Einstein's Dream

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Abstract

I discuss Einstein's dream of a **Final Theory** and recent attempts to realize it.

Introduction

Einstein, the founder of modern cosmology, hoped to be able to explain ***everything*** by a geometrical theory of curved spacetime together with fields on it [Dongen]. Complex, dynamic patterns of bumps and ripples in the geometry and fields were to be the underlying reality for what we usually call light, matter, animals and people. His program had reasonable success with bulk light and matter. For instance he was able to obtain the Newtonian Gravitational Theory as a first approximation, and higher order approximations resolved the anomaly in the perihelion shift of the orbit of Mercury (a major open problem in celestial mechanics for over fifty years). Most of the models, simulations, and discussions of **The Big Bang Model** basically use this approach (see: [Weinberg-1]).

In #1 I will discuss a recent version of Einstein's **Unified Field Theory Program**. **If** this theory had been empirically adequate, it would have provided a completely satisfactory[1] **Final Theory**. The extreme irony is that one of the main places his **Unified Field Theory Program** fails is in Einstein's own **EPR** type situations. In #2 I will discuss a quantization program for the theory introduced in #1. **If** this program could be worked out, it would provide a mathematically precise version of **The Standard Models of Particle Physics and Cosmology** and hence **The Final Theory**[2]**.** In #3 I discuss my program of **Interlocking Worlds** and its relationship to **Pragmatic Pluralism.** I also mention the possibility that the Universe is **tangled**.

#1 **Unified Field Theories**

The singularities of **ordinary** General Relativity [Hawking-Ellis] can be avoided by considering the (mathematically well-defined) Einstein-Yang-Mills-Dirac System[3] [Finster-Hainzl] which is (heuristically) the super-classical [Varadarajan] limit of the (**<u>not</u>** mathematically well-defined[4,5]) Standard Model. This system has complete solutions without singularities, solitons, and a Cyclic Universe solution. (The system has negative energy density; hence doesn't satisfy the positivity conditions in the Penrose-Hawking Singularity Theorems.) The E-Y-M-D equations provide an alternative approach to a Cyclic Universe which Penrose [Penrose] has recently been advocating. They also imply that the massive compact objects now classified as Black Holes are actually Quark Stars, possibly with event horizons, but without singularities.[6] A Super version [Varadarajan] of the above-including super-neutrinos-might be needed to explain Dark Matter. The E-Y-M-D is also a totally geometricized theory as a non-commutative geometry [Connes] [Connes-Marcolli]; the charge e and the mass m of the electron are geometric invariants of the non-commutative geometry analogous to pi. Unfortunately, there are quantum phenomena, such as EPR, for which this **beautiful** theory doesn't make adequate predictions. (One can still have a Block Universe: see [Goldstein], [Nottale].)

#2 Quantum Field Theories

One would like to be able to apply deformation quantization [Sternheimer] to the Einstein-Yang-Mills-Dirac System to get a mathematically rigorous Standard Model in a Curved Noncommutative Spacetime. This would obviate any **necessity** of going further to a full quantum gravity theory. The deformation parameter would be Planck's constant and the associated Planck Length would be the scale at which the manifold structure of spacetime gives way to a rougher structure of a noncommutative geometry.

Quantization Program for E-Y-M-D

1. Extend the results of [Flato] from the Maxwell-Dirac system to the Einstein-Yang-Mills-Dirac system.

2. Show that the positivity condition in the Penrose-Hawking singularity theorem **[**Hawking-Ellis**]** is violated for the E-Y-M-D system. Construct smooth solutions to E-Y-M-D [Finster-Smoller-Yau-1] [Finster-Smoller-Yau-2] having Dark Stars. Construct Cyclic Universe solutions to the E-Y-M-D system [Finster-Hainzl].

3. The History of the Universe:

i. Derive the approximate history of the universe from the E-Y-M-D system both analytically and *via* computer simulation.

ii. Compare with the theoretical results in [Weinberg-1].

iii. Compare with the observations reported in [Weinberg-1].

4. Show that the solution space for the E-Y-M-D system, **F**, is a reasonable infinite dimensional super-sympletic manifold [Varadarajan].

5. The space of solutions **F** needs to be quotiented by a big infinite dimensional group (or, in the super-theory case, a big infinite dimensional supergroup). One hopefully gets a reasonable infinite dimensional noncommutative sympletic geometry, or possibly, a reasonable infinite dimensional noncommutative Poisson Algebra [Pflaum] [Uchino], which we now need to deformation quantize[7] [Sternheimer] to

obtain a mathematically rigorous definition of Q-E-Y-M-D (quantum version of E-Y-M-D).

6. Derive the history of the universe from Q-E-Y-M-D and compare with observation.

Unfortunately, this program seems to be beyond our current capabilities. Already in 1990 Dito [Dito-1] showed how to deformation quantize the free field. In spite of having the successful Glimm-Jaffe [Glimm-Jaffe] program of constructive quantum field theory for $(\phi^4)_2$ (completed by ~1975), in over 20 further years not even $(\phi^4)_2$ has been deformation quantized [Dito-2] [Dito-Sternheimer] (but see [Rivasseau-1] [Rivasseau-2]).

The current situation in QFT is philosophically very interesting. By ~1950 Feynman, Schwinger and Dyson had shown that one could renormalize the perturbation expansion [Schweber]. This was extended to Yang-Mills theories by t'Hooft ~1970 [t'Hooft] [Cao]. But ~1950 Dyson had given arguments that the series was divergent. Schweber [Schweber] relates that while this led to Dyson leaving QED, this didn't seem to bother Schwinger at all; he didn't even seem to care if one couldn't renormalize all the terms of the perturbation expansion! In any case, by using just the low order terms of the perturbation expansion, physicists were eventually able to get <u>**12**</u> digit agreement between theory and experiment! Physicists believe that the perturbation expansion is an asymptotic expansion for the Scattering Matrix, **S**. But **S** is <u>***only***</u> at present defined empirically. Their opinion is buttressed by the 12 digit agreement between theory and experiment, and by the fact that one can actually **<u>prove</u>** that the Feynman Perturbation Expansion is an asymptotic expansion for the rigorously defined asymptotic **S** matrix in $(\mathrm{phi}^4)_2$ [Glimm-Jaffe].

Besides the above perturbation expansions, one also has finite lattice 'approximations' [t'Hooft], Euclidean techniques [Gibbons-Hawking], and the super-classical limit 'approximations' of #1. Physicists seem reasonably satisfied with this situation. They sometimes express the belief that *eventually* we'll have full **non-perturbative** QFT, or, at least, of some *deeper* theory, such as a non-perturbative string theory. I'm not holding my breath! In fact, I don't expect to live to see such a culmination. It's interesting to contemplate the opposite assumption; namely, either because we're not clever enough or because it simply doesn't exist, that we'll never have such non-perturbative theories. One would then have to ***settle*** for a variety of 'approximations' having some vague relationships to one another, but without any central, rigorous theory! Thus, right in the heart of fundamental physics we would find the same situation we'll be discussing next in #3.

#3 Interlocking Worlds, Pragmatic Pluralism, and Tangled Hierarchies

The central message that Bohr and von Neumann taught us about the Standard Quantum Logic is that it can be viewed as a **manifold of interlocking perspectives that cannot be <u>embedded</u> into a single perspective** [Edwards-2].[8] Hence, the perspectives cannot be viewed as perspectives on **<u>one</u> real**

world. So, even considering **one world** as a methodological principle breaks down in the quantum micro-domain. The issue I'm pondering is the inadequacy of only talking about appearances and not going beyond appearances to some sort of world. Appearances are very complicated, confused, etc. Worlds are both simpler and more inclusive. I have no problem merely assuming some sort of world if it **<u>works</u>**! That is, simplifies our conceptions. Think of what happened to Chew's S-matrix approach; it lacked powerful enough heuristics to get anywhere.

To give you an idea of what I'm talking about concerning ***interlocking worlds*** consider the following definition of a ***quantum phase space*** associated to a quantum system described by a non-commutative C* algebra **B.** First replace **B** by its diagram-in the sense of category theory (see: [Edwards-2]) -of **commutative** sub-C* algebras {**A**}. Then apply the functor **D** which replaces a commutative C* algebra **A** with its maximal ideal space **D**(**A**)-a compact topological space-to {**A**} to get {**D**(**A**)}. This diagram is ***the quantum phase space*** of a quantum system. This is the type of gadget which could be described as ***interlocking worlds.*** Of course***,*** in general, we'll have to deal with more **loosely, vaguely** defined diagrams. The above could be a **<u>precise</u>, <u>toy</u>** model of Cartwright's ***dappled world*** [Cartwright]**.** It is identical to Hawking's recent notion of ***model dependent realism*** [Hawking-Mlodinow]**.** It also formalizes William James' notion of the ***pluralistic universe*** [James].

My understanding of the history of science leads me to expect the opposite of what Pierce expected; in the long run our network of scientific theories don't converge on the truth, but, instead, become richer and more complex, requiring intellectual ecology to comprehend. The network of scientific theories moves towards looking like a rainforest! A good example is optics. Geometrical optics goes back to at least Euclid; wave optics to Huygens; then we have Maxwell's electro-magnetic theory and its' classical competitors the Feynman-Wheeler theory and the Maxwell-Dirac theory. Onwards to Einstein's photon theory and later to quantum electro-dynamics. If one examines an optician's handbook one will find many ad hoc theories; similarly for treatises on computer graphics.

The following collection of theories exhibits an even more frightening possibility, **tangled hierarchies**: The Theory of Mind; Neuroscience; Molecular Biology; Particle Physics; Quantum Theory. Sometimes I feel that only Hegel is ***crazy enough*** to incorporate quantum theory.

Endnotes

1. **Completely satisfactory** that is to fundamental theoretical physicists! While there is a plurality of viewpoints within the sciences, the gap between the sciences and the humanities is much larger than that within the sciences. All the sciences attempt to take a value neutral, I-it, approach to their respective subject matters. This is as true in anthropology as it

is in physics. An anthropologist qua scientist attempts to describe man and his culture in unemotional language similar to the astronomer's description of the planets and their motion. But the use of such language is often found very objectionable by non-scientists. The astronomer has succeeded in demoting the planets from gods to merely large chunks of rock. Humanists are often afraid that science will have a similar "success" with man.

2. **The Final Theory.** If we're lucky, the LHC will discover not only the Higgs Boson, but also supersymmetry. This might provide super-neutrinos as the explanation for dark matter. I don't believe that dark energy requires any explanation. The cosmological constant is a natural constant in the Einstein Field Equations, and its positivity doesn't require a material explanation. Since the super-standard model in a curved spacetime, SSMCST, violates the positivity condition in the Penrose-Hawking singularity theorem, there might be satisfactory complete solutions. This would obviate any need to quantize gravity. Economic conditions might make the LHC the end of the line for big particle accelerators. High energy cosmic ray astronomy might not yield anything not explicable by the SSMCST. We would then have a final theory which in principle predicts all phenomena we can produce or observe. Of course, in practice, it would even have great difficulty predicting the properties of protons. Here I'm not even objecting to the fact that it wouldn't be mathematically well-defined. This would be the completion of Newton's hopes at the end of his preface to

The Principia. In **practice**, we'd still have pluralism even after having attained a final theory.

3. By 1930 Einstein had already considered the Einstein-Maxwell-Dirac System [Dongen].

4. **The current situation in Fundamental Physics.** Even QED in Minkowski spacetime is not yet mathematically well-defined; there is even a $1,000,000 prize for making it so. The fantastic 12 digit agreement between the theoretical predictions of QED and the experimental results is a miracle. It would be good for physicists to acknowledge the unsatisfactory mathematical structure of this prediction. It starts from a mathematically undefined Feynman Integral, proceeds by making many very complicated manipulations, and ends up with a formal series that Dyson showed to be divergent! Physicists think of it as an asymptotic expansion, but they have no mathematical proof of this. I often joke that this agreement of theory and experiment is a new proof of the existence of God and that she loves physicists! There is no evidence that Wald's axioms, like Wightman's before him, will eventually include QED and/or the Standard Model. They may be beautiful, but irrelevant! It's still not clear whether we'll ever have a consistent QED or SM. Most physicists don't seem to be overly concerned by this issue and have retreated to being satisfied with effective field theories all the way down. Furthermore, effective field theory philosophy is merely a cover for having in practice abandoned the ideal of unity in exchange for the practice of applied mathematics.

5. **Mathematics & Logic.** My mathematics colleagues almost never think about mathematical logic (see: ***The Ideal Mathematician***, Phillip J. Davis & Reuben Hersh, http://people.maths.ox.ac.uk/bui/ideal.pdf,
for what is simultaneously the funniest and most profound description of mathematicians!). Mathematical logic is almost never taught in mathematics departments- it's taught in computer science departments and philosophy departments- and when it is, it is taught in a purely technical way with no concern for history or philosophy. Mathematicians still live in Cantor's paradise-or even Eilenberg's paradise-in spite of Russel's paradox; they simply learn not to make certain moves that lead to trouble (as long as the referee doesn't complain, ***what, me worry*!**). The various formalizations for avoiding Russell's paradox also hamstring one from making certain moves which are usually safe and powerful. So, mathematicians work informally and have always done so; **there is almost no trace of mathematical logic in most of the history of modern mathematics!** I'm not saying that mathematicians are aware of what I just said; most are totally unaware of these issues and simply working in a successful research tradition. Similarly for theoretical physicists!

6. The large mass quark stars would still have an event horizon; thus, be indistinguishable from Black Holes to external observers. But, theoretically, it undermines one of the main arguments for going to a full theory of quantum gravity such as string theory. (Wheeler [Wheeler], already in 1963, stressed the singularities as motivation for his quantum geometrodynamics.)

7. **Quantizing non-commutative geometries.** When I first started studying QFT in the early 60's I was puzzled about the rational for assuming the vanishing of anti-commutators for spin 1/2 fields at space-like separated points. By 1979 [Edwards-1] I understood this assumption in the context of super-theory, where operators lying in a graded-commutative sub-algebra are assumed to be ***compatible***. Thus, the usual Wightman QFT of spin 1/2 fields provides a graded-***<u>non</u>***-commutative algebra, and passing to the associated diagram-in the sense of category theory [Edwards-2]-of graded-commutative sub-algebras provides an example of a super-quantum theory. Analogously, ***<u>diagrams of non-commutative algebras (geometries) are what non-commutative quantum theory is about</u>***. One now seeks to construct such diagrams via some sort of deformation quantization of non-commutative Poisson algebras. In particular, one would like to show that they can be related to interesting theoretical physics-or, to at least, string theory. One simple construction is to start with a non-associative algebra and pass to the diagram of its associative sub-algebras. One also has an interesting **Gleason Problem** [Held]: Does there exist a non-associative algebra whose diagram of associative sub-algebras **<u>cannot</u>** be embedded into an associative algebra?

8. Bohr's approach to quantum theory would lead to a ***<u>disjoint</u>*** collection of perspectives. The **<u>very strong</u>** assumption that the

perspectives **interlock** in a way determined by the closed subspaces of a Hilbert Space is **the substitute** for the even stronger classical assumption of a ***real*** world-that is, that the perspectives embed into a single perspective (Gleason's Theorem [Held] proves that the Standard Quantum Logic does **not** so embed! I believe that Gleason's Theorem should be as well-known as Gödel's Theorem.).

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