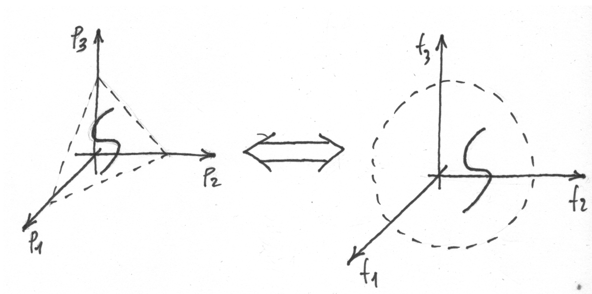
I search to obtain a dynamics for a probabilistic system: a system with a finite number of variables $P_i \geq 0$ such that $\sum_i P_i = 1$.

The dynamics of the probabilistic system is on a a face of a octahedron, but it is complex to require a dynamics on one face (the general solutions $\frac{dP_i}{dt} = F(P_1, \cdots, P_n)$ tend to cover the whole octahedron faces).

I simplify the problem using the probability amplitude f_i such that $P_i = f_i^2$, so that the probabilities are defined as positive.



The dynamics of the system is:

$$\frac{df_i}{dt} = a_i + \sum_i a_{ij} f_j + \sum_i a_{ijk} f_j f_k + \cdots$$

so that it is simple to obtain the normalization:

$$0 = \frac{d}{dt} \sum_i P_i = \frac{d}{dt} \sum_i f_i^2 = 2 \sum_i f_i \frac{df_i}{dt} = \sum_i a_i f_i + \sum_{ij} a_{ij} f_i f_j + \sum_{ijk} a_{ijk} f_i f_j f_k + \cdots$$

for each arbitrary values of the amplitudes this polynomial must be zero (even for points near the octahedron surfaces), so that

$$\begin{aligned}
&a_i = a_{ii} = a_{iii} = 0 \\
&a_{ij} + a_{ji} = 0 \\
&a_{iij} + a_{iji} + a_{jii} = 0 \\
&a_{ijk} + a_{ikj} + a_{jik} + a_{kij} + a_{jki} + a_{kji} = 0
\end{aligned}$$

so that $\sum_P a_{P(i,j,k,\cdots)} = 0$, so that the sum of the coefficients with the permutation of the indices is null.

The amplitudes dynamics is on a sphere, and if the initial amplitude is on a unitary sphere, then the probability dynamics is normalized to one.

It is possible to use more complex dynamics, for example $P_i = f_i^{2n}$, or $P_i = f_i f_i^*$ (using a quantum mechanics analogy), but this simple solution is interesting.