Symmetry of Covariance & Exchange: The Two Body Electron Equation

Paris Samuel Miles-Brenden

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Introduction

The interior and effective representational algebra of the raising and lowering operators for the spin eigenstates via their commutation relationship so instantiated by real orbital motion of the electrons in the two body Dirac electron equation violates the Pauli Exclusion Principle representationally when exchange is not considered simultaneously to its consequent effective spin flip from orbital motion. When these effects are included they are representative of an effective artifact of Bosonization upon the Fermions at the same strength as that of the ordinary Coulomb repulsion between them. A way of visualizing and interpreting this symmetry principle, is that were the two electron states in spin and orbital to be anything but independent locally and globally they would not be simultaneous eigenstates and hold fast to the net antisymmetry of the Pauli exclusion principle as fermions.

Hypotheses

1.) *Rotations of the electrons upon the spin of the two electrons under exchange are clockwise and counterclockwise when viewed from above or below.*

2.) *These rotations are generative under exchange of an effective raising and lowing operator upon their individual spins by the commutation relationship of the spin algebra.*

3.) *Relativity holds fast to an objective artifact of relativistic frame transformation which is inertial in both frames of the electrons and there is no substantive existence of a tertiary or privileged observer.*

From this it follows that the interior phase gauge symmetry is broken and a coordinate dependence to the orbitals occurs that must be compensated for in the two body Dirac equation.

1.) *Since the representation is physical for the electrons in their own given frames, the relationship that exists for the orbitals of the electrons and their given spins, exists as an 'excess' coordinate dependence that would violate the Pauli exclusion principle unless it is corrected for the sake of global to local relativistic considerations.*

2.) *Correcting for this coordinate dependence results in a state for which the spins will continue to follow the Pauli exclusion principle as fermions with the charge wave function, but in doing so, a portion of the electromagnetic interaction becomes of a small but real attractive interaction which is equivalent to a weak bosonization of the states.*

Proof by Dual Contradiction

As proof of this, consider the exchange of the spins or orbitals as separately from one another. We have illustrated a methodology by which they are representationally and therefore physically strictly dependent upon each other when the coordinate connection is not included. As a result, both wave functions are complete eigenstates with both symmetries and there is no strict dependence in the coordinate space when the coordinate connection is included, in order to adhere to both the Pauli Exclusion Principle and be completely antisymmetric in both spin and charge as attributes of the wave function. This is necessary so that the total eigenstate in spin and wave function under exchange are totally antisymmetric in space and time.

Hence what is found can be put more generally as the confluence of two concepts and principles:

1.) *The Pauli Exclusion Principle*
2.) *Objective Global and Local Inertial States*

The final and proper way to put this is in a case by case basis, and an argument by contradiction. Given that spin and wave function are in a product relationship in the conventional Dirac equation for the electron; where the spin is a unique decomposition and factoring of the manner in which to put the Lorentz invariant frame the frame of rest, we proceed by analysis of the Pauli Exclusion Principle and the eigenstate condition with global and local considerations of simultaneously meeting these provisions.

This is most easily imagined as two vectors of projection; one for the spin, and one for the orbital wave function. They can be imagined as initially of oppositional character, and of the same magnitude of covariant and contravariant extension for our purposes, and it is considered that we comparatively assess the projection in the Hilbert space of these vectors and one forms, under the action of the orbital to representationally raise and lower the spin of their individual electron states.

There are two operations that are mutually exclusive to be considered. From the perspective of one electron within its own state, the other antipodal electron state seen globally to locally is either of contravariantly and covariantly similar magnitude on both vectors and one forms as projections, or of differing magnitude, but while preserving the contravarying and covarying contraction.

The state where the magnitude differs by a scale and its inverse, for what was an eigenstate a priori will no longer be one for the projection of the spin portion of the eigenstate and that of the orbital will not meet the product relationship locally and globally. However here the Pauli Exclusion Principle can be satisfied, for the area relationship is the same, and hence their projections into each other can remain purely antisymmetric in spin and orbital momentum.

The state where the magnitude changes scale for both projections is an admissible eigenstate, because the weights of the covarying orbital momentum and contravarying spin are equivalent within the basis, and hence if it was initially an eigenstate in the Dirac electron equation, it will continue to be so. However, since the projection as measured by their combination (the spin and orbital) will determine an area that differs, the projection of the eigenstates into each other cannot be purely antisymmetric any longer locally and globally and the Pauli Exclusivion principle is violated.

Thus, it appears that either of the two electrons comparatively must attribute the same equivalent weight to charge and spin to satisfy the dual conditions that are the eigenstate condition and the Pauli Exclusion Principle both locally and globally; and that to preserve these conditions relativistically it is required that a correction to the orbital momentum be introduced, which is:

$$\Gamma_\mu = \partial_\mu \log \gamma^\nu \tag{1}$$

This correction is nothing but the gauge connection for the sake of the orbital momentum as a consequence of a co-evolving spin of the electron. This correction comes out as a logarithmic differential of the spin for the conjugate representation in the two body electron equation, and it is the same whether we consider the spin to be evolving and raising and lowering from the quantum perspective, or when viewed by way of relativity, as a direct consequence of preserving the inertial property of the state both locally and globally under inertial considerations.

The reconciliation of the local with the global properties and the correction for the sake of relativity is nothing more than the regularization of the renormalization group flow for the null principle of quantum mechanics; and it is indeed the intermediate gauge in which the electrons are in inertial states.

$$D_\mu = \partial_\mu + \alpha A_\mu + \Gamma_\mu \tag{2}$$

Where Γ_μ and $\partial_\mu + \alpha A_\mu$ are components of the momentum D_μ which separately anticommute and commute; thus rendering a non-zero exchange under local and global commutation or anticommutation relation with the prefix γ^μ in the new electron equation:

$$(i\gamma^\mu D_\mu - m)\Psi = 0 \tag{3}$$

To prove that the two body electron equation has a lower energy than that of the single body electron equation and the fact that the global and local operators of position and momentum exist within a state lacking a center of uncertainty is now as simple as pointing to the relation that is absent; that of a commonly null exclusively local and global eigenstate eigenvector condition of four dimensional nature or four coordinate nature on that of the Pauli Exclusion Principle; with that of instead it's replacement by a colocally everywhere local and global condition of the same nature.

This can be proven; as the vanishing locally and globally with a given constant offset in the two body electron equation with that of the midpoint of displacement on either side of the relation of the two components described above for that of the generalized four momentum D_μ; that of it's replacement by a comoving basis provided by the auxiliary electron; and the substitution of the offset of the eigenvalue; eigenvector equation with that of it's reduction under symmetry to that of a finite offset under what is in effect either anticommutation and commutation. One is reminded of the Spin Statistics Theorem here; with the principle:

"Extrinsic modifications to a given equation under antisymmetry of operators and symmetry of operators have symmetric and antisymmetric consequences for Fermionic eigenstates." This is entirely consistent with the interpretation of what a Fermion means; and what properties operators and eigenstates of such possess.

Consequentially:

$$(i\gamma^\mu D_\mu - m)(i\gamma^\mu D_\mu - m)\Psi_A \Psi_B = 0 \tag{4}$$

This equation is the two body electron equation with the gauge covariant differential known as the four energy momentum; for which either such given corrected relation of the four energy momentum is corrected fully as contracted; and through which the discrepancy of intrinsic and extrinsic mass is known in it's corrected form.

$$(-\gamma^\mu D_\mu \gamma^\mu D_\mu - i2m\gamma^\mu D_\mu + m^2)\Psi_A \Psi_B = 0 \tag{5}$$

This is the equation written out in full form; and demonstrated as a full eigenvalue eigenvector equation. When this is translated into component form it is a reexpression as:

$$(-\gamma^\mu D_\mu \gamma^\mu D_\mu - m^2)\Psi_A \Psi_B = 2im\gamma^\mu D_\mu \Psi_A \Psi_B \tag{6}$$

After reorganization of terms and a process of reduction to two new terms there is a reexpression once again by the following factoring:

$$(i\gamma^\mu D_\mu - m)(i\gamma^\mu D_\mu + m)\Psi_A \Psi_B = 2im\gamma^\mu D_\mu \Psi_A \Psi_B \tag{7}$$

Which means that two electrons are the generator under the anticommutation and commutation relationship of their subsidiary operators of a full notion of particle and antiparticle product relationship with a mass gap equivalent to the splitting equivalent to each of their reduction's in energy at the relativistic accomodated energy level of the full energy momentum of either one such particle.

This explains a mass energy gap; for that of the two body electron equation which is a real energy lowering; of what is understood when imagined as the absence of one electron in it's surrounding notion as in the presence of the other electron as an positron; for what is of presence is of absence with matter; and together forming a solid whole of which the energy momentum is lower by a double accounting for that of either electron.

The final explanation is consistent with QED which is that without evaluation of field diagrams; the electron positron renormalization of the photon propogator is a full energy expectation of the same apportionment on $\gamma^\mu A_\mu$ for that of either the electron mass and photonic light energy mass for what is known as the relation of that which is necessary to have agreement between the photonic energy momentum carried by the electron and it's own non-self-energy lowering in energy momentum (incidentally explaining the non-occurrence of self energy).

This comes about by consideration of the separation into two sum renormalization processes under disconnected and decoupled tadpole (electron positron) diagrams; for which the energy mometum is of the relation of an intermediary gauge boson of which is the carrier of the force; for what are disconnected and repulsive interactions become of the other sign.

This has the entire description as the same as the above equations for electron and positron; but is seen more clearly when these equations are fully written out:

$$(i\gamma^{\mu}(\partial_{\mu}+\alpha A_{\mu}+\Gamma_{\mu})-m)(i\gamma^{\mu}(\partial_{\mu}+\alpha A_{\mu}+\Gamma_{\mu})+m)\Psi_{A}\Psi_{B}=2im\gamma^{\mu}D_{\mu}\Psi_{A}\Psi_{B} \tag{8}$$

With the re-writing as:

$$(\partial_{\mu}^{2}+\alpha^{2}A_{\mu}^{2})\Psi_{A}\Psi_{B}=(2m\gamma^{\mu}D_{\mu}+im^{2}-\Gamma_{\mu}^{2})\Psi_{A}\Psi_{B} \tag{9}$$

This equation can be re-written as:

$$\partial_{\mu}^{2}+\alpha^{2}A_{\mu}^{2}=\Delta \tag{10}$$

Which expresses the photon propogator with the energy momentum of the electron particle in balance with the gap; the energy momentum squared; and that of the gauge connection energy momentum due to the curved space; known as a 'field momentum energy' of spin. This exists as spin is an intrinsic kinetic energy momentum of the subatomic particle known as the electron. So; here we define Γ_{μ} as:

$$\Gamma_{\mu}=\partial_{\mu}\log\gamma^{\nu} \tag{11}$$

Leaving for the gap on the right hand side:

$$\Delta=2m\gamma^{\mu}D_{\mu}+im^{2}-\Gamma_{\mu}^{2} \tag{12}$$

And the equation for the left hand side as the relation re-written from before and above on $\Psi_{A}\Psi_{B}$:

$$(\partial_{\mu}+A_{\mu})(\partial_{\mu}-A_{\mu})\Psi_{A}\Psi_{B}=\Delta \tag{13}$$

Which is the Klein Gordon equation for two photons of energy momentum gap equivalent up to a discrepancy to the lost field energy momentum of the electrons forming a pair.