding energy, while vacuum polarization tends to increase it, but the sum of both effects gives a repulsive action. At atomic levels, this causes the well-known "Lamb shift" on the order of a few 10^{-5} eV. This is of intermediate importance between the fine and the hyperfine structure shifts. Moreover, it acts on the spin g-factor of the electron, leading to the so-called "anomalous magnetic moment".

As the effects of QED increase with the intensity of the electric field that the electrons are exposed to, one can expect these corrections to become much stronger for an electron localized near the nucleus. Observations on heavy atoms can already give an idea of the size of these corrections. For example, for the ground state of a H-like uranium ion (U^{91+}), one observes a Lamb shift of almost ½ keV [29]. Effective calculations using QED and other considerations for near-nuclear effects are beyond the scope of this paper, but will certainly be involved in future work.

There is another possibility of weakening, that for interactions involving the spin. Indeed, about the solutions of Dirac equation for a free electron, one can note [30] that a velocity transverse to the spin affects the direction of the spin. To summarize: as the general expressions of the Dirac spinors involve the momentum **p** of the particle, if its spin at rest was in an eigenstate, it will become a linear combination of eigenstates when $\mathbf{p} \neq 0$; in fact, when $|\mathbf{p}|$ increases, the direction of the spin "bends" in the direction of **p**, and as the velocity approaches c, the spin tends to be aligned with the helicity. Except for the cited reference, the phenomenon seems practically passed over in the literature. In the case of a relativistic bound electron, one would therefore expect a weakening of the interactions and a source of new resonances involving the electron spin ($\boldsymbol{s}_1 \bullet \boldsymbol{s}_2$ and $\boldsymbol{l} \bullet \boldsymbol{s}$). But, because of the orbital acceleration experienced, it is much more difficult to evaluate possible "bending" effects in the case of a bound electron than a free one. Both effects have consequences for spin interactions, and mostly without a signature in the expression of the wavefunction. In the literature, it seems that authors [24] note rather a strengthening of the *SO* interaction for relativistic *s* electrons of heavy atoms at the same time as a shrinkage of the considered orbital, thus expressing a strengthening of the binding energy. Concerning the spin-spin interaction, there are works about relativistic calculations of spin-spin constants (principally in nuclear resonance spectra). For example, in a very complete and complex study [31] involving possible interactions between electrons, the authors consider multi-electron atoms. Nevertheless, it is difficult to deduce a numerical weakening of the *SS* interactions near the nucleus from it. In future work, we will reexamine assumptions and more strictly apply relativity in the nuclear region.

Under these conditions, and for the present, we make simple simulations of weakening for V_{eff}, E_{SSR} and V_4, while computing localization of a LME near the nucleus. For V_{eff}, we first consider a weakened version (due to QED effects) of the static Cb potential, noted V_{Cbw} and defined in the following way:

we choose a radius $r_1 > r_0$, where r_0 is the charge radius ~ 0.84 F, and we suppose

- at $r = r_1$, $V_{Cbw}(r) \sim -\alpha c\hbar/r$, the usual Cb potential V_{Cb}
- at $r = r_0$, $V_{Cbw}(r) = K\, V_{Cb}$, where K is a coefficient < 1

To simplify, we suppose a linear weakening of V_{Cb} when going from r_1 to r_0, and we do not specify what happens for $r < r_0$, since we are only interested in values of LME outside the nucleus.

So, $V_{Cbw}(r)$ can be defined by :

- $V_{Cbw}(r) = V_{Cb}(a\, r + b)$, when $r_0 < r < r_1$, where $a = (1- K)/(r_1 - r_0)$ and $b = 1 - a\, r_1$
- $V_{Cbw}(r) = V_{Cb}(r)$ for $r \geq r_1$. In fact, it is unnecessary to consider this case.

Next we deduce the expression of a lessened version of V_{eff} (i.e., V_{effw}) by using V_{Cbw} for V in the expression (3), i.e. $V_{effw} = \gamma\, V_{Cbw} + (V_{Cbw})^2/2mc^2$. For the expression of energy E, we take $E = E_H + V_{effw} + E_{SSR}/C + V_4/D$, where C and D are constants >1. We made several computations with various values of r_1, K, C, and D, resulting in LME outside the nucleus. Here we report only the following example.

For $r_1 = 2.5$ F, K=0.55, C=1.8 and D = ½, we have a LME at $r \sim 1.6$ F, where $E \sim -5$ MeV. Moreover, $\gamma \sim 235$, and the potential energy is ~ -125 MeV, corresponding to the potential well at the LME. The "dynamic" potential wall due to the HUR is order ~ 18 MeV at ~ 8F. In *Fig.* 2, we plot the curve of E as a function of the radius taken in the interval [1.3 F, 2.5 F].

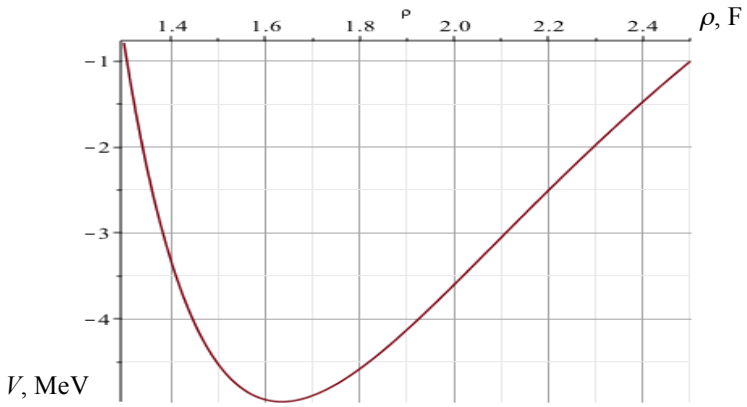

Fig 2. Plot of E (in MeV as function of ρ in F) based on *weakenings* of the potentials *inside the nuclear region* (see text)

While looking at the indicated parameters (K, C, D), the weakening of the energies can seem small; nevertheless for r = 1.6 F, i.e. at the LME, the weakening $V_{eff} - V_{effw}$ is equal to ~ -49 MeV; the weakening of E_{SSR} is ~ 22 MeV, and for V_4 , it is ~ 6 MeV. In fact, these weakenings are considerable in absolute values.

5. Analysis of the results and conclusions

-1. We can make a first important remark: contrary to what was observed in the Barut-Vigier works, where the region near the nucleus was characterized by a pre-eminence of the magnetic interactions, whereas the "atomic region" was characterized by the electric interactions, now *both types of interactions are entangled in the nuclear region* and they have similar importance.

A pure mathematical exercise (where values are non significant) is shown in Fig. 3.

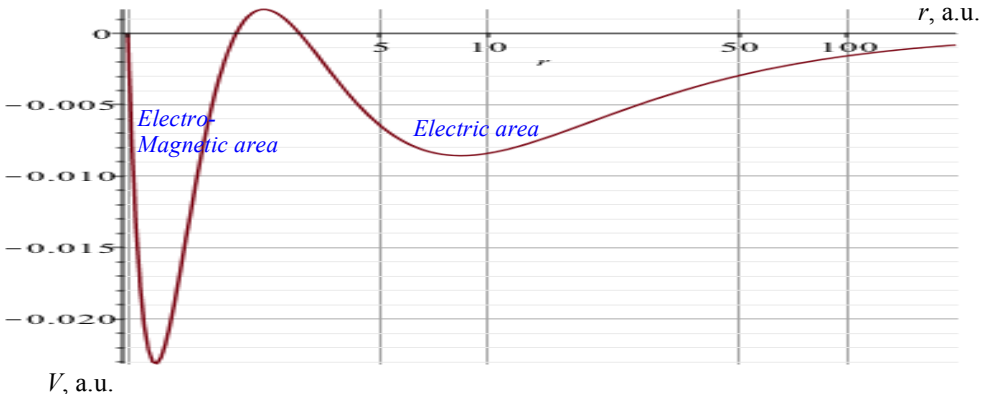

Fig 3. Simulation of two potential wells

-2. The computations have been carried out with coarse approximations: we used $\gamma \sim \underline{\lambda}_c/r$, but one could as well multiply/divide this expression by 2 or by π. This is only a size order, which must be taken into account. Nevertheless,

(i). By combining in some ways the different potentials, we easily find a local minimum of energy outside the nucleus, for very deep electron orbits.

(ii). This LME is associated with a potential well, which is strong enough to respect the HUR.

-3. The combinations of potentials, which yield a positive result, lead to the following choices and requirements for future studies:

(i). As the confined electrons are highly relativistic, we have to take into account the effective Coulomb potential V_{eff}.

(ii). We discard the Spin-Orbit interaction and this amounts to confine the angular momentum to l=0. As a consequence, the term expressing the centrifugal barrier also vanishes.

(iii). Concerning the Spin-Spin interaction, the repulsive version seems more propitious to the existence of EDO's. (The attractive version hints at quarks and higher-energy nuclear resonances.) Either way, this interaction makes it necessary to take into account the magnetic moment and spin of the proton and not just assume a point charge. Under these conditions, the Dirac equation for one particle in an "external" electric field is not adequate to the task.

(iv). We take into account the repulsive "diamagnetic" potential energy, V_4, which is caused by an interaction between the magnetic moment of the electron with the charge of the proton (and not the symmetric interaction) and involves the squared norm of the magnetic vector potential of the electron. The only constraints are to take into account the electron spin and its magnetic moment in the near presence of the nuclear electric-charge field.

For future continuation of the study on EDO's, the points cited above could lead us to use at least relativistic 2-body quantum equations involving spins. Moreover, works on QED are necessary to evaluate the radiative corrections evoked above in 4.2.2 (ii), which have certainly a significant action on the strength of the different interactions near the nucleus. In a QED context, it seems that the Two Body Dirac Equation of (Dirac) Constraint Dynamic [32] [33] could usefully be tested, for the following reasons:

-(i).The initial two-body Dirac equation involves 16-component spinors, since they have the dimension of tensor products of 4-D spinors. Nevertheless, the Constraint Dynamic allows one to reduce it to a 4-component equation, if one mass is (relatively) very large as is the case for the proton mass relative to the mass of the electron.

-(ii).This method is fully relativistically covariant and it eliminates some problems of non-interaction for highly-relativistic particles [34].

-(iii) The equations lead to the expression, in a consistent relativistic way, of numerous magnetic interactions at short distance as "quasi-potentials," including all the possible interactions.

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Appendix A: Electron deep orbits (EDOs)

Beginning in 1920, with Rutherford's suggestion of electrons combining with protons to form neutral particles within the nucleus, a decade followed where the concept of a "nuclear electron" was acceptable [35]. The concept fell out of favor toward the end of the decade with the Heisenberg Uncertainty Relation (HUR), the Klein Paradox, and the fact that the intrinsic spin of the proton, electron and neutron do not add up. This was despite developments of the relativistic Klein-Gordon and Dirac equations. The next few years saw the discovery of the neutron and the development of proton-neutron models of the nucleus. Thus, the concept of a nuclear electron became unnecessary and was largely forgotten.

Periodically, in later decades, the concept of the deep-electron orbits was revisited for various reasons, but outside of the neutron. In this case, the objection of the intrinsic-spin disparity no longer was of concern. However, other (mathematical) problems, along with the HUR, became the dominant mode of rejection. Early

(1994 and 1995) in the development of cold fusion, after electron screening was identified as a major requirement, EDOs were analyzed in the Klein-Gordon and Dirac equations to a much greater extent than previously. Unfortunately, this work was all but ignored for over a decade.

In 2005, the EDO of the Klein-Gordon equation was again explored. However, this time it was in the context of Randall Mills' fractional orbits rather than in that of cold fusion. Nevertheless, it opened the eyes of one of the present authors (AM) when he viewed the paper in 2009. This revelation of relativistic quantum mechanics predicting deep-electron orbits began the series of papers on the topic in Sections 2 and 3 below from Meulenberg, Sinha, and Paillet. Section 1 contains the EDO papers from other authors that have contributed to this concept in cold fusion.

1. EDO work within the context of cold fusion

J.A.Maly, J.Vavra, "Electron Transitions on Deep Dirac Levels I," Fusion Technology, vol. 24, p. 307, 1993
J. Maly and J. Va'vra, "Electron Transitions on Deep Dirac Levels II," Fusion Technology, vol. 27, January pp. 59-70, 1995
J. Va'vra, "On a possibility of existence of new atomic levels, which were neglected theoretically and not measured experimentally," presented at Siegen University, Germany, November 25 (1998)
J. Naudts, "On the hydrino state of the relativistic hydrogen atom," http://arxiv.org/abs/physics/0507193
J. Va'vra, "A new way to explain the 511 keV signal from the center of the Galaxy and some dark matter experiments," Astronomy & Astrophysics (2013) http://arxiv.org/abs/1304.0833
V.K. Ignatovich. "A Missed Solution for an Atom: A Gate Toward Cold Nuclear Fusion." Infinite Energy Magazine, issue 117, pp. 33-36 (2014)
V.K. Ignatovich, "A Singular Solution for a Hydrogen Atom as a Way Toward Cold Nuclear Fusion," Open Science Journal of Modern Physics, 2015 - academia.edu

2. EDO work by A. Meulenberg and K. P. Sinha
"Tunneling Beneath the ^{4}He∗ Fragmentation Energy," AM-KPS, 239th ACS Nat. Meeting *JCMNS* 4
"From the Naught Orbit to He4 Ground State" AM, ICCF-16, *JCMNS* 10
"Deep-electron orbits in Cold Fusion," AM-KPS, ICCF-16, *JCMNS* 13
"Deep-Orbit-Electron Radiation Emission in Decay from 4He* to 4He," AM-KPS, ICCF-16, *JCMNS*13
"Femto-atoms and Transmutation," AM, ICCF-16, *JCMNS* 13
"Lochon and Extended-Lochon Models for LENR in a Lattice," AM-KPS, *Infinite Energy Magazine,* pp. 29-32, Issue 112, November/December 2013
"Femto-Atom and Femto-Molecule Models of Cold Fusion," *AM, Infinite Energy Magazine,* pp. 41-45, Issue 112, November/December 2013
"Femto-Helium and PdD Transmutation," AM, ICCF-18, *JCMNS* 15
"Deep-orbit-electron radiation absorption and emission," AM, ICCF-18, *JCMNS* 15
"Pictorial description for LENR in linear defects of a lattice, A. M. ICCF-18, *JCMNS* 15

3. EDO work by J-L. Paillet and A. Meulenberg
"Arguments for the Anomalous Solutions of the Dirac Equations". JLP-AM, *JCMNS 18*
"Basis for Electron Deep Orbits of the Hydrogen Atom". JLP-AM, *ICCF19, JCMNS 19*
"Nature of the Deep Dirac Levels". AM-JLP, *ICCF1, JCMNS 19*
"Basis for femto-molecules and -ions created from femto-atoms". AM-JLP, *ICCF19*. *JCMNS 19*
"Electron Deep Orbits of the Hydrogen Atom". JLP-AM, *11th Int. W., Airbus, Toulous*e. *JCMNS 23*
"Special Relativity, the Source of the Electron Deep Orbits". JLP-AM, *Found. of Phys. 47(2*)
"Relativity and Electron Deep Orbits of the Hydrogen Atom". JLP-AM *RNBE I, Avignon, JCMNS 21*
"Advance on Electron Deep Orbits of the Hydrogen Atom". JLP-AM, ICCF20, Sendai.
"Implications of the electron deep orbits for cold fusion and physics". AM-JLP, *ICCF20*.
"Physical reasons for accepting the Deep-Dirac Levels". AM-JLP, *ICCF20*.

Appendix B: Application of Electron Deep Orbit (EDO) models to cold fusion predictions and experimental results

Why an EDO model – It was created to explain the D+D => ^{4}He results of CF.

1. It does so by transferring energy (mass) from a nucleus to a bound relativistic electron orbiting within femto-meters of the nucleus. *This transfer occurs (for different reasons) prior to, during, and after fusion*.

2. The DO electron, in forming a femto-H atom, eliminates the Coulomb barrier of a hydrogen nucleus.
3. The extra kinetic energy of the DO electrons lowers the mass defect Q of the fusing deuteron pair to below the ^{4}He* fragmentation or other excited-nucleon levels.
4. If fragmentation or gamma decay is not possible, other decay modes must lower ^{4}He* to ^{4}He.
5. Application of this model and its consequences readily explain most, or all, CF experimental results.

What is the EDO model – electrons are Coulomb-bound in deep orbits about a nucleus

1. The existence of deep orbits is predicted by the **relativistic** Klein-Gordon and Dirac equations.
2. For H, the predicted orbits are in the femtometer range with a binding energy $|BE| \geq 507$ keV.
3. Kinetic energy of DO electrons are predicted to be in the KE = 1 MeV range (e.g., [36] and references therein) and the 100 MeV range (above).
4. H or ^{4}He with DO electrons are *femto-atoms*, which are near-nuclear-size neutral objects with properties to explain most of CF experimental results.
5. KE = 1 MeV DO electrons violate Heisenberg Uncertainty Relation (a problem not yet resolved). The ~100 MeV electrons do not.
6. Nuclear mass reduction is key to D-D => ^{4}He CF results:
    - A change of 1 MeV may not be noticeable. ^{4}He with a DO electron would appear to be H with strangely shifted spectral lines; so, EDOs would be confirmed, but cause(s) of the exact shift might not be obvious.
    - A change of 100 MeV would be clearly identifiable in spectroscopic data as a nuclear mass shift.
7. Known example of such energy/mass transfer – energy conservation in the atomic hydrogen system. Emitted photons from decaying electrons carry away excess Coulomb energy from the nuclear-mass.

EDO model predictions – How does the model fit with Cold Fusion models and experiment?

1. It is a natural extension of Sinha's Lochon model that has published calculations of interaction probabilities for the D^+ - D^- fusion reaction in a solid-state lattice.
2. It is a natural consequence of both the linear-H molecule model and Takahashi's Tetrahedral Symmetric Condensate
3. It works for both D-D and H-H cold fusion results and explains the observed differences.
4. It predicts transmutation results consistent with observed in CF results.
5. It predicts the CF results of nuclear energy transfer to the lattice without the energetic particles or gamma radiation of hot-fusion and neutron-activation experiments.
6. It predicts selective attraction of femto-atoms to radioactive isotopes for nuclear waste remediation

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