Spinors with Deformed Lorentz Transformations

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In deformed special relativity with commuting coordinates transforming according special relativity and deformed plane waves the field equations and interactions in coordinate space remain unchanged. However in momentum space Lorentz transformations of spinors, the Dirac equation and helicity spinors are modified. The calculation of some simple tree level scattering amplitudes is discussed.

1. Introduction

Deformed or doubly special relativity (DSR) is a modification of special relativity (SR) with two invariant scales, a velocity scale *c* (speed of light) and a fundamental length scale ℓ (proportional to the Planck length), see the reviews in [1] and references therein. Recently [2] (see also [3]) we discussed the possibility to use commuting coordinates transforming according SR with deformed plane waves and a standard field theory in coordinates space.

From the Dirac equation in coordinate space with commuting SR coordinates and deformed plane waves one obtains the deformed Dirac equation in momentum space. With modified Lorentz transformations of rest frame spinors the deformed spinor solutions and relations between them can be derived. Then solutions of the deformed massless Dirac equation and the modification of the spinor helicity formalism in DSR are investigated. The calculation of some simple tree level amplitudes in DSR is discussed.

2. Commuting SR Coordinates and Dirac Equation in Momentum Space

We begin with a deformed dispersion relation written in the form

$$f(p)^2 = F^2 E^2 - G^2 \boldsymbol{p}^2 = m^2 \tag{1}$$

with the functions $F, G(E, \boldsymbol{p}^2, \ell)$ preserving rotational symmetry and $f^{\mu}(p) = \left(F\, p^0, G\, p^i\right)$. We use here the metric $(+,-,-,-)$ and $\hbar = c = 1$. Specific DSR models can be found in [1] and for $G = F$ in [4].

Recently [2] we considered in the case $G = F$ commuting coordinates transforming according SR without momentum dependence in their transformation, but with deformed plane waves. We repeat here shortly some of the results. The dispersion relation (1) for $G = F$ is invariant under the transformation of the momenta $p'^{\mu} = A \Lambda^{\mu}_{\nu} p^{\nu}$, if $F' = F / A$. The commuting SR coordinates ξ_{μ} transform as $\xi'_{\mu} = \overline{\Lambda}^{\nu}_{\mu} \xi_{\nu}$. $\Lambda, \overline{\Lambda}$ are standard, standard inverse Lorentz transformations. The boost and rotation generators are

$$M_{\mu\nu} = F(p_{\nu}\xi_{\mu} - p_{\mu}\xi_{\nu}) \tag{2}$$

and the commutators between them remain standard, as can be seen by introducing the auxiliary SR momenta $\pi_{\mu} = F\, p_{\mu}$. Noticing $F' = F / A$, the invariants built from these coordinates and the momenta become

$$F^2 p^{\mu} p_{\mu} = inv\,,\; F\, p^{\mu} \xi_{\mu} = inv\,,\; \xi^{\mu} \xi_{\mu} = inv \tag{3}$$

From here on denote the commuting SR coordinates ξ_μ as x_μ and the derivatives with respect to them as $\partial^\mu = \partial / \partial x_\mu$. Plane waves are therefore deformed as $\exp(-iF\, p\cdot x)$. The advantage of these commuting SR coordinates is clearly, that the field equations in coordinate space and the interactions with a form dictated by gauge invariance remain in the standard form.

Now consider the general dispersion relation in (1). The Klein Gordon equation is $(\partial^\mu\partial_\mu + m^2)\varphi(x) = 0$, inserting deformed plane waves of the general form $\varphi(x) = \varphi_0 \exp(-\mathrm{i}\, f(p)\cdot x)$ gives again the above dispersion relation (1). Similarly the Dirac equation in coordinate space is $(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi(x) = 0$, the deformed plane wave solutions for particles and antiparticles are given by the following ansatz

$$\psi_s(x) = u_s(p)\exp(-\mathrm{i}\, f(p)\cdot x)\ ,\ \chi_s(x) = v_s(p)\exp(+\mathrm{i}\, f(p)\cdot x) \tag{4}$$

where explicitly $f(p)\cdot x = F\, p^0 x_0 + G\, p^i x_i = F\, E\, t - G\, \boldsymbol{p}\cdot\boldsymbol{x}$. The Dirac equation then becomes deformed in momentum space and the solutions for the spinors u_s, v_s have been discussed in [5]. Here we take a slightly different approach, more adapted to the compact notation in the textbooks [6],[7]. We work in Weyl representation with $\sigma^\mu = \bar\sigma_\mu = (1, \sigma^i)$ and $\bar\sigma^\mu = \sigma_\mu = (1, -\sigma^i)$, where σ^i are the Pauli matrices and the Gamma matrices are $\gamma^\mu = \{\{0\ \sigma^\mu\}\{\bar\sigma^\mu\ 0\}\}$. Inserting the deformed plane waves from (4) in the standard Dirac equation in coordinate space gives the deformed Dirac equation in momentum space, where $f(p)\cdot\sigma = F\, p_0\sigma^0 + G\, p_i\sigma^i$

$$\begin{pmatrix} -m & f(p)\cdot\sigma \\ f(p)\cdot\bar\sigma & -m \end{pmatrix} u_s(p) = 0\ ,\ \begin{pmatrix} -m & -f(p)\cdot\sigma \\ -f(p)\cdot\bar\sigma & -m \end{pmatrix} v_s(p) = 0 \tag{5}$$

One can obtain the solutions for u_s, v_s by boosting from a rest frame. As mentioned above the algebra between rotations and boosts remains unchanged for the commuting SR coordinates and therefore one can write the pure boost transformation of a spinor as

$$\Lambda_{\frac{1}{2}} = \exp\left(-\frac{\beta_i}{2}\begin{pmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{pmatrix}\right) = \begin{pmatrix} \cosh\left(\frac{\beta}{2}\right)\cdot 1 - n_i\sigma^i \sinh\left(\frac{\beta}{2}\right) & 0 \\ 0 & \cosh\left(\frac{\beta}{2}\right)\cdot 1 + n_i\sigma^i \sinh\left(\frac{\beta}{2}\right) \end{pmatrix}$$

With the hyperbolic relations $\cosh(\beta/2) = \sqrt{(\cosh(\beta)+1)/2}$, $\sinh(\beta/2) = \sqrt{(\cosh(\beta)-1)/2}$ together with $\cosh(\beta) = f(p^0)/m = F\, p^0/m$ one can write the modified Lorentz transformation of a spinor similarly to [6],[7] in the very compact form

$$\Lambda_{\frac{1}{2}} = \begin{pmatrix} \sqrt{\frac{f(p)\cdot\sigma}{m}} & 0 \\ 0 & \sqrt{\frac{f(p)\cdot\bar\sigma}{m}} \end{pmatrix} \tag{6}$$

$$\begin{aligned} \sqrt{f(p)\cdot\sigma} &= (f(p)\cdot\sigma + m)/\sqrt{2(f(p^0)+m)} \\ \sqrt{f(p)\cdot\bar\sigma} &= (f(p)\cdot\bar\sigma + m)/\sqrt{2(f(p^0)+m)} \end{aligned} \tag{7}$$

which is in agreement with the expression given in [5]. Furthermore we note some useful relations showing amongst other things that (7) is correct:

$$\begin{aligned} & f(p)\cdot\sigma\, f(p)\cdot\sigma = 2f(p^0)\, f(p)\cdot\sigma - m^2 \\ & f(p)\cdot\bar\sigma\, f(p)\cdot\bar\sigma = 2f(p^0)\, f(p)\cdot\bar\sigma - m^2 \\ & f(p)\cdot\sigma\, f(p)\cdot\bar\sigma = m^2 = f(p^+)f(p^-) - f(\tilde p^+)f(\tilde p^-) \\ & f(p^\pm) = F\, p^0 \pm G\, p^3\ ,\ f(\tilde p^\pm) = G(p^1 \pm i\, p^2) \end{aligned} \tag{8}$$

The rest frame solutions of (5) are

$$u_s(0)=\sqrt{m}\begin{pmatrix}\xi_s\\ \xi_s\end{pmatrix},v_s(0)=\sqrt{m}\begin{pmatrix}\eta_s\\ -\eta_s\end{pmatrix} \text{ with } \xi_s,\eta_s=\begin{pmatrix}1\\0\end{pmatrix}\text{or}\begin{pmatrix}0\\1\end{pmatrix}$$

The boosted solutions now become with (6)

$$u_s(p)=\Lambda_{\frac{1}{2}}\,u_s(0)=\begin{pmatrix}\sqrt{f(p)\cdot\sigma}\,\xi_s\\ \sqrt{f(p)\cdot\bar{\sigma}}\,\xi_s\end{pmatrix}\ ,\ v_s(p)=\Lambda_{\frac{1}{2}}\,v_s(0)=\begin{pmatrix}\sqrt{f(p)\cdot\sigma}\,\eta_s\\ -\sqrt{f(p)\cdot\bar{\sigma}}\,\eta_s\end{pmatrix}\tag{9}$$

With (7) and (8) one can show, that they fulfil the deformed Dirac equation in (5). Their normalisation and relations between these spinor solutions can be derived with (8) and (9), see also [5], and we write here some of them.

$$\bar{u}_r(p)u_s(p)=2m\delta_{rs}\ ,\ \bar{v}_r(p)v_s(p)=-2m\delta_{rs}\ ,\ \bar{u}_r(p)v_s(p)=\bar{v}_r(p)u_s(p)=0$$
$$\bar{u}_r(p)\gamma^\mu u_s(p)=\bar{v}_r(p)\gamma^\mu v_s(p)=2f^\mu(p)\delta_{rs}\ ,\ \sum_s u_s(p)\bar{u}_s(p)=\gamma^\mu f_\mu(p)+m\ ,\ \sum_s v_s(p)\bar{v}_s(p)=\gamma^\mu f_\mu(p)-m$$

3. Deformed Massless Dirac Equation and Spinor Helicity

Amplitudes in particle scattering can be calculated most easily in the spinor helicity formalism see [7],[8],[9]. For very high energies particles can be considered as massless. Therefore one has to investigate the solutions of the massless Dirac equation. The deformed massless Dirac equation in momentum space is given by (5) with $m=0$ and is identical for *u* and *v* spinors. For outgoing antifermions and fermions, with $\pm$ denoting their helicity, it reads

$$\begin{pmatrix}0 & f(p)\cdot\sigma\\ f(p)\cdot\bar{\sigma} & 0\end{pmatrix}v_\pm(p)=0\ ,\ \bar{u}_\pm(p)\begin{pmatrix}0 & f(p)\cdot\sigma\\ f(p)\cdot\bar{\sigma} & 0\end{pmatrix}=0\tag{10}$$

Helicity is defined as $h=\boldsymbol{\pi}\cdot\boldsymbol{S}/|\boldsymbol{\pi}|=(G\,\boldsymbol{p}\cdot\boldsymbol{S})/|G\,\boldsymbol{p}|=\boldsymbol{p}\cdot\boldsymbol{S}/|\boldsymbol{p}|$ and for the modified dispersion relation (1) remains the same as in SR if $G>0$. This may be taken as an argument for considering the spatially isotropic modified dispersion relation in (1) and not a relation with different factors G_i in front of every p^i.

As usual one can represent all four vectors by bispinors with the matrices $\sigma,\bar{\sigma}$. For an arbitrary four vector a_μ we write $\bar{a}=a^{\alpha\dot{\alpha}}=a_\mu\sigma^\mu$ and $\underline{a}=a_{\dot{\alpha}\alpha}=a_\mu\bar{\sigma}^\mu$, in order to avoid the notation with dotted and undotted indices. The dot product between two four vectors is then $a\cdot b=a^\mu b_\mu=\frac{1}{2}tr\{\bar{a}\cdot\underline{b}\}$. For $f(p)$ in (10) this leads to the following expressions

$$\bar{f}(p)=f(p)\cdot\sigma=\begin{pmatrix}f(p^-) & -f(\tilde{p}^-)\\ -f(\tilde{p}^+) & f(p^+)\end{pmatrix}\ ,\ \underline{f}(p)=f(p)\cdot\bar{\sigma}=\begin{pmatrix}f(p^+) & f(\tilde{p}^-)\\ f(\tilde{p}^+) & f(p^-)\end{pmatrix}\tag{11}$$

$$f(p^\pm)=F\,p^0\pm G\,p^3\ ,\ f(\tilde{p}^\pm)=G(p^1\pm i\,p^2)$$

For $G=F$ one has $f(p)=F(p)\,p$ and for SR simply $f(p)=p$ and $F=G=1$. The solutions of (10), where angle (square) spinors denote negative (positive) helicity states, similar to the SR case can be written in the following compact form, where again (un)dotted indices are not displayed. The index four denotes a four component Dirac spinor and is often omitted, since it is in most cases clear from the context whether a spinor has two or four components.

$$|p\rangle_4=\begin{pmatrix}|p\rangle\\0\end{pmatrix},|p]_4=\begin{pmatrix}0\\|p]\end{pmatrix},\langle p|_4=\begin{pmatrix}\langle p| & 0\end{pmatrix},[p|_4=\begin{pmatrix}0 & [p|\end{pmatrix}\tag{12}$$

Note that $\langle p\,q\rangle_4=\langle p\,q\rangle,[p\,q]_4=[p\,q]$ and $\langle p\,q]_4=[p\,q\rangle_4=0$.

The deformed two component spinors in (12) are

$$|p\rangle = \frac{z}{\sqrt{f(p^-)}}\begin{pmatrix} f(p^-) \\ -f(\tilde{p}^+) \end{pmatrix} \ , \ |p] = \frac{z^{-1}}{\sqrt{f(p^-)}}\begin{pmatrix} f(\tilde{p}^-) \\ f(p^-) \end{pmatrix} \tag{13}$$

$$\langle p| = \frac{z}{\sqrt{f(p^-)}}\begin{pmatrix} f(\tilde{p}^+) & f(p^-) \end{pmatrix} \ , \ [p| = \frac{z^{-1}}{\sqrt{f(p^-)}}\begin{pmatrix} f(p^-) & -f(\tilde{p}^-) \end{pmatrix}$$

They are denoted in the same form as usually in spinor helicity, but are different from the corresponding spinors in SR. Even for $G = F$ with $f(p) = F(p)\,p$ they differ from the SR spinors by a factor $\sqrt{F(p)}$. One sees easily by using (11) and the modified dispersion relation in the form $f(p^+)\cdot f(p^-) - f(\tilde{p}^+)\cdot f(\tilde{p}^-) = 0$, that the four component spinors in (12) are solutions of the deformed massless Dirac equation in (10). With (13) one can rewrite (11) as

$$\overline{f}(p) = f(p)\cdot\sigma = |p\rangle[p| \, , \ \underline{f}(p) = f(p)\cdot\overline{\sigma} = |p]\langle p| \, , \ \not{f}(p) = f_\mu(p)\gamma^\mu = |p\rangle_4 [p|_4 + |p]_4 \langle p|_4 \tag{14}$$

The spinor products $\langle p\ q\rangle$ and $[p\ q]$ are different from the SR ones, even for $G = F$ they differ by the factor $\sqrt{F(p)F(q)}$, but their antisymmetry remains true as well as $\langle p\ p\rangle = [p\ p] = 0$. We note the relation

$$f(p)\cdot f(q) = \frac{1}{2} tr\left\{\overline{f}(p)\cdot \underline{f}(q)\right\} = \frac{1}{2}\langle p\ q\rangle[q\ p] \tag{15}$$

Special attention must be paid to the addition of momenta. One knows that for the auxiliary SR momenta defined as $\pi = f(p)$ the standard momentum conservation $\pi_{tot} = \pi_1 + \pi_2$ is valid. This translates into $f(p_{tot}) = f(p_1) + f(p_2)$ and a nonlinear law for p_{tot}. Squaring this relation together with the deformed massless dispersion relations $f(p_i)^2 = 0$ gives $f(p_{tot})^2 = 2 f(p_1)\cdot f(p_2)$. The deformed Mandelstam variables $\tilde{s}_{i\,j}$ together with (15) are defined as

$$\tilde{s}_{i\,j} = \left(f(p_i) + f(p_j)\right)^2 = 2f(p_i)\cdot f(p_j) = \langle i\ j\rangle[j\ i] \tag{16}$$

For $G = F$ with $f(p_i) = F(p_i)p_i$ they are given by $\tilde{s}_{i\,j} = F(p_i)F(p_j)s_{i\,j}$, where $s_{i\,j} = 2p_i\cdot p_j$ are the SR like Mandelstam variables.

Conservation of the all outgoing auxiliary momenta $\sum f(p_i) = 0$ can be written as $\sum\langle p\ i\rangle[i\ q] = 0$ in terms of the deformed spinors, see (14a). Many other relations of spinor helicity as for example the Schouten or Fierz identity remain valid, but are now expressed by deformed spinors, and we don't repeat them here. However a momentum p, whenever appearing freely must be replaced by $f(p)$. Two examples of this rule are the following identities

$$[p|\gamma^\mu|p\rangle_4 = [p|\overline{\sigma}^\mu|p\rangle = 2f^\mu(p) = \langle p|\sigma^\mu|p] = \langle p|\gamma^\mu|p]_4 \, , \langle p|f_\mu(k)\gamma^\mu|q]_4 = \langle p\ k\rangle[k\ q] \tag{17}$$

Similarly the deformed massless propagators are for spin one bosons $-i\eta_{\mu\nu}/f(p)^2$ and for fermions $i\not{f}(p)/f(p)^2$.

In analogy to the momentum vector the current vectors $j^\mu_{-+} = \langle p|\gamma^\mu|k]_4 = \langle p|\sigma^\mu|k]$ and $j^\mu_{+-} = [p|\gamma^\mu|k\rangle_4 = [p|\overline{\sigma}^\mu|k\rangle$ can be represented as bispinors by $\overline{j} = j\cdot\sigma$ and $\underline{j} = j\cdot\overline{\sigma}$, which again are different from the SR case.

$$\overline{j}_{-+} = 2|p\rangle[k| \ , \ \overline{j}_{+-} = 2|k\rangle[p| \ , \ \underline{j}_{-+} = 2|k]\langle p| \ , \ \underline{j}_{+-} = 2|p]\langle k| \tag{18}$$

Polarisation vectors of a massless vector bosons with momentum p are defined as $\varepsilon^{+\mu}(p) = \langle r|\gamma^\mu|p]/\sqrt{2}\langle r\ p\rangle$ and $\varepsilon^{-\mu}(p) = -\langle p|\gamma^\mu|r]/\sqrt{2}[r\ p]$, where $|r\rangle$ and $|r]$ are massless reference spinors, and can be written as bispinors

$$\overline{\varepsilon}^+_p = \sqrt{2}\frac{|r\rangle[p|}{\langle r\ p\rangle} \, , \underline{\varepsilon}^+_p = \sqrt{2}\frac{|p]\langle r|}{\langle r\ p\rangle} \ , \ \overline{\varepsilon}^-_p = -\sqrt{2}\frac{|p\rangle[r|}{[r\ p]} \ , \ \underline{\varepsilon}^-_p = -\sqrt{2}\frac{|r]\langle p|}{[r\ p]} \tag{19}$$

They obey $\varepsilon_p^{\pm}\cdot\varepsilon_p^{\mp}=-1$, $\varepsilon_p^{\pm}\cdot\varepsilon_p^{\pm}=0$, $\varepsilon_p^{\pm}\cdot f(p)=\frac{1}{2}tr\{\overline{\varepsilon}_p^{\pm}\cdot \underline{f}(p)\}=0$. In the case $G=F$ they are identical to the SR polarisation bispinors. Shifting r by something proportional r does not change the polarization, shifting $r\rightarrow r+p$ gives $\overline{\varepsilon}_p^- \rightarrow \overline{\varepsilon}_p^- + \frac{\sqrt{2}}{[p\ r]}\overline{f}(p)$. The products between polarisation bispinors are as usual, the products with the auxiliary momenta are from (14)

$$\varepsilon_i^+\cdot f(p_j)=\frac{1}{\sqrt{2}}\frac{\langle j\ r_i\rangle[i\ j]}{\langle r_i\ i\rangle}\ ,\ \varepsilon_i^-\cdot f(p_j)=\frac{1}{\sqrt{2}}\frac{\langle i\ j\rangle[r_i\ j]}{[r_i\ i]} \tag{20}$$

Field theory with gauge invariant interactions remains undeformed in coordinate space by using the commuting SR coordinates. An interaction between bosons and fermions in coordinate space is $\mathcal{L}_I=g\,\overline{\Psi}\,\gamma^\mu\Psi\,A_\mu$ yielding terms of the form $j_{1^\pm 2^\mp}\cdot\varepsilon_3^\pm$. The interaction between bosons in nonabelian gauge theories involving derivatives is given by $\mathcal{L}_I=-i\,g\sqrt{2}\,\partial^\mu A^\nu A_\mu A_\nu$. Note that coordinate derivatives acting on deformed plane waves $A_\mu=\varepsilon_\mu\exp(-i\,f(p)\cdot x)$ lead to contractions of the form $\varepsilon_i\cdot f(p_j)\ \varepsilon_k\cdot\varepsilon_l$. Thereby calculations of amplitudes should in general run as in SR but with deformed spinors.

A very simple example is the fermion antifermion photon three point amplitude [9].

$$A_3(1_e^-,2_{\bar{e}}^+,3_\gamma^-)=e\,j_{1^-2^+}\cdot\varepsilon_3^-=\frac{e}{2}tr\left\{2|1\rangle[2|\cdot-\sqrt{2}\frac{|r]\langle 3|}{[r\ 3]}\right\}=-e\sqrt{2}\frac{\langle 3\ 1\rangle[2\ r]}{[r\ 3]}$$

After multiplying with $\langle 1\ 2\rangle/\langle 1\ 2\rangle$ and using auxiliary momentum conservation in the form $\langle 1\ 2\rangle[2\ r]+\langle 1\ 3\rangle[3\ r]=0$ one obtains equation (21). Here the last equality gives the corresponding expression for $G=F$, where A_3^{SR} is the SR amplitude:

$$A_3(1_e^-,2_{\bar{e}}^+,3_\gamma^-)=e\sqrt{2}\frac{\langle 3\ 1\rangle\langle 1\ 3\rangle[3\ r]}{\langle 1\ 2\rangle[r\ 3]}=e\sqrt{2}\frac{\langle 1\ 3\rangle^2}{\langle 1\ 2\rangle}=\sqrt{\frac{F(p_1)F(p_3)^2}{F(p_2)}}A_3^{SR} \tag{21}$$

As a further example we consider the process $e^+e^-\rightarrow\gamma\gamma$, see [8]. The spin averaged cross section derived there, becomes with (16) for $G=F$

$$\left\langle|\mathcal{T}|^2\right\rangle=2e^4\left(\left|\frac{\tilde{s}_{13}}{\tilde{s}_{14}}\right|+\left|\frac{\tilde{s}_{14}}{\tilde{s}_{13}}\right|\right)=2e^4\left(\left|\frac{F(p_3)p_1\cdot p_3}{F(p_4)p_1\cdot p_4}\right|+\left|\frac{F(p_4)p_1\cdot p_4}{F(p_3)p_1\cdot p_3}\right|\right) \tag{22}$$

Similarly in tree amplitudes for gluon scattering one can replace in the final cross section the SR Mandelstam variables by the DSR ones. For example in four gluon scattering in the case $G=F$ with $\tilde{s}_{ij}=F_i F_j s_{ij}$ and $F_i=F(p_i)$, one obtains a modified expression for the squared and averaged amplitude [7].

So one gets deviations from the usual results only in the extreme high energy regime, where the deformation function becomes important. The problem here is of course, that for higher energies processes with loops must be considered, which presumable would dominate the deformation effects.

Massive spinors with momentum p_m can also be defined in the deformed case with a corresponding massless momentum p and reference momentum r. With $f(p)^2=0$, $f(r)^2=0$ one gets

$$f(p_m)=f(p)+\frac{m^2}{2f(p)\cdot f(r)}f(r)\ ,\ f(p)=f(p_m)-\frac{f(p_m)^2}{2f(p_m)\cdot f(r)}f(r) \tag{23}$$

satisfying $f(p_m)^2=m^2$ and $f(p_m)\cdot f(r)=f(p)\cdot f(r)$. The solutions of the deformed massive Dirac equation $\left(\gamma^\mu f_\mu(p_m)-m\right)u(p_m)=0$, $\left(\gamma^\mu f_\mu(p_m)+m\right)v(p_m)=0$ are

$$
\begin{aligned}
u_{+}(p_m) &= |p\rangle + \frac{m}{[p\ r]}|r] = \frac{\left(\gamma^{\mu} f_{\mu}(p_m) + m\right)|r]}{[p\ r]} \\
u_{-}(p_m) &= |p] + \frac{m}{\langle p\ r\rangle}|r\rangle = \frac{\left(\gamma^{\mu} f_{\mu}(p_m) + m\right)|r\rangle}{\langle p\ r\rangle} \\
v_{+}(p_m) &= |p] - \frac{m}{\langle p\ r\rangle}|r\rangle = \frac{\left(\gamma^{\mu} f_{\mu}(p_m) - m\right)|r\rangle}{\langle p\ r\rangle} \\
v_{-}(p_m) &= |p\rangle - \frac{m}{[p\ r]}|r] = \frac{\left(\gamma^{\mu} f_{\mu}(p_m) - m\right)|r]}{[p\ r]}
\end{aligned}
\tag{24}
$$

Here the massless spinors have four components. The conjugate massive spinors can be obtained analogously.

4. Summary

In summary we have considered spinor fields in DSR theories based on commutative SR coordinates with deformed plane waves. The field theory and interaction structure in space-time remains unchanged in terms of these coordinates, while the Lorentz transformations and spinor solutions in momentum space are modified. Solutions of the massless deformed Dirac equation and their properties are investigated. With a modified spinor helicity formalism it is possible, to calculate some simple tree level scattering amplitudes in DSR. Of course several problems still remain to be solved, as for example the calculation of processes with loops.

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