Newton's $E = mc^2$ Two Hundred Years Before Einstein? Newton = Einstein at the Quantum Scale

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August 7, 2017

Abstract

The most famous Einstein formula is $E = mc^2$, while Newton's most famous formula is $F = G\frac{mm}{r^2}$. Here we will show the existence of a simple relationship between Einstein's and Newton's formulas. They are closely connected in terms of fundamental particles. Without knowing so, Newton indirectly conceptualized $E = mc^2$ two hundred years before Einstein. As we will see, the speed of light (which is equal to the speed of gravity) was hidden within Newton's formula.

Key words: $E = mc^2$, energy, kinetic energy, mass, gravity, relativity, Newton and Einstein.

1 Did Newton Discover" $E = mc^2$ Two Hundred Years Before Einstein?

The Italian geologist and industrialist, Olinto de Pretto, speculated three years before Einstein that the old, well-known[1] formula $E = mv^2$ had to be equal to $E = mc^2$ when something moved at the speed of light, where v is the object's speed, c is the speed of light, m is the mass, and E is the energy. In his 1904 book Electricity and Matter", Thomson [1] presented what he called a kinetic energy formula for light, which he described[2] as $E = \frac{1}{2}mc^2$.

Einstein [2] is still likely the first to mathematically prove" the $E = mc^2$ relationship between energy and mass. Today, moreover, we know that the kinetic energy formula and rest mass formula are not the same. Here, however, we will show that Newton basically conceived the same mathematical relationship between energy and matter, as hidden in his gravity formula (see [3]) two hundred years before Einstein; if only he had simply known the radius" and mass of a fundamental particle.

The rest mass of any fundamental particle can be written as

$$m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}, \tag{1}$$

where $\bar{\lambda}$ is the reduced Compton wavelength. For example, the rest mass of an electron is given by

$$m_e = \frac{\hbar}{\bar{\lambda}_e}\frac{1}{c} \approx 9.10938 \times 10^{-31} \text{ kg}. \tag{2}$$

On this basis, we obtain the following relationship between energy mass:

$$E = mc^2 = G\frac{m_p m_p}{\bar{\lambda}}, \tag{3}$$

where m_p is the Planck mass (see [4]) and $\bar{\lambda}$ is the reduced Compton wavelength of the fundamental particle m (this is actually the particle's extended radius", see [5, 7]). That is, the rest energy embedded in any fundamental particle is equal to the gravity (energy) between two Planck masses separated by the reduced Compton wavelength of the fundamental particle of interest.

Moreover, the kinetic energy of a fundamental particle is given by

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[1] $E = mv^2$ was suggested by Gottfried Leibniz in the period 16761689.

[2] In the original formula he used notation $E = \frac{1}{2}MV^2$, but Thomson clearly stated where V is velocity with which light travels through the medium...".

$$E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 = G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}} - G\frac{m_p m_p}{\bar{\lambda}}. \tag{4}$$

And when $v << c$, we can use the first term of a series expansion and closely approximate the kinetic energy by

$$E_k \approx \frac{1}{2}mv^2 = \frac{1}{2}G\frac{m_p m_p}{\bar{\lambda}}\frac{v^2}{c^2}. \tag{5}$$

Furthermore, we must have

$$\begin{aligned} \frac{mc^2}{c^2} &= G\frac{m_p m_p}{c^2\bar{\lambda}} \\ m &= G\frac{m_p m_p}{c^2\bar{\lambda}}. \end{aligned} \tag{6}$$

And what we can call the rest force of a fundamental particle is given by

$$F = \frac{mc^2}{\bar{\lambda}} = G\frac{m_p m_p}{\bar{\lambda}^2}. \tag{7}$$

In other words,

$$Newton = \frac{Einstein}{r},$$

where r is the extended radius of the fundamental particle in question (the reduced Compton wavelength of that particle).

Moving on, the relativistic force must be

$$F = \frac{mc^2}{\bar{\lambda}\left(1-\frac{v^2}{c^2}\right)} = G\frac{m_p m_p}{\bar{\lambda}^2\left(1-\frac{v^2}{c^2}\right)}. \tag{8}$$

Based on Haug's recent research, the maximum velocity of a Planck mass particle (mass gap particle) surprisingly is zero. The Planck mass particle is at rest in any reference frame and therefore the only invariant particle. This can only happen if the Planck mass particle only lasts an instant (see [10, 8]). The Planck mass particle is the collision point between two photons (the building blocks of photons), and it only lasts for one Planck second as measured with Einstein-Poincare synchronized clocks. This means that the force for any particle moving at its maximum velocity must be

$$F = m_p a_p = m_p\frac{c^2}{l_p} = \frac{m_p c^2}{l_p} = G\frac{m_p m_p}{l_p^2}. \tag{9}$$

A Planck mass particle should not be confused with two Planck masses: we can have a heap of protons making up a Planck mass, yet this is not a Planck mass particle. A Planck mass particle has a reduced Compton wavelength of l_p. For non-Planck masses, it seems that one perhaps needs to perform a relativistic adjustment for gravity, but this is likely not the case for gravity between light particles (photons).

Furthermore, we must have the following relationship for relativistic momentum

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}\frac{v}{c^2}. \tag{10}$$

Based on this, the relativistic energymomentum relation can then be written as

$$
\begin{aligned}
E^2 &= p^2c^2 + (mc^2)^2 \\
E &= \sqrt{p^2c^2 + (mc^2)^2} \\
E &= \sqrt{\left(G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}\frac{v}{c^2}\right)^2 c^2 + \left(G\frac{m_p m_p}{\bar{\lambda}}\right)^2} \\
E &= \sqrt{G^2\frac{m_p^2 m_p^2}{\bar{\lambda}^2\left(1-\frac{v^2}{c^2}\right)}\frac{v^2}{c^2} + G^2\frac{m_p^2 m_p^2}{\bar{\lambda}^2}} \\
E &= G\frac{m_p m_p}{\bar{\lambda}}\sqrt{\frac{1}{1-\frac{v^2}{c^2}}\frac{v^2}{c^2} + 1} \\
E &= G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}\sqrt{\frac{v^2}{c^2} + \left(1-\frac{v^2}{c^2}\right)} \\
E &= G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}.
\end{aligned}
\quad (11)
$$

Table 1 summarizes the mathematical relationship between special relativity and Newtonian gravity (Newton-inspired formulas).

	Einstein = Newton
Mass	$m = G\frac{m_p m_p}{c^2\bar{\lambda}}$
Relativistic mass	$\frac{m}{\sqrt{1-\frac{v^2}{c^2}}} = G\frac{m_p m_p}{c^2\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}$
Energy	$E = mc^2 = G\frac{m_p m_p}{\bar{\lambda}}$
Relativistic energy	$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}$
Kinetic energy	$E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 = G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}} - G\frac{m_p m_p}{\bar{\lambda}}$
Kinetic energy ($v << c$)	$E_k \approx \frac{1}{2}mv^2 = \frac{1}{2}G\frac{m_p m_p}{\bar{\lambda}}\frac{v^2}{c^2}$
Force	$F = \frac{mc^2}{\bar{\lambda}} = G\frac{m_p m_p}{\bar{\lambda}^2}$
Relativistic force	$F = \frac{mc^2}{\bar{\lambda}\left(1-\frac{v^2}{c^2}\right)} = G\frac{m_p m_p}{\bar{\lambda}^2\left(1-\frac{v^2}{c^2}\right)}$
Relativistic energy–momentum relation	$E = \sqrt{p^2c^2 + (mc^2)^2} = G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}$

Table 1: The table shows some simple mathematical relationships between the Einstein special relativity formulas and Newton-inspired formulas.

Table 2 illustrates the relativistic limit for the Einstein and Newtonian formulas. This is based on the maximum velocity for anything with a rest mass $v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}$.

	Einstein = Newton
Relativistic mass	$m_{max} = \frac{m}{\sqrt{1-\frac{v_{max}^2}{c^2}}} = G\frac{m_p m_p}{c^2 \bar{\lambda}\sqrt{1-\frac{v_{max}^2}{c^2}}} = G\frac{m_p m_p}{c^2 l_p} = m_p$
Relativistic energy	$E_{max} = \frac{mc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} = G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v_{max}^2}{c^2}}} = G\frac{m_p m_p}{l_p} = m_p c^2$
Kinetic energy	$E_{k,max} = \frac{mc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - mc^2 = G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v_{max}^2}{c^2}}} - G\frac{m_p m_p}{\bar{\lambda}} = m_p c^2 - mc^2$
Relativistic momentum	$p = \frac{m v_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}} = G\frac{m_p m_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}\frac{v}{c^2}$
Relativistic force	$F_{max} = \frac{mc^2}{\bar{\lambda}\left(1-\frac{v_{max}^2}{c^2}\right)} = G\frac{m_p m_p}{\bar{\lambda}^2\left(1-\frac{v_{max}^2}{c^2}\right)} = G\frac{m_p m_p}{l_p^2} = \frac{m_p c^2}{l_p}$

Table 2: The table shows the relativistic maximum limit for energy, kinetic energy and force based on Haug's maximum velocity formula.

2 The Speed of Gravity and Light Hidden Within the Newton Gravitational Formula?

It is often assumed that the speed of gravity in Newton gravitational theory is instantaneous or alternatively that the Newton formulas contains no information about the speed of gravity. Here we will question this view and claim that the speed of gravity in the Newton formula must be the speed of light. The speed of light seems to be hidden within Newton's gravitational constant. Haug [9, 10] has recently suggested that the Newton gravitational constant is a composite constant given by

$$G = \frac{l_p^2 c^3}{\hbar}, \tag{12}$$

where l_p is the Planck length, $\hbar$ is the reduced Planck constant and c is the speed of light. One could easily conclude that knowing big G is necessary for calculating the Planck length, and that big G therefore must be more fundamental, and that the Planck length must be a derived constant. However, it is fully possible to find the Planck length while having no knowledge of big G (see [11]). Still, Newton could not have discovered this, since he was unaware of the Planck constant and the Planck length at that time.

This means the Newton gravitational formula can be re-written as

$$F = G\frac{Mm}{r^2} = \frac{l_p^2 c^3}{\hbar}\frac{M\frac{m}{m_p}\frac{\hbar}{l_p}\frac{1}{c}}{r^2} = \frac{Mc^2}{r}\frac{m}{m_p}\frac{l_p}{r} \tag{13}$$

This formula is somewhat different than the formula (equation 7) we have described for fundamental particles. The reason for this is that we now have generalized the formula for objects of any size. From this formula, we can see that the speed of light is embedded in the Newton formula. Newton actually knew the speed of light quite accurately. In his 1704 book Opticks [12], Newton, based on Rømer's findings, calculated that it would take seven to eight minutes for light to travel from the Sun to the Earth. However, Newton did not link the speed of light to the speed of gravity; on the contrary, he seems to have believed that gravity was an instantaneous force. Still, the speed of light (gravity) is hidden in his gravitational formula.

We in no way claim that Newton knew that his gravitational formula embedded in the gravitational constant contained the speed of light. But Newton was not able to measure the gravitational constant in his formula, and he did not know that such measurements are actually indirectly dependent on the speed of light as well as the Planck length and the Planck constant. The Newton gravitational constant was first indirectly measured quite accurately in 1798 by Cavendish [13], who was also unaware that it was a composite constant.

If Newton had known the Planck length and the Planck constant, he could hypothetically have found the speed of light from gravitational observations, such as the moon's orbital velocity. If he had done so, he also could have determined that the speed of gravity was likely identical to the speed of light.

3 Alternative Way of Writing the Gravitational Formulas for Any Object

The analysis above leads us to an alternative way to write the many well-known gravitational formulas. Table 3 shows the standard gravitational formulas of Newton (and general relativity for bending of light). We see that all alternative formulas contain the mass ratio multiplied by the radius ratio. This is in our view much more intuitive than the standard formulas containing big G. In mathematical atomism the Planck length is an indivisible particle's diameter and the Planck mass particle's radius. Furthermore, as mentioned above, the Planck length can be measured independent of any knowledge of big G.

The more intuitive re-written formulas support our view that big G is a composite constant. And because it is a composite constant, it actually removes intuition when used without knowledge of what it actually is. The discovery of the Newton gravitational constant was naturally a great achievement. In no way do we claim that the gravitational constant is unimportant. However, we claim that it is a composite constant that consists of even more fundamental entities, namely the Planck length, the reduced Planck constant and the speed of light.

	Standard way	**Alternative way**
Gravitational force	$F = G\frac{Mm}{r^2}$	$F = \frac{Mc^2}{r}\frac{m}{m_p}\frac{l_p}{r}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{r}}$	$v_o = c\sqrt{\frac{M}{m_p}\frac{l_p}{r}}$
Escape velocity	$v_e = \sqrt{\frac{2GM}{r}}$	$v_e = c\sqrt{2\frac{M}{m_p}\frac{l_p}{r}}$
Bending of light	$\delta = \frac{4GM}{c^2 r}$	$\delta = 4\frac{M}{m_p}\frac{l_p}{r}$
Red shift	$z(r) \approx \frac{GM}{c^2 r}$	$z(r) \approx \frac{M}{m_p}\frac{l_p}{r}$
Acceleration field	$g = \frac{GM}{r^2}$	$g = \frac{c^2}{r}\frac{M}{m_p}\frac{l_p}{r}$

Table 3: The table shows the standard and alternative methods of writing gravitational formulas without using big G. All alternative formulas contain the mass ratio multiplied by the radius ratio. Again, we believe these formulas are more intuitive than the traditional formulas containing big G.

4 Conclusion

We have presented some interesting mathematical relationships between Einstein's special relativity formulas and Newton's formulas. It seems like the Newtonian gravity formulas are likely non-relativistic. Still, they are more than that, as they also seem to be the relativistic limit based on Haug's maximum velocity formula. We do not claim that Newton knew about $E = mc^2$ directly. But indirectly, Newton essentially had an energymass relationship formula that was valid for any fundamental particle. He merely lacked the concept of reduced Compton wavelength and the radius for fundamental particles. Furthermore, we have seen that all the gravitational formulas contain the mass ratio multiplied by the radius ratio. By writing the gravitational formulas in this way, we avoid big G and obtain much more intuitive formulas linked to something that is conceptually easier to understand. People can grasp the concepts of mass ratio, radius ratio and to some extent pure energy, but what does the gravitational constant represent? This is hard to grasp because, as we argue, it is a composite constant.

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