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# Interval Neutrosophic Fuzzy Stochastic Multi-Criteria Decision-making Methods Based on MYCIN Certainty Factor and Prospect Theory

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**Abstract**

For the stochastic multi-criteria decision-making problem, in which the information on criteria's weights is incomplete and the indices value of alternatives are in the form of interval neutrosophic fuzzy numbers, an interval neutrosophic random decision-making approach based on WYCIN certainty factor and prospect theory is proposed. According to the score function of interval neutrosophic fuzzy numbers and prospect theory, the MYCIN certainty factors of different alternatives in different indices are obtained. And the trust degrees of different indices are determined by using the grey incidence analysis. Then, the certainty factor fusion method in different indices is deduced to get the optimal alternative. And the best alternative is got by using the method. Finally, an example shows the feasibility and validity of this method.

**Key words:** MTYCIN Certainty Factor, Grey Incidence Analysis, Prospect Theory, Interval Neutrosophic Fuzzy Number, Decision Making.

## 1. INTRODUCTION

Since Zadeh (1962) put forward the concept of fuzzy sets, fuzzy multi-attribute decision-making problem got widely researches. In traditional fuzzy set, the membership of the element is the value between 0 and 1. On the basis of fuzzy sets, Atanassov (1986) had added a new parameter, which is the nonmembership, and defined the definition of intuitionistic fuzzy sets. Torra (2010) defined a new fuzzy set, which allowed the membership of the element to be a finite set between 0 and 1. Subsequently, Qian et al. (2013) and Zhu (2012) had defined generalized hesitant fuzzy set and dual hesitant fuzzy set. Although the fuzzy set could better deal with the problem of decision in the fuzzy information, but in real life there still exists the unable to deal with the uncertain information. Therefore, Smarandache (1999) provided the concept of neutrosophic set, and pointed out that neutrosophic set could be independently expressed truth-membership degree, indeterminacy-membership degree and falsity-membership degree. All the factors described by the single-valued neutrosophic set are very suitable for human thinking due to the imperfection of knowledge that human receives or observes from the external world. Obviously, the neutrosophic components are best fit in the representation of indeterminacy and inconsistent information, while the intuitionistic fuzzy set cannot represent and handle indeterminacy and inconsistent information. Hence, the single-valued neutrosophic set has been a rapid development and a wide range of applications.

For multi-attribute decision making problem of neutrosophic set, Ye (2014) prospected a method based on correlation degree and cross entropy of single-valued neutrosophic set. After that he constructed neutrosophic fuzzy prioritized weighted averaging operator and neutrosophic fuzzy prioritized weighted geometric operator and their multicriteria decision-making method, in which the criteria are in different priority level. For the problem of multiple attribute decision making with interval neutrosophic sets, Chi and Liu established the attribute weight determination model based on deviation maximization and proposed an extension of TOPSIS method. In real decision-making, a single numerical value does not accurately depict the indeterminacy and inconsistent information. So Wang (2008) defined interval neutrosophic numbers, and gave logic operation, the distance and similarity of them. However, except using weighted sum of attribute value to decide in the decision making process, the decision makers usually use individual experience and collecting data to do evidential reasoning, and find the best alternative. Here we call them as evidential reasoning decision making. For the stochastic multi-criteria decision-making problem, in which the information on criteria's weights is incomplete and the indices value of alternatives are in the form of interval neutrosophic fuzzy numbers, an interval neutrosophic random decision-making approach based on WYCIN certainty factor and prospect theory is proposed. The aim of the method is to help the people making the decision efficiently facing risk.

## 2. PRELIMINARIES

### *2.1. Neutrosophic Sets*

From philosophical point of view, Smarandache originally presented the concept of a neutrosophic set $A$ in a universal set $X$ .

Definition 1 Let X be a universe of discourse, for any $x \in X$ , the set $A = \{x(T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ is called a neutrosophic set, which is characterized independently by a truth-membership function $T_A(x)$ , an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$ . The functions $T_A(x)$ , $I_A(x)$ and $F_A(x)$ in $X$ are real standard or nonstandard subsets $]0^0, 1^+[$ , such that $T_A(x): X \to [0,1]$ , $I_A(x): X \to [0,1]$ and $F_A(x): X \to [0,1]$ . Then, the sum of $T_A(x)$ , $I_A(x)$ and $F_A(x)$ satisfies the condition $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$ .

Obviously, it is difficult to apply the neutrosophic set to practical problems. To easily apply it in science and engineering fields, Wang et al. (2010) introduced the concept of a single-valued neutrosophic set as a subclass of the neutrosophic set and gave the following definition.

Definition 2 Let X be a universe of discourse, for any $x \in X$ , the set $A = \{x(T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ is called a interval neutrosophic set, which is characterized independently by a truth-membership function $T_A(x)$ ,an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$ . The functions $T_A(x)$ , $I_A(x)$ and $F_A(x)$ in $X$ are real standard or nonstandard subsets $[0,1]$ , and the sum of $T_A(x)$ , $I_A(x)$ and $F_A(x)$ satisfies the condition $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$ .

For convenience, the interval neutrosophic number are denoted by $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ , where $[T^L, T^U] \subseteq [0,1], [I^L, I^U] \subseteq [0,1], [F^L, F^U] \subseteq [0,1]$ and $0^- \le \sup[T^L, T^U] + \sup[I^L, I^U] + \sup[F^L, F^U] \le 3^+$ .

Definition 3 Let $x = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $y = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two interval neutrosophic numbers, then Hamming distance of them is

$$d(x, y) = \frac{|T_1^L - T_2^L| + |T_1^U - T_2^U| + |I_1^L - I_2^L| + |I_1^U - I_2^U| + |F_1^L - F_2^L| + |F_1^U - F_2^U|}{6}$$

Scoring function and accuracy function are two very important indicators in the interval neutrosophic numbers sorting method. For an interval neutrosophic set, the truth-membership value is bigger, and the interval neutrosophic set is bigger; the indeterminacy-membership value is smaller, and the interval neutrosophic set is bigger; similarly the falsity-membership value is smaller, and the interval neutrosophic set is bigger. So the scoring function of the interval neutrosophic number is defined as the following:

Definition 4 Let $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ be an interval neutrosophic number, the scoring function of $x$ can be defined by

$$S(x) = \frac{T^L + T^U}{2} + 1 - \frac{I^L + I^U}{2} + 1 - \frac{F^L + F^U}{2} .$$

Definition 5 Let $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ be an interval neutrosophic number, the accuracy function of $x$ can be defined by

$$H(x) = \frac{T^L + T^U}{2} + 1 - \frac{I^L + I^U}{2} + \frac{F^L + F^U}{2} .$$

Definition 6 Let $x = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $y = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two interval neutrosophic numbers, the scoring function and accuracy function of $x$ and $y$ are $S(x), H(x)$ and $S(y), H(y)$ , so

(1)If $S(x) > S(y)$ , then $x > y$ ;

(2)If $S(x) = S(y)$ and $H(x) > H(y)$ , then $x > y$ ;

(3)If $S(x) = S(y)$ and $H(x) = H(y)$ , then $x = y$ .

***2.2. Prospect Theory***

Definition 7 (interval neutrosophic random variable) When the value of random variable $\xi$ is interval neutrosophic number and corresponding probability is known, the random variable $\xi$ is called interval neutrosophic random variable.

In the process of risk decision, people often is deviating from the "rational". Kahneman, etc. also provides a lot of evidence, which suggests that the decision making process often run counter to the expected utility theory, and puts forward a new model framework based on the prospect theory.

The prospect theory mainly considers the value function and decision making weight. The prospect value is defined by $W=\sum_{i=1}^{n} h(p_i)c(x_i)$, where $h(p)=\frac{p^r}{(p^r+(1-p)^r)^{1/r}}$ is the increasing function of probability. The decision making weight is defined by $c(x)=\begin{cases} x^{\delta}, & x\ge 0 \\ -\sigma(-x)^{\beta}, & x\le 0 \end{cases}$, which is the value function of $x$, and is the decision maker's subjective value.

**3. MYCIN UNCERTAINTY FACTORS**

Definition 8 Let $h$ be a random suppose variable and $e$ be a evidence random variable, then $CF(h/e)=MB(h/e)-MD(h/e)$ is MYCIN uncertainty factor, where

$$MB(h/e)\begin{cases} \frac{P(h/e)\vee P(h/e)-P(h)}{1-P(h)}, & P(h)\ne 1 \\ 1, & P(h)=1 \end{cases}, MD(h/e)\begin{cases} \frac{P(h)-P(h/e)\wedge P(h/e)-}{1-P(h)}, & P(h)\ne 0 \\ 1, & P(h)=0 \end{cases}.$$

$CF(h/e)$ indicates when the evidence $e$ is true, it is the trust degree for the assumption $h$. The value range is $[0,1]$.

Specially, if $CF(h/e)=1$, this indicates the assumption $h$ is true under the evidence $e$; if $CF(h/e)=-1$, this indicates the assumption $h$ is false under the evidence $e$; if $CF(h/e)=0$, this indicates the assumption $h$ is uncertain.

In this paper, we consider the index system and alternative system separately as a set of information with different assumptions. However, when the evidence $e$ uncertainty is true, let $CF(e)$ be the trust degree of the evidence $e$, and the value range is $[-1,1]$. The value $CF(e)$ is bigger; the trust degree of the evidence $e$ is bigger. Specially, when $CF(e)=1$, this indicates the evidence $e$ is true. When $CF(e)=-1$, this indicates the evidence $e$ is false.

Definition 9 Let $CF_T(h/e)=CF(h/e)CF(e)$ be substantial uncertainty factors.

If you want to get the substantial uncertainty factor, you must obtain the trust degree credibility $CF(e_i)$ of the evidence $I_i$.

Grey system theory is a very good tool to deal with poor information. In the general decision making problems, the numbers involved solutions and general fewer attributes is small which is in accordance with article of the grey system modeling. In order to calculate the trust degree of each evidence (indicators), the paper uses the grey correlation to get the uncertainty trust degree of the indicators, according to the Li Peng.

Firstly we calculate the uncertainty trust degree. Let the matrix $S=(s_{ij})_{m\times n}$ and denote $\overline{s_i}=\frac{1}{n}\sum_{j=1}^{n}s_{ij}$, $j=1,2,\cdots,m$.

Definition 10 Let the uncertainty degree of indicators $I_j$ with $q$ order be

$$DOI(I_j)=\frac{1}{m}[\sum_{i=1}^{m}(r_{ij}^{q})]^{\frac{1}{q}},$$

where $r_{ij}=\frac{\min|g_{ij}-\overline{g}|+\xi\max|g_{ij}-\overline{g}|}{|g_{ij}-\overline{g}|+\xi\max|g_{ij}-\overline{g}|}$, $i=1,2,\cdots,n$ is grey average correlation. The general let $\xi=0.5$.

According to the definition 9, we calculate the trust degree $CF(e_j) = 1 - DOI(I_j), j = 1, 2, \cdots, n$ of the indicators $I_j$ by using the Hamming distance. So we can get the substantial uncertainty factors $CF_T(h_i/e_j) = CF(h_i/e_j)CF(e_j)$.

Before making decision, we need evidence fusion for substantial uncertainty factors of each alternative under the different index (evidence). The fusion method is the followings:

Definition 11 If the generating function $F(x)$ exists dual function $f$ satisfying $F(x \cdot y) = f(F(x), F(y))$, then $F(x)$ can be called the synthetic function.

Theorem 1 If $e_1$ and $e_2$ are conditional independence about $h$ and $\overline{h}$, and the function $F$ is a synthetic function, then $CF_T(h/e_1, e_2) = F(\frac{P(e_1, e_2/h)}{P(e_1, e_2/\overline{h})})$ is a synthetic function of uncertainty factor, which means existing function $f: [-1,1]^2 \to [-1,1]$ satisfies $CF_T(h/e_1, e_2) = f(CF_T(h/e_1), CF_T(h/e_2))$.

Theorem 2 The evidence synthetic function getting by $F_1$ has the following properties

(1) Commutative law $CF_T(h/e_1, e_2) = CF_T(h/e_2, e_1)$;

(2) Associative law $CF_T(h/(e_1, e_2), e_3) = CF_T(h/e_1, (e_2, e_3))$.

Inference 1 If $e_1$, $e_2, \cdots, e_m$ are conditional independence about $h$ and $\overline{h}$, then

$$CF_T(h/e_1, e_2, \cdots, e_m) = f(CF_T(h/e_1, e_2, \cdots, e_{m-1}), CF_T(h/e_m))$$

Meaning $CF_T(h/e_1, e_2, \cdots, e_m) = \dfrac{CF_T(h/e_1, e_2, \cdots, e_{m-1}) + CF_T(h/e_m)}{1 + CF_T(h/e_1, e_2, \cdots, e_{m-1}) \cdot CF_T(h/e_m)}$.

The inference provides a fusion method of substantial uncertainty factors under more evidence reasoning. It can be seen that the fusion of substantial uncertainty factors is very simple. MYCIN uncertainty factor method has three excellent properties of D-S evidence theory :(1) the commutativity; (2) the associativity; (3) the polarizability.

## 4. DECISION-MAKING METHOD BASED ON MYCIN UNCERTAINTY FACTOR

In this section, we develop an approach based on the MYCIN uncertainty factor to deal with multiple attribute decision making problems with random interval neutrosophic information.

In a multiple attribute decision-making problem with random interval neutrosophic information, there is a set of alternatives $A = \{A_1, A_2, \cdots, A_m\}$ which satisfies a set of attributes $C = \{C_1, C_2, \cdots, C_n\}$ and under the state $Z = \{Z_1, Z_2, \cdots, Z_s\}$ where the probability of $k$ th state is $p_k$. An alternative on attributes is evaluated by the decision maker, and the evaluation values are represented by the form of random interval neutrosophic numbers. Then, we can establish a random interval neutrosophic decision matrix under the different state $D_1 = (d_{ij1})_{m\times n}, \cdots, D_s = (d_{ijs})_{m\times n}$, where $[T_{ijk}{}^L, T_{ijk}{}^U] \subseteq [0,1]$ indicates the degree that the alternative $A_i$ satisfies the attribute $C_j$ under the state $Z_k$, $[I_{ijk}{}^L, I_{ijk}{}^U] \subseteq [0,1]$ indicates the degree that the alternative $A_i$ is uncertain about attribute $C_j$ under the state $Z_k$, $[F_{ijk}{}^L, F_{ijk}{}^U] \subseteq [0,1]$ indicates the degree that the alternative $A_i$ does not satisfy the attribute $C_j$ under the state $Z_k$ with $0^- \le T_{ijk}^U + I_{ijk}^U + F_{ijk}^U \le 3^+$ for $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$ and $k = 1, 2, \cdots, s$.

In the following, we apply the MYCIN uncertainty factor method and the score and accuracy functions to a multiple attribute decision-making problem with interval neutrosophic random information, which can be described as the following procedures:

Step1 By using the score function, the interval neutrosophic numbers decision matrix can be convert to score function matrix.

According to the definition 4 , the interval neutrosophic numbers are transformed into real numbers, then we obtain the real number decision matrix $S_1 = (s_{ij1})_{m\times n}, \cdots, S_s = (s_{ijs})_{m\times n}$.

Step2 Calculate the prospect matrix. The focus on determining the prospect decision matrix is the gap between expectations and results, not the result itself. So the choice of the reference point is the key. According to the definition of interval neutrosophic numbers and the score function, the reference point chooses 0. Then we obtain the prospect value of various alternative under attributes: $W_{ij}=\sum_{s=1}^{k}h(p_s)c(S_{ijs})$ , where $h(p_s)=\dfrac{p_s^r}{(p_s^r+(1-p_s)^r)^{\frac{1}{r}}}$ , $c(S_{ijs})=\begin{cases}S_{ijs}^{\delta}, & S_{ijs}\geq 0\\ -\sigma(-S_{ijs}^{\delta})^{\beta}, S_{ijs}^{\delta}\leq 0\end{cases}$ . So get the prospect decision matrix $W=(W_{ij})_{m\times n}$ .

Step3 Determine the substantial uncertainty factor. By the relationship between score function and MYCIN uncertainty factors and the characteristics of the prospect theory, the prospects of decision matrix $W=(W_{ij})_{m\times n}$ replace MYCIN uncertain factor matrix, meaning $CF(h_i/e_j)=W_{ij}$ . Then calculate the uncertainty degree $DOI(I_j)$ of attribute $I_j$, and get the certainty degree $CF(e_j)=1-DOI(I_j)$ and substantial uncertainty factor matrix $CF_T=(CF_T(h_i/e_j))_{m\times n}$ where $CF_T(h_i/e_j)=CF(h_i/e_j)\cdot CF_T(e_j)$ .

Step4 Fuse the evidence information using the fusion formula according to the evidence theory. Choose the optimal solution by the essence of uncertain factor maximization principle.

## 5. ILLUSTRATIVE EXAMPLES

In this section, an illustrative example of a major aviation equipment project investment problem adapted for a multiple attribute decision-making problem is provided to demonstrate the application and effectiveness of the developed multiple attribute decision-making method under interval neutrosophic stochastic environment.

There are four investment programs $A=\{A_1,A_2,A_3,A_4\}$ . In the four investments the decision makers consider three attributes $C=\{C_1,C_2,C_3\}$ . Respectively it is promoting national security, promoting the military technology, driving the development of related industries and research and development time. Because the future of the international situation is uncertain, there are three kinds of possible $Z=\{Z_1,Z_2,Z_3\}$ , stable international environment, the surrounding local small war, a large-scale war. The three states probabilities which the experts evaluate are $0.6,0.3,0.1$ . The experts give three interval neutrosophic numbers decision making matrixes:

Under the international environment $Z_1$, stable decision matrix is：

$$D_1=\begin{bmatrix}([0.2,0.3],[0.3,0.4],[0.4,0.5]) & ([0.5,0.6],[0.3,0.4],[0.2,0.3]) & ([0.6,0.7],[0.4,0.5],[0.1,0.2])\\ ([0.4,0.5],[0.2,0.3],[0.1,0.2]) & ([0.3,0.4],[0.3,0.4],[0.1,0.2]) & ([0.5,0.6],[0.3,0.4],[0.2,0.3])\\ ([0.7,0.8],[0.5,0.6],[0.2,0.3]) & ([0.1,0.2],[0.2,0.3],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4],[0.1,0.2])\\ ([0.5,0.6],[0.3,0.4],[0.2,0.3]) & ([0.4,0.5],[0.5,0.5],[0.2,0.4]) & ([0.2,0.3],[0.3,0.4],[0.4,0.5])\end{bmatrix}$$

Under the surrounding local small war $Z_2$, stable decision matrix is：

$$D_2=\begin{bmatrix}([0.4,0.5],[0.3,0.4],[0.2,0.3]) & ([0.3,0.4],[0.3,0.4],[0.2,0.3]) & ([0.3,0.4],[0.3,0.4],[0.1,0.2])\\ ([0.3,0.5],[0.3,0.4],[0.2,0.3]) & ([0.5,0.6],[0.2,0.3],[0.1,0.2]) & ([0.5,0.6],[0.2,0.3],[0.1,0.2])\\ ([0.7,0.8],[0.5,0.6],[0.1,0.2]) & ([0.2,0.3],[0.2,0.3],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4],[0.2,0.2])\\ ([0.5,0.6],[0.3,0.3],[0.2,0.3]) & ([0.3,0.4],[0.2,0.3],[0.2,0.3]) & ([0.5,0.6],[0.2,0.3],[0.1,0.2])\end{bmatrix}$$

Under a large-scale war environment $Z_3$, stable decision matrix is：

$$D_3=\begin{bmatrix}([0.6,0.7],[0.2,0.3],[0.1,0.2]) & ([0.4,0.5],[0.3,0.4],[0.4,0.5]) & ([0.4,0.5],[0.3,0.4],[0.1,0.2])\\ ([0.4,0.5],[0.3,0.4],[0.3,0.4]) & ([0.6,0.7],[0.2,0.3],[0.1,0.2]) & ([0.5,0.5],[0.2,0.3],[0.3,0.3])\\ ([0.5,0.5],[0.2,0.2],[0.1,0.2]) & ([0.4,0.5],[0.2,0.3],[0.3,0.4]) & ([0.3,0.4],[0.3,0.4],[0.2,0.3])\\ ([0.5,0.6],[0.3,0.3],[0.2,0.3]) & ([0.2,0.3],[0.1,0.2,[0.2,0.3]) & ([0.6,0.7],[0.2,0.3],[0.1,0.2])\end{bmatrix}$$

The decision makers try to decide which investment program is the best.

Step 1 We translate the interval neutrosophic decision making matrix into score matrix by using score function.

$$D_1=\begin{bmatrix}1.45 & 1.95 & 2.02\\ 2.05 & 1.85 & 1.95\\ 1.95 & 1.55 & 2.05\\ 1.95 & 1.65 & 1.45\end{bmatrix},\ D_2=\begin{bmatrix}1.85 & 1.75 & 1.85\\ 1.80 & 2.15 & 2.15\\ 2.05 & 1.65 & 2.00\\ 2.00 & 1.85 & 2.15\end{bmatrix},\ D_3=\begin{bmatrix}2.25 & 1.65 & 1.95\\ 1.75 & 2.25 & 1.95\\ 2.15 & 1.85 & 1.75\\ 2.00 & 1.85 & 2.25\end{bmatrix}.$$

Step 2 Catculate the prospect decision matrix.

We use the parameter values of Tversky[3] to calculate the prospect decision matrix: $\delta = \beta = 0.88$, $\sigma = 2.25, \gamma = 0.61$. So we get the prospect matrix

$$W = \begin{pmatrix} 0.659 & 0.687 & 0.723 \\ 0.736 & 0.734 & 0.737 \\ 0.769 & 0.613 & 0.733 \\ 0.754 & 0.650 & 0.658 \end{pmatrix}.$$

Step 3 Catculate the uncertainty degree of the attributes: $DOI(I_1) = 0.365$, $DOI(I_2) = 0.336$, $DOI(I_3) = 0.357$. The we get the substantial uncertainty factor matrix

$$CF_T = \begin{pmatrix} 0.417 & 0.455 & 0.465 \\ 0.466 & 0.485 & 0.474 \\ 0.487 & 0.406 & 0.471 \\ 0.478 & 0.430 & 0.422 \end{pmatrix}.$$

Step 4 Fuse the evidence information by the fusion formula, and get the best investment program.

$$CF_T = CF_T(h_1/e_1, e_2, e_3) = 0.733,$$

$$CF_T = CF_T(h_2/e_1, e_2, e_3) = 0.755,$$

$$CF_T = CF_T(h_3/e_1, e_2, e_3) = 0.746,$$

$$CF_T = CF_T(h_4/e_1, e_2, e_3) = 0.753.$$

Based on substantial maximizing principle of uncertainty factor, the order investment programs is $A_2 \succ A_4 \succ A_3 \succ A_1$.So $A_2$ is the optimal solution.

## 6. CONCLUSIONS

This paper presented a new method of the interval neutrosophic stochastic multi-attribute decision based on MYCIN uncertain factor and the prospect and the prospects theory. The advantage of the proposed method is more suitable for solving multiple attribute stochastic decision-making problems with interval neutrosophic information because “it can more realistically reflect and describe the real decision-making process.” And this method gives play to the fast and effective characteristics of uncertainty factors in evidence reasoning which makes the decision result more scientific. The example shows that the new method is reasonable and feasible.

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