An approach for analyzing time dilation in the TSR (v9. 2018-06-22)

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Abstract. We present an approach to analyze time dilation in the theory of special relativity, starting out from the Lorentz transformation. The concepts of symmetry and simultaneity are essential in these investigations. We also stress the importance of the observational principle, *i.e.*, the location of clocks used for the clock comparisons of the two reference frames (RFs) moving relative to each other. For a specific RF we may follow just a single clock (SC), or we can use multiple clocks (MC) to follow a single clock on the other RF. In addition to these standard cases, we consider an approach, utilizing an auxiliary RF, which – in combination with symmetry considerations – provides a consistent definition of 'simultaneity at a distance'. We use the overall approach to discuss the travelling twin paradox; (providing corrections of previous versions).

Key words: Lorentz transformation, symmetry, simultaneity, auxiliary reference frame, perspective of RF, travelling twin.

1 Introduction

The present work presents basic features of time dilation in special relativity. In particular, we explore the use of the Lorentz transformation (LT) for one spatial parameter. We stress the importance of specifying which clocks to apply for the clock comparisons between the two inertial reference frames (RFs). We will refer to the specification of clocks as the *observational principle*, and a graphical presentation of the LT proves useful to get the overall picture.

Symmetry is important when we discuss time dilation within special relativity. It may appear paradoxical that we have complete symmetry between the two RFs, but at the same time we will 'take the perspective' of one of them, apparently destroying symmetry. For instance the common statement, 'moving clock goes slower', represents such an apparent paradox, which is handled somewhat differently in the literature. Some authors apply the expression 'as seen' by the observer on the other reference system, perhaps indicating that it is an apparent effect, not a physical reality. Actually, Giulini, [1] in his Section 3.3 states: 'Moving clocks slow down' is 'potentially misleading and should not be taken too literally'. Others stress that 'everything goes slower' on the 'moving system', not only the clocks; truly stating that the time dilation represents a physical reality also under the conditions of special relativity, (*i.e.* no gravitation *etc.*) However, for instance Pössel [2] points out that it is the procedure related to clock comparison (*i.e.* the 'observational principle') that decides which reference system has the 'time' which is 'moving faster', resp. 'slower'.

Simultaneity at a distance is another important issue. Using synchronized clocks of a specific RF we can define simultaneity of events 'in the perspective' of any RF, but the various RFs will give different specifications of simultaneity. However, we will introduce an *auxiliary RF,* which – in combination with symmetry requirements – provides a useful tool for specifying simultaneity at a distance.

We present an approach to meet these challenges in the analysis of time dilation, and apply this approach in a discussion of the 'travelling twin' example, (under the strict conditions of TSR).

Actually, some authors also question the validity of the theory of special relativity (TSR) and the LT, (*e.g.* see McCausland [3], Phipps [4], Robbins [5]); and perhaps we should include Serret [6]. In particular Ref. [3] reviews various controversies on the topic (related to H. Dingle) during several decades, and gives many references. Ref. [5] also treats the Bergson-Einstein controversy, dating back to 1922. The scope of the present work, however, is more restricted, accepting the validity of the TSR as a premise. Our objective is mainly to investigate the logical implications of the LT and thereby provide an approach for analyzing relative time and simultaneity within the framework of the TSR.

2 The Lorentz transformation and some special cases

We first specify basic notation and assumptions, and then discuss some aspects of the LT.

2.1 Basic notation and assumptions

We start out from the standard theoretical experiment: Two co-ordinate systems (inertial reference frames), pointing in the same direction are moving relative to each other at speed, v. We consider just one space co-ordinate, (x-axis), and investigate the relation between space and time parameters, (x, t) on one RF, and the corresponding parameters (x_v, t_v) on the other. Thus, we have the following *basic simultaneity*: At time (*i.e. clock reading*), t and position, x on the first system, we observe that time equals t_v and position equals x_v on the other. We further make the following specifications:

- There is a complete *symmetry* between the two co-ordinate systems.
- On both RFs there is an arbitrary number of identical, synchronized clocks, located at any position where it is required.
- We choose the *perspective* of one of the RFs, say K. *Simultaneity in the perspective* of K, means that all clocks on this RF is considered to show identical values, t. Further, the clock readings on another RF are given as a function of the position, x on this K. Then we refer to K as the *primary* system, and the other RF as the *secondary* system.
- Throughout we let SC refer to a RF utilizing a 'single clock' (or the 'same clock'), for the time comparisons with other RF(s). Similarly, MC will refer to a reference frame, which utilizes 'multiple clocks' (at various locations) for time comparisons.

2.2 The Lorentz transformation

Our two RFs are here denoted K_1 and K_2. Now let K_1 be the primary RF. Then the parameters of K_1 are denoted (x, t), and the parameters of K_2 are denoted (x_v, t_v). In this notation the LT takes the form

$$t_v = t_v(x) = \frac{t-(vx)/c^2}{\sqrt{1-(v/c)^2}} \tag{1}$$

$$x_v = \frac{x-vt}{\sqrt{1-(v/c)^2}} \tag{2}$$

These formulas include the length contraction along the x-axis (inverse Lorentz factor):

$$k_x = \sqrt{1-(v/c)^2} \tag{3}$$

Fig.1 provides an illustration of this time dilation formula. Here we give the clock reading ('time') on both RFs in the perspective of K_1. So the figure illustrates an instant when time equals t all over this RF. The horizontal axis gives the position x on K_1 at which the clock measurements are carried out. The vertical axis gives the actual clock readings. So as time on K_1 equals t at any position, x, the clock readings on K_2 at this instant, $t_v = t_v(x)$, depend on x; see decreasing straight line. We will often write $t_v(x)$, rather than t_v to stress this dependence on x. Note that the time dilation is provided by the slope of this line, entirely given by v, (*cf.* (3)).

2.3 Standard special cases (observational principles)

Now focusing on time dilation, *cf. eq.* (1), there are various interesting special cases (*observational principles*). First, if a specific clock located at the origin $x_v = 0$ on K_2 is compared with the passing clocks on K_1. These clocks on K_1 must have position $x = vt$, and we directly get the relation

$$t_v(vt) = t\sqrt{1-(v/c)^2} \tag{4}$$

which equals the 'standard' time dilation formula. Further, when a specific clock at the origin, $x = 0$, on K_1 is used for comparisons with various passing clocks on K_2, we must choose $x = 0$ and thus get

$$t_v(0) = t \,/\sqrt{1-(v/c)^2} \tag{5}$$

as the relation between t and t_v. We specify the special cases (4), (5) in Fig. 1; also see Ch. 3 below.

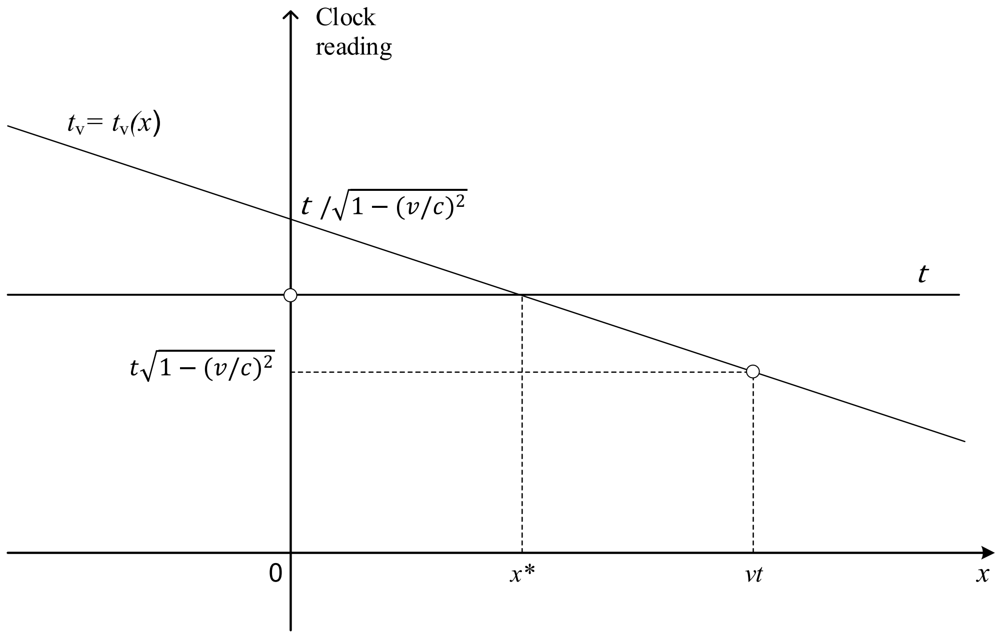

Figure 1. Clock readings in the perspective of K_1. Thus, 'time' all over K_1 equals, t. while clock readings, t_v(x) on the other RF, K_2 is given as a function of the position, x on K_1. (Two small circles indicate the origins of the RFs, where the 'basic clocks' are located.)

2.4 The symmetric case

There is another interesting special case of the LT, (1), (2). We can ask which values of x (and thus x_v) will result in t_v(x) = t. We easily find that this equality is obtained by choosing $x = x^*$, where

$$x^* = \frac{c^2}{v}\left(1-\sqrt{1-(v/c)^2}\right)t = \frac{vt}{1+\sqrt{1-(v/c)^2}} \tag{6}$$

In other words (*cf.* Fig. 1):

$$t_v(x^*) = t$$

Further, by this choice of x we get $x_v = -x^*$. Thus, at this position also x_v = - x, providing a nice symmetry. So when we choose the observational principle, (6), then absolutely everything is symmetric, and it should be no surprise that we get $t_v = t$. Now observe that we can write (6) as

$$x^* = w^* t \tag{7}$$

where

$$w^* = \frac{c^2}{v}\left(1-\sqrt{1-(v/c)^2}\right) \tag{8}$$

This is a velocity, and basically we interpret it as the speed at which the point of observation 'moves' along the x-axis; (note that we say 'point of observation' and not 'clock'; as we apply different clocks on K, and thus we are not referring to a moving object). However it can also refer to a velocity of a moving object. According to standard results of TSR, *e.g.* Refs. [7] - [9], two velocities v_1 and v_2 will add up to v, given by the formula

$$v = v_1 \oplus v_2 \overset{\text{def}}{=} \frac{v_1+v_2}{1+\frac{v_1}{c}\frac{v_2}{c}} \tag{9}$$

So if we now define the operator ⊕ by (9), it follows from (8) that

$$w^* \oplus w^* = v \tag{10}$$

This says that if B moves relative to A at velocity w^*, and C moves relative to B at velocity w^*, then C will move relative to A at velocity v. This is an additional feature of w^* which we will also use later.

2.5 A note on symmetry and initial conditions.

We recall that Fig. 1 provides clock readings on K_2 as a function of the location (and a specific 'time') on K_1. However, this is not a symmetric representation of these results. It gives the corresponding clock readings *in the perspective of* K_1 (as we assume clock readings to equal t all over K_1). In the perspective of K_2, (now letting (x, t) be the parameters of K_2, the picture would obviously be (replacing v by $-v$) as given in fig. 2(a), there replacing v by $-v$.

Fig. 2(b) indicates how we could provide a completely symmetric presentation of relating the clock readings at identical positions. We may introduce a new auxiliary RF as our primary RF, K, and let the 'original' RFs move relative to this at speed w^* and $-w^*$, respectively, (which ensures that the original RFs still move relative to each other at speed, v). So in Fig. 2(b) the comparison of clock readings on K_1 and K_2 are done at positions, x on K when all clocks on K reads t. As our main argument for providing figures like Figs. 1-2 is to illustrate simultaneous clock readings at various positions, we find Fig. 2(b) most informative. Note that we also in Fig. 2(b) have inserted some dashed vertical lines to indicate simultaneous clock readings. However, at this point we do not indicate specific x-values on K.

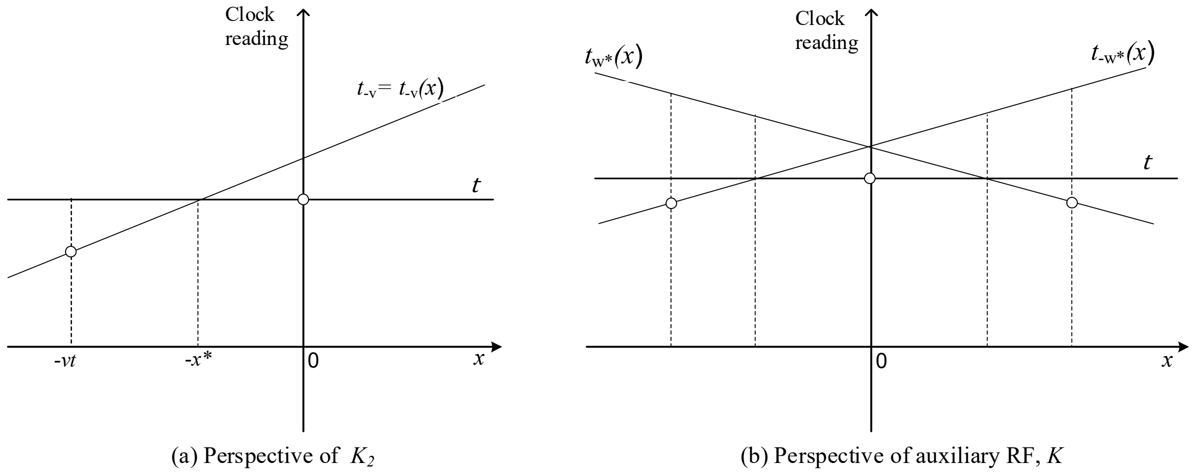

Figure 2.Clock readings. (a) In the perspective of K_2. (b) In the perspective of an auxiliary RF.

We point out that the relations as illustrated in Figs. 1-2 are valid for any $t \geq 0$; thus also for $t = 0$. That is the figures are valid also for $t = 0$, and a considerable part of the secondary RFs will have negative clock readings when $t = 0$.

At this point we add a comment on the initial conditions, (*i.e.* at our 'point of initiation'). There are three such conditions: (i) Synchronization) of clocks on K_1, (carried out in advance), (ii) Synchronization of clocks on K_2, and (iii) synchronization of one clock at K_1 and one clock on K_2. We carry out the first two synchronizations using light rays, but the third one is performed utilizing two clocks simultaneously located at the same position ('basic simultaneity'). Usually we apply the clocks at the origins of the RFs, simultaneously being at the same location, $x = 0$, when $t = 0$. However, the synchronization at $t = 0$ could as well be carried out at another position, (time dilation always given by the slope of the lines).

3 "The moving clock": SC *vs*. MC

We now take a closer look at the observational principles given by (4) and (5). These *eqs.* relate clock readings when one of the RFs has a single clock located *at the origin* of its RF. Thus, we can combine *eqs.* (4) and (5) into one single formula:

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2} \qquad (11)$$

Here t^{SC} is the clock reading of the specific clock located at the origin (either of K_1 or K_2). Further t^{MC} is the clock reading of the clock at the same location, but on the other frame. So, for instance, t^{SC} replaces $t_v(x)$ in *eq.* (4) and *t* in *eq.* (5); while t^{MC} replaces *t* in *eq.* (4) and $t_v(0)$ in *eq.* (5), This is clearly demonstrated in Fig. 1. At both these locations it is the SC that gives the lower value.

Note: Extended notation

Note that the notation of *eq.* (11), using t^{SC} and t^{MC}, shall not replace the more general notation, $t_v = t_v(x)$ and *t,* as used in (1). The new terms t^{SC} and t^{MC} shall just help us to realize the symmetry of two specific positions. Thus, we will continue to use the general notation, but at the two positions, $x = 0$ and $x = vt$, we may in addition apply the essential result, (11).

Also, observe that we in (11) have dropped the subscript, *v* in the time parameters. This just means that (11) is valid irrespective of which RF is chosen as the primary. However, we may add a subscript *v* to either t^{SC} or t^{MC}, to indicate which systems we choose as the primary/secondary RF. Thus, using the notation (t_v^{SC} , t^{MC}) means that we 'follow' a fixed SC at the origin of the secondary RF, and using notation (t^{SC}, t_v^{MC}) means that we 'follow' a SC on the primary RF.

Before we leave (11) some further comments are relevant. First note that (11) is more than an efficient way to write the two *eqs.* (4) and (5). By *eq.* (11) we stress that (4) and (5) actually represent the same result, and is thus more informative than (4) and (5) alone. Actually, the choice of which RF shall apply a SC is crucial, and it introduces an asymmetry between the two RFs.

We also note that observers on both RFs will agree on the result (11). Thus, it is somewhat misleading to apply the phrase 'as seen' regarding the clock readings. All time readings are objective, and all observers (observational equipment) on the location will 'see' the same thing.

Further, we have the formulation 'moving clock goes slower'. It is true that an observer on one RF, observing a *specific clock* (on the other RF) passing by, will see this clock going slower, when it is compared to his own clocks. However, we could equally well take the perspective of the single clock, considering this to be at rest, implying that the clocks on the other RF are moving. The point is definitely not that clock(s) on one RF are moving and clocks on the other are not. It is the observational principle that decides which of the two clocks initially at the origin, which we observe to move slower. Therefore, the talk about the 'moving clock' could be rather misleading.

We should stress that relations like *eq.* (11) are well known. For instance, *eq.* (3-1) of Smith [10] equals our *eq*. (11), and our concepts SC and MC correspond to the concepts of 'proper' and 'improper' time used in that book. Actually, 'coordinate time' is another term used for 'improper time'. However, we will further highlight the insight provided by (11).

We also note that it is not required to have one RF to be SC (having 'proper' time), and the other to be MC (having 'improper' time). We may at the same time have clock(s) on *both* RFs observed to 'go slower'. The equation (11) just says that if we follow a specific clock (here located at the origin), we will observe that this goes slower than the passing clocks on the other RF.

This point in my opinion also gives an answer to 'Dingle's question'. Dingle [11] raises the question of symmetry regarding the travelling twin paradox: "Which of the two clocks in uniform motion does the special theory require to work more slowly? This is an important question, which according to the discussion in McCausland [12] so far has not been given a satisfactory answer.

However, it is clearly not the case that the clock(s) on *one* of the two RFs go(es) slower than the clock(s) on the other. We could very well choose to follow *both* the two clocks being at the origin at time 0; which will give that both RFs have a clock 'going slower'. So, the result is fully symmetric with respect to the two RFs. The question is not which RF has a clock that 'goes slower'; it is rather which observational principle we chose.

4 Using an auxiliary reference frame of symmetry

We proceed to investigate the important question of simultaneity at a distance. We primarily elaborate on the fundamental result (11). However, we will now treat the two reference frames in a symmetric way, and denote them K_1 and K_2. In addition, we introduce an auxiliary RF, K, which we chose as our primary RF; thus, we make our observations 'in the perspective' of this auxiliary K.

To get a completely symmetric situation we let K_1 have velocity $-w^*$ with respect to K, and K_2 have velocity w^* with respect to K. Here we define w^* by (8), and according to (10) it then follows that the velocity between K_2 and K_1 equals v.

Next we specify the observational principle. We choose to operate the auxiliary reference frame as MC, and so both K_1 and K_2 are SC.[1] Thus the single clocks at the origins of K_1 and K_2 are at any time compared with various clocks along K. Now we can apply the relation (11) between the auxiliary RF, K and the two RFs K_1 and K_2, giving, (*cf.* the Note, *Alternative notation* in Chapter 3):

$$t_{w^*}^{SC} = t^{MC}\sqrt{1-(w^*/c)^2} \tag{12a}$$

$$t_{-w^*}^{SC} = t^{MC}\sqrt{1-(-w^*/c)^2} \tag{12b}$$

Here $t_{-w^*}^{SC}$ is the clock reading of the clock at the origin of K_1, and $t_{w^*}^{SC}$ is the reading of the clock at the origin of K_2. It directly follows that

$$t_{-w^*}^{SC} = t_{w^*}^{SC} \tag{13}$$

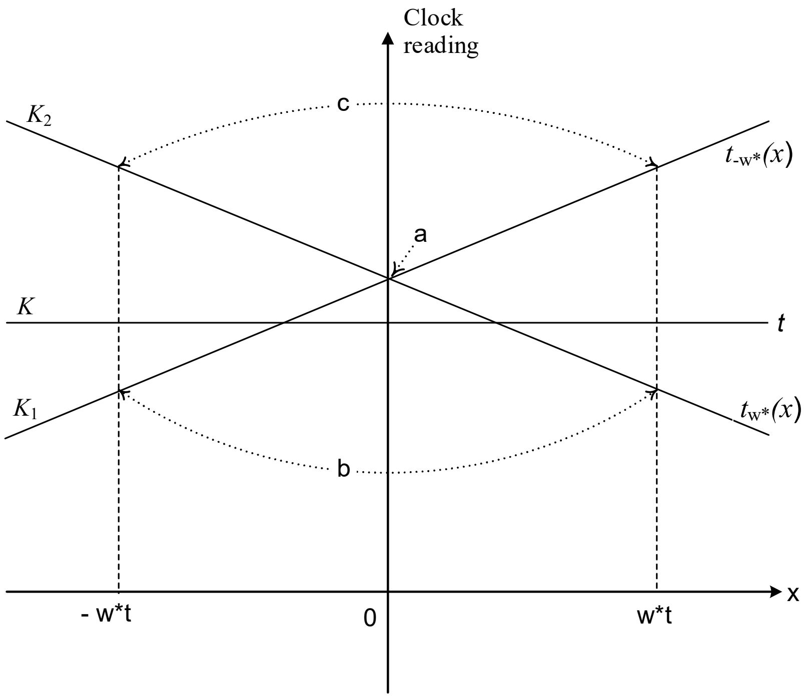

Figure 3 Clock measurements ('time') in the perspective of the auxiliary reference frame, *K*, where the reference frames K_1 and K_2 have velocity $-w^*$ and w^*, respectively, relative to *K*. (Here *w* represents 'position' on *K*.)

[1] Alternatively we could let the auxiliary reference frame, K operate as SC; but then we would just obtain the same result as given in Section 2.5, and this is therefore of limited interest.

Thus, in the perspective of K these are now the simultaneous readings of the clocks located at the origins of the two 'main' reference frames, K_1 and K_2, (moving relative to each other at speed, v).

We illustrate these results in Fig. 3. Like Fig 2(b) this gives a symmetric picture with respect to two RFs, by also introducing a third RF, K, and taking the perspective of this new one. Fig. 3 gives a snapshot of the clock measurements at an instant when all clocks on K reads t; *cf.* horizontal line marked t.

The horizontal axis refers to the positions, x on the auxiliary RF, K. The lines $t_{-w^*}(x)$ and $t_{w^*}(x)$ give the time measured on clocks on K_1 and K_2, respectively, as a function of x (and time t) on K. We focus on three positions on K, *i.e.* x equal to $-w^*t$, 0 and w^*t. These three values correspond to the origins of the three RFs, K_1, K and K_2, respectively.

We note that the letter a in this figure indicates the simultaneous clock readings of reference frames K_1 and K_2, observed at the origin of K. At this position the clocks on K_1 and K_2 show the same time, and are simultaneously located at the same location, $x = 0$. For *these* measurements the reference frame K is a SC system, and its clock will appear slower than the corresponding clocks on K_1 and K_2: we observe the line t falling below the point a.

As stated, the two points marked b correspond to the SC time readings at the origins of K_1 and K_2. Thus, using the extended notation introduced in Chapter 3, we have $t_{-w^*}^{SC} = t_{-w^*}(-w^*t)$ and $t_{w^*}^{SC} = t_{w^*}(w^*t)$. According to our result (13), these are identical. Thus, the clock on K_1 at the position $-w^*t$ and the clock on K_2 at the position $x^* = w^*t$ give identical clock readings.

These origins have moved apart after time 0; and the events that these two clock readings are equal are not simultaneous neither in the perspective of K_1 nor in that of K_2. However, *eq.* (13) tells that *in the perspective of the auxiliary RF* we have two simultaneous events. When we also have this symmetric situation, this simultaneity 'in the perspective of' becomes interesting, and not surprising. Rather, I would postulate that this symmetric 'simultaneity at a distance' indeed represents a valid form of simultaneity. This is not a strong assumption, but rather a consequence of the complete symmetry we have postulated. Claiming that the one of the two events b would occur prior to the other would represent a contradiction.

In conclusion, this is the most significant result obtained by using the auxiliary RF: We manage to establish a simultaneity of events at K_1 and K_2 'at a distance'. It is a key question in a proper handling of time dilation to achieve this. We also refer to the further discussion on simultaneity in Hokstad [13].

Finally, Fig. 3 also have two clock readings marked with the letter c. These exhibit the same type of symmetric simultaneity as the points b. The only difference is that the time readings at c will *not* correspond to the origins of the two main RFs, but to the location on the other frame at the same position.

The observant reader might realize that the experimental set-up given here (Fig. 3) is well suited for handling the travelling twin paradox, which we discuss in the next chapter.

5 Example: The travelling twin

We now utilize the framework provided in the previous chapters to analyze the so-called travelling twin example, which goes back to Langevin [14]. As stated for instance in Mermin [9] the travelling twin paradox shall illustrate that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together.

5.1 A numerical example. The outward travel and simultaneity.

Ref. [9] (in Chapter 10) gives the following numerical example: "If one twin goes to a star 3 light years away in a super rocket that travels at 3/5 the speed of light, the journeys out and back each takes 5 years in the frame of the earth. Since the slowing-down factor is $\sqrt{1-(3/5)^2} = 4/5$ the twin on the rocket will age only 4 years on the outward journey, and another 4 years on the return journey. When she gets back home, she will be 2 years younger than her stay-at-home sister, who has aged the full 10 years." So the claim is that the referred difference in ageing occurs during the periods of the journey with a constant speed; *i.e*. under the conditions of the TSR, (ignoring the acceleration/deceleration periods). After all, the whole argument relies on the Lorentz transformation! Thus, our discussion will restrict to the periods of constant velocity.

First looking at the travel *to* the star, I fully agree with the above claim; which is also illustrated in Fig. 4(a). Here K_1 is the RF of the earth/star (earthbound twin), and we have also introduced a RF, K_2 of the travelling twin; (We could equip him with additional rockets at appropriate (constant) distances and synchronized clocks to provide his RF.) The SC of the travelling twin will – when compared with passing clocks of the earth/star– observe that his own goes slower than these, and at arrival, the clocks show 4 and 5 years respectively. However, in exactly the same way the earthbound twin will observe that his clock goes slower than the passing clocks of K_2. In particular, when his own clock shows 5 years, the passing clock of his travelling twins RF show 5/0.8 = 6.25 years. This is just an example of Fig. 1.

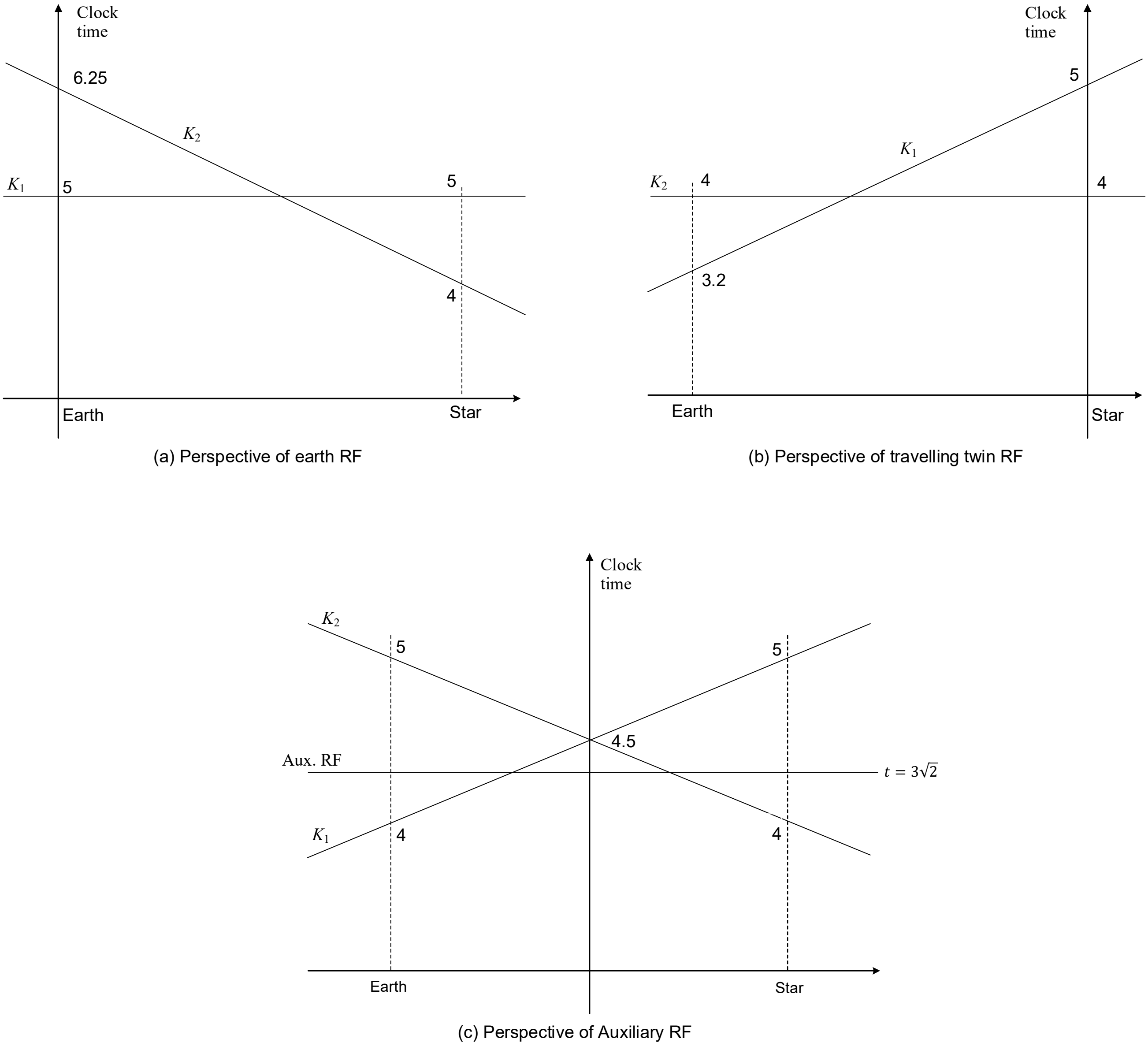

Figure 4 Clock readings by the arrival at the star, from different perspectives.

Fig 4(b) demonstrates that if we take the perspective of the travelling twin – and his earthbound twin moves relative to him at velocity ($-v$) – then his arrival at the star is 'simultaneous' with the event that the clock of his earthbound twin shows $4\cdot 0.8 = 3.2$ years. So they agree on the degree of time dilation at both locations (factor 0.8), but they dramatically disagree about simultaneity.

Therefore, in Fig. 4(c) we include the perspective of the auxiliary RF of symmetry introduced in Ch. 4. The detailed calculations are given in Appendix A.1, but we mention that this implies that both twins travel away (in opposite directions) from the origin of the auxiliary RF at speed $w^* = c/3$. Quite obviously, also here the clocks of K_1 and K_2 at the point of arrival at the star read 5 and 4 years, respectively. (These are actual clock readings, which will not change whatever perspective we choose, as long as we carry out the observation 'on location'.) So the difference here is the chosen symmetry, obviously giving that the clock readings of 5 years for the earthbound twin and 4 years for the clock of the RF to the travelling twin passing the earth are simultaneous with the arrival at the star. This is a fully symmetric situation, and we follow both twins having positions as a SC; *cf.* discussion in Ch. 4. From Fig 4(c) we also see that at the origin of the auxiliary RF, both K_1 and K_2 show time 4.5 years at this moment; while we can show that the clock time on the auxiliary RF equals $3\sqrt{2} \approx 4.24$ years (see Appendix A.1).

Thus, whatever perspective we choose, the clock of the travelling twin shows just 4 years when he arrives, while the clock of the RF of the earth/star at this location shows 5 years. The choice of perspective will not affect this fact.

However, there is also a question of simultaneity. How shall we specify the event on earth being simultaneous with this event of arrival at the star? In addition to the above argument, we give a rather thorough discussion in Appendix A.2. As seen, we strongly argue that the consistent answer is that the arrival at the star is simultaneous with the clock on the earth showing 4 years. Actually, considering the three answers suggested by Fig. 4(a)-(c), it is the result of Fig. 4(c) only, which gives a symmetric solution, and as we have chosen a symmetric model (with respect to the two RFs) we should also choose definitions (of simultaneity) which give a symmetric answer to our problem. We should explain any deviation from symmetry, but here no such argument is given. Actually there are authors that claim that the arrival to the star is 'simultaneous with' the clock on the earth showing 5 years. I consider this rather flawed. Further, see discussions on simultaneity in my related paper, [13].

5.2 The 'reunion' of the twins[2]

We have concluded on two questions. First, The two clocks on the star will show 4 and 5 years by the arrival, independent of perspective. Next this event on the star should be considered 'simultaneous' with the event on the earth that the clock of the earthbound twin shows 4 years and that of the travelling twin shows 5 years. These are two essential results. However, they may look contradictory; so what can we conclude regarding the clocks by the 'reunion' of the twins at the earth?

The return travel will require an additional line of arguments. In order to model the return travel, we introduce a new RF, K_3 at velocity –v relative to K_1; (K_3 having the same orientation of its x-axis as the other two RFs). Further, at the exact moment when the travelling twin arrives to the star, this new RF has its origin at the same location, and we synchronize these two clocks, (showing 4 years). Of course, also the RF K_3 - like K_1 and K_2 - have its synchronized clocks, as required.

This means that in our model considerations, we refrain from treating the return of the twin himself, but restrict to consider the return of his clock, (or actually, a clock synchronized with his clock 'on location'). Then we will ask what this third clock (on K_3) will show by its arrival to the earth. We realize that in order to apply the model of TSR we must make some idealizing assumptions, and this assumption keeps the essence of our thought experiment, and it is also sufficiently precise to provide the answer to our main question, (as provided by the TSR model).

[2] Section 5.2 in the previous version of the paper was flawed, and has been completely rewritten.

Thus, our model for the return travel involves the two RFs, K_1 and K_3. Now we must take a thorough look at the initiating conditions for our new RF, K_3. As stated, we synchronize the clock at its origin with that of the travelling twin at his arrival to the star; *i.e.* reading 4 years, and at that time and location the clock on K_1 shows 5 years.

Now we get a figure similar to Fig. 2 to illustrate the return travel. We do not take all details here, but Appendix A.3 provides a discussion to show how we can handle this in a proper way; (in particular see Fig. 5 in that Appendix). In particular, we note the distinct discontinuity of the clock readings of the travelling twin's RF along K_1 at the moment when he switches from using the RF, K_2 to the RF, K_3.

The arguments of Appendix A.3 may seem unnecessary complex, as the conclusion directly follows from the fundamental result that a single clock (SC) – when compared to passing clocks on another RF- always goes slower at the specified rate, (Chapter 3). However, it is somewhat reassuring to see this being confirmed in this way.

Finally, there are of course various ways to bring the twins together, and this will determine which will be the younger/older. An alternative to the standard approach, discussed above, of simply turning the travelling twin, is to let the earthbound twin set out to the star, which raises the question of *simultaneity*; *i.e.* at what time shall he start *his* journey. This choice will of course determine the ages of the twins by reunion, and, as we have seen, there are several solutions regarding this simultaneity.

5.3 Concluding comments on the travelling twin

This so-called paradox is indeed thoroughly discussed in the literature. Shuler [15] informs that about 200 per reviewed academic papers with *clock paradox* or *twin*(*s*) *paradox* in their title can be identified since 1911, most of them since 1955. He comments: "An outside observer might reasonably conclude there is deep conviction that matter *should have been* settled, along with a nagging suspicion that *it is not.* Further: "Though the correct answer has never been in doubt the matter of *how to explain* the travelling twins appears be far from settled". He also refers to the following statement: "On the one hand, I think that the situation is well understood, and adequately explained in plenty of textbook. On the other hand… there are complementary explanations which take different points of view on the same underlying space-time geometry (though, alas, the authors don't always seem to realize this, which rather undermines my assertion that the effect is well enough understood)".

Also Debs and Redhead [16] give a thorough discussion on this case. They refer to the two asymmetries that have been the basis for most of the standard explanations. The first group of arguments focuses on the effect of different standards of simultaneity, and secondly one can designate the acceleration as the main reason for the differential aging. However, regarding the last group of arguments they write "… since we are dealing with flat space-time, we regard the reference to general relativity in this context as decidedly misleading"; a statement in which I agree.

A main conclusion of our discussion above is that the outward travel is rather trivial: Both twins will observe that his own clock goes slower (than passing clocks on the twins RF). Thus, by the arrival to the star, the 'travelling twin' will observe that his clocks shows 4 years, as compared to reading 5 years of the clock of the earthbound system (*i.e.* the star).

The return travel is a bit more challenging. However, we introduce a new RF going in the 'opposite direction', giving a completely new set of clock readings for the RF of the travelling twin; (the old and new clock reading coincides just at one point: the star). This distinct discontinuity of the clock readings of the travelling twin's RF is sufficient to explain that the returning clock reads 8 years, as that on the earth reads 10 years; (following our numerical example).

This result may seem contradictory to our claim regarding 'simultaneity at a distance': From symmetry reasons we have claimed that 'clock on the star reads 4 years' is simultaneous with 'clock on the earth reads 4 years'. However, this does not seem that crucial; it is the changing of the travelling twin's RF, which completely explains the ageing difference (irrespective of simultaneity).

Note that we focus on the clock readings, and not the actual ageing process; in particular, as we cannot carry out this sudden change of RF in practice, (we should need a third twin located on this third RF). Therefore, it is not easy to get the answer valid under the idealised assumptions of TSR model.

6 Summary and conclusions

Starting out from the Lorentz transformation (LT) we discuss the approach for analyzing time dilation in the theory of special relativity (TSR). We also comment on the phrasing used for communicating essential aspects of the theory.

First we stress the importance of the *observational principle* for the observed time dilation. Here the specification of the observational principle means that we state the location of the clocks used for time comparisons of the RFs. In order to give the overall picture it is also useful to provide a *graphical presentation* of corresponding clock readings at various positions of the involved reference frames (RFs). Further, we suggest writing out the main observational principle ('time dilation' formula) as

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2}$$

Here t^{SC} is the clock reading of a fixed 'single clock' (SC), and t^{MC} is the clock reading of the various 'multiple clocks' (MC) passing by on the other RF. This formulation is informative; stressing that time dilation is not related to the RF but to the choice of observational principle. In particular we find the phrase 'moving clock goes slower' potentially misleading. Multiple clocks on any RFs will observe that any single (specific) clock on the other RF goes slower. This has nothing to do with whether we consider the SC or the MCs to be the moving one(s). Although this being well-known phenomenon, we like to stress this fact

Another observational principle is to perform all clock comparisons at the midpoint between the origins of the two main RFs. At this position, we get identical clock readings at the two frames. Therefore, when we apply this observational principle - being symmetric with respect to the two RFs - we also get a symmetric result, (no time dilation!).

This midpoint is also relevant if we include an *auxiliary RF*. A main reason for introducing this is to provide a sensible definition of simultaneity at a distance. We obtain this by taking the perspective of this auxiliary RF, and then follow a SC on both the main RFs,

We also apply the suggested approach to the travelling twin paradox. The travelling twin's abrupt change of RF by his arrival to the star implies an asymmetry, which explains the difference in ageing of the two twins. We also discuss the question of simultaneity at a distance: Which event (clock reading) on the earth is simultaneous with the arrival (and turning) of the travelling twin?

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Appendix A Some material regarding the travelling twin example

We here present some additional material regarding the travelling twin example, thoroughly discussed in Ch. 5. We first provide details of the numerical calculations. In this Appendix we let

t_1 = reading on the clock of the earthbound twin

t_2 = reading on the clock of the travelling twin

Similarly, the distance between earth and the 'star' is denoted $x_1 = 3$ light years, and since the rocket has speed, $v = (3/5)c$, we get $\sqrt{1-(v/c)^2} = 4/5$. Further, $w^* = c/3$ (*eq.* (8)) is the speed between any twin and the auxiliary RF. Further, $\sqrt{1-(w^*/c)^2} = \frac{2}{3}\sqrt{2} \approx 0.94$.

A.1 The numerical calculation

It directly follows that in the RF of the earth/star, the rocket reaches the star at time, $t_1 = x_1/v = 5$ years. Further, the LT gives that at the arrival at the star the clock reads $t_2 = t_1\sqrt{1-(v/c)^2} = 4$ years. So it does follow from the LT that the two clocks at the star shows 4 and 5 years at the arrival. We note that the above presentation describes the travelling twin as a 'SC system', and so in this description the earthbound twin is located on a 'MC system'. Therefore, we can just look at *eq.* (11) and Fig. 1 to get this result. The results are illustrated in Fig. 4(a) and 4(b); also showing the 'simultaneous' clock readings on the earth; *cf.* Apendix A.2 below.

Now we elaborate on this numerical example when we introduce an auxiliary symmetric RF, see illustration in Fig. 4(c), and also Fig. 3, which represents the situation when the travelling twin has reached his point of destination. We apply *eq.* (1), here inserting w^* for *v*, and obtain the following clock readings of the three RFs. Observe that we utilize the notation of Fig. 3 with numerical values as inserted in Fig. 4(c):

i. The auxiliary reference frame (primary). Time is constant, *t*, (see horizontal line in figures).
ii. Earthbound twin. Time as function of *x*: $t_{-w^*}(x) = (\sqrt{2}/4)\cdot(3t + x/c)$.
iii. Travelling twin. Time as function of *x*: $t_{w^*}(x) = (\sqrt{2}/4)\cdot(3t - x/c)$.

We here stress that *x* in these expressions represent the position of the auxiliary RF. In Fig. 3 we now let the observational point *b,* correspond to position $x = w^*t = ct/3$. The clock of the travelling twin at his arrival shows 4 years. Thus $t_{w^*}(tw^*) = 4$, which gives $t = 3\sqrt{2} \approx 4.24$ years. Thus, we find that at this time (of the arrival)

$$t_{w^*}(x) = 4.5 - (\sqrt{2}/4)\cdot(x/c).$$

(and a similar expression for $t_{-w^*}(x)$). Thus we have completely specified the clock readings by the arrival, in the perspective of the auxiliary RF. Further by inserting $x = 0$ it directly follows that the point *a* corresponds to 4.5 years. Similarly, the point *c* of the figure, ($x = ct/3$; $t = 3\sqrt{2}$), will corresponds to 5 years; in full agreement with the given example.

A.2 Simultaneity at a distance

We consider the following question: Which event on the earth should we consider simultaneous with the arrival of the travelling twin of the star. The LT does not give a definite answer regarding such a simultaneity 'at a distance', and thus we elaborate on the main options regarding this simultaneity.

Recall that we assume there is also of a RF of the travelling twin with the required number of clocks. Say, he is equipped with rockets at appropriate and fixed distances from his own rocket, all moving with constant speed in the same direction as himself, and all equipped with a synchronized clock showing time, t_2. Whether this is practically feasible is not relevant here. We refer to the model of the TSR, and investigate what this theory tells about clock readings, *if* we provide such an arrangement.

To proceed, we also introduce a symmetric auxiliary RF, *K*, with velocities $\pm w^*$, respectively, relative to the RFs of the two twins; (with w^* given in (8)). Then we can consider simultaneity in the perspective of each of these three RFs. Starting with the arrival at the star, where $t_2 = 4$, $t_1 = 5$, we will identify the simultaneous event on the earth from these three perspectives; also illustrated in Fig. 3.

We present the result in Table 1. Note that on the earth it is the earthbound twins clock that acts as SC, that is here t_1 being a SC time reading, and t_2 a MC time reading. Thus, *eq.* (11) gives the result, $t_2 / t_1 = 1/0.8 = 1.25$, for all observations on the earth, whatever instant we consider after departure. Therefore, at this location it is the clock on the earth that always 'goes slower'.

First, in the perspective of the travelling twin, the clock reading of his own clock equals 4 years. So when the clock of his RF (showing 4 years) passes the earth, the clock on the earth reads $0.8{\cdot}4 = 3.2$ years; see perspective 1 in Table 1.

Next, in the perspective of the earthbound twin, we have calculated that his clock located at the star, reads 5 years by the arrival of his twin. But when his own clock on earth shows 5 years, the passing clock of the travelling twin's reference frame then shows $5{\cdot}1.25 = 6.25$ years; see perspective 2. (So if this is the relevant answer, we should expect the return of the twin brother after 12.5 years.)

Table 1. Various clock readings (light years) at/on the earth, *potentially* 'simultaneous to' the arrival of the travelling twin at the star; (so, at the star we have $t_1 = 5$, $t_2 = 4$).

Clock reading at earth	Perspective of		
	1.Travelling twin	2.Earthbound twin	3.Auxiliary RF (symmetric)
Earthbound twin system (***t_1***)	3.2	5	4
Travelling twin system (***t_2***)	4	6.25	5

The third possibility is to apply the perspective of the symmetric auxiliary RF. In this perspective, we treat both clocks belonging to the twins as SC. Then we get the following symmetric result regarding simultaneity: The arrival at the star occurs when both twins observe that their own clock shows 4 years, and the adjacent clock on the other RF shows 5 years. By these direct measurements, they observe that the other twin at this moment apparently has aged more than himself by a factor 1.25. This gives a completely symmetric and consistent answe. In addition to the symmetry, it is an important point here that in options 1 and 2 of Table 1 we directly follow the clock of just *one* twin, while we in option 3 follow *both* these clocks. (It is a feature of the auxiliary RF that it allows us to treat the RFs of *both* twins as SC, and thus to establish a symmetric 'simultaneity at a distance'.)

Appendix A.3 A note on the return travel

Fig. 5 below illustrates the result of introducing a third RF with 'opposite' direction; *cf.* the introduction of a RF, K_3 to bring back the clock of the travelling twin. Here we give clock readings in the perspective of the earthbound twin, now with parameters (t, x). The figure specifies the time $t = t_0$ (in our example 5 years) on K_1, and the position vt_0 on this RF corresponds to the location of the star.

Further, $t_v(x) = \frac{t-(vx)/c^2}{\sqrt{1-(v/c)^2}}$ gives the clock readings on the RF of the outward traveling twin (*i.e.* K_2). Next, $t^*(x)$ is the clock readings of the returning RF (*i.e.* K_3). We observe the distinct discontinuity when we change RF of the travelling twin from K_2 to K_3.

We see that at the given instant, $t^*(x)$ is chosen so that the clock reading at its origin coincides with that of the travelling twin, $t_v(x)$, (see Section 5.2). Further it is parallel to $t_{-v}(x)$, which corresponds to a RF setting out from the earth at the same instant and same speed as the travelling twin, but in the opposite direction. Now we simply give $t^*(x)$ as

$$t^*(x) = t_{-v}(x) + (t_v(vt_0) - t_{-v}(vt_0))$$

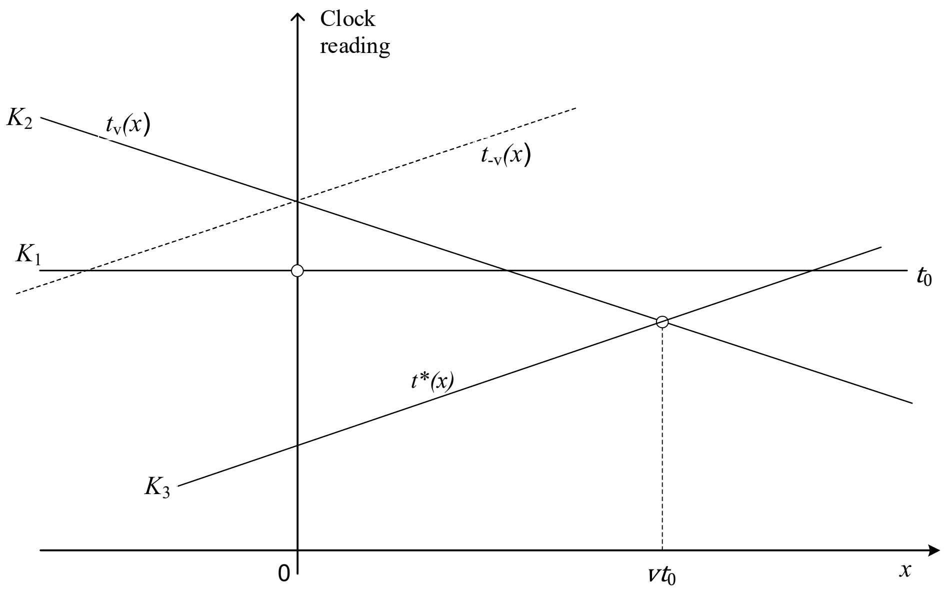

Figure 5 Clock readings at the point of return ($t = t_0$).

It quite easily follows that

$$t^*(x) = \frac{t + (vx)/c^2}{\sqrt{1-(v/c)^2}} - 2\frac{(v/c)^2}{\sqrt{1-(v/c)^2}}t_0$$

By this choice of time on K_3 we obtain that at time $t = t_0$ (and position $x = vt_0$) we have

$$t^*(\mathrm{v}t_0) = 2\sqrt{1-(v/c)^2}t_0;$$

This equals 4 years in our numerical example, which specifies that the travelling twin has aged 4 years by the start of his return.

Next, by inserting $t = 2t_0$ (*i.e.* the time of the return to the earth), and also letting $x = 0$ (*i.e.* specifying the position of the earth), we find that $t^*(x)$ equals

$$t^*(0) = 2\sqrt{1-(v/c)^2}t_0.$$

Thus the total time elapsed on K_2 and K_3 by the return, equals $2\sqrt{1-(v/c)^2}t_0$, (that is $2 \cdot 0.8 \cdot 5 = 8$ years in our numerical example); and the 'returning clock' shows just 8 years.