International Journal of Applied Research 2016; 2(5): 143-150

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2016; 2(5): 143-150
www.allresearchjournal.com
Received: 01-03-2016
Accepted: 02-04-2016

I Arockiarani
Nirmala College for Women, Coimbatore, Tamilnadu, India.

C Antony Crispin Sweety
Nirmala College for Women, Coimbatore, Tamilnadu, India.

Correspondence
I Arockiarani
Nirmala College for Women, Coimbatore, Tamilnadu, India.

Rough neutrosophic set in a lattice

I Arockiarani, C Antony Crispin Sweety

Abstract
In this paper, we examine the relationship between rough fuzzy neutrosophic sets and lattice theory. We introduce the notion of Rough fuzzy neutrosophic set and Rough fuzzy neutrosophic lattice (resp Rough fuzzy neutrosophic ideals). Further, we discuss about fuzzy neutrosophic rough set corresponding to a rough set and define the terms and conditions for fuzzy neutrosophic rough lattice. We also prove that a fuzzy neutrosophic rough set A in X is a fuzzy neutrosophic rough lattice iff it's level rough sets $(\underline{R}(A)_{(\alpha,\beta,\gamma)}, \overline{R}(A)_{(\alpha,\beta,\gamma)})$ is a rough sub lattice of X.

Keywords: Rough set, rough fuzzy neutrosophic set, fuzzy Neutrosophic rough sets.

1. Introduction

In 1982, Pawlak [6] introduced the concept of rough set, as a formal tool for modeling and processing incomplete information in information systems. This concept is fundamental to the examination of granularity in knowledge. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. After Pawlak, there have been many models built upon different aspect, i.e, universe, relations, object and operators by many scholars [3, 4, 5, 10, 12]. Various notions that combine rough sets and fuzzy sets, vague set and intuitionistic fuzzy sets are introduced, such as rough fuzzy sets, fuzzy rough sets, generalized fuzzy rough sets, rough vague sets. The theory of rough sets is based upon the classification mechanism, from which the classification can be viewed as an equivalence relation and knowledge blocks induced by it be a partition on universe.

One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache [8]. Neutrosophic sets described by three functions: Truth function indeterminacy function and false function that are independently related. The theories of neutrosophic set have achieved great success in various areas such as medical diagnosis, database, topology, image processing, and decision making problem. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness.

Recently many researchers applied the notion of fuzzy neutrosophic sets to relations, group theory, ring theory, lattice theory etc. In this paper we studied relationship between rough sets and fuzzy neutrosophic sets. Here we give the rough approximation of fuzzy neutrosophic set and introduced rough fuzzy neutrosophic sub lattices, ideals etc. Also we defined fuzzy neutrosophic rough sets, fuzzy neutrosophic rough sub lattices, and ideals and studied their properties

2. Preliminaries:

Definition 2.1[2] A Neutrosophic set A on the universe of discourse X is defined as

A = $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$,

Where $T, I, F: X \to]^-0,1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.3: [2]
A Fuzzy Neutrosophic set A on the universe of discourse X is defined as
A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \to [0, 1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2.2: [2] A neutrosophic set A is contained in another neutrosophic set B. (i.e.,)
A$\subseteq$ B $\Leftrightarrow T_A(x) \le T_B(x), I_A(x) \le I_B(x), F_A(x) \ge F_B(x)$,$\forall$ x$\in$ X.

Definition 2.4: [2]
The complement of a neutrosophic set (F, A) denoted by (F, A)^c and is defined as

(F, A)^c = (F^c, $\rceil$A)

Where $T_{F^c}(x) = \mathrm{F}_{\mathrm{F}}(x)$, $I_{F^c}(x) = 1 - \mathrm{I}_{\mathrm{F}}(x)$,

$F_{F^c}(x) = \mathrm{T}_{\mathrm{F}}(x)$.

Definition 2.5: [2]
Let A and B) be two neutrosophic sets over the common universe U. A is said to be neutrosophic subset of B if A $\subset$ B and $T_A(x) \le T_B(x), I_A(x) \le I_B(x), F_A(x) \ge F_B(x)$ $\forall$ E$\in$ A, x $\in$ U.

Definition 2.6: [2]
Two neutrosophic sets (F,A) and (G,B) over the common universe U are said to be equal if (F,A) $\subseteq$ (G,B) and (G,B) $\subseteq$ (F,A).We denote it by (F,A) = (G,B).

Definition 2.7: [2]
Let X be a non empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$

are fuzzy neutrosophic sets. Then

$$A \tilde{\cup} B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$$

$$A \tilde{\cap} B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$$

Definition 2.2: [3] Let U be any non-empty set. Suppose R is an equivalence relation over U. For any non-null subset X of U, the sets
$A_1(X) = \{x: [x]_R \subseteq X\}$
$A_2(X) = \{x: [x]_R \cap X \neq \emptyset\}$
are called lower approximation and upper approximation respectively of X and the pair
S= (U, R) is called approximation space. The equivalence relation R is called indiscernibility relation. The pair A(X) = (A_1(X), A_2(X)) is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing x

3. Rough Fuzzy Neutrosophic Sets In A Lattice
In this section we define rough fuzzy neutrosophic set and some of their operations. Further, we introduce Rough fuzzy neutrosophic lattices (RIFL) and ideals and study certain properties of them.

Definition 3.1: Let U be a non-null set and R be an equivalence relation on U. Let A be a neutrosophic set in U with the truth value $T_A(x)$, indeterminate value $I_A(x)$ and false value $F_A(x)$. The lower and the upper approximations of A in the approximation (U, R) denoted by $\underline{R}(A)$ and $\overline{R}(A)$ are respectively defined as follows:

$$\overline{R}(A) = \{x, T_{\overline{R}(A)}(x), I_{\overline{R}(A)}(x), F_{\overline{R}(A)}(x) / y \in [x]_R, x \in U\}$$

$$\underline{R}(A) = \{x, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x) / y \in [x]_R, x \in U\}$$

where:
$T_{\underline{R}(A)}(x) = \wedge_{y \in [x]R} T_A(y)$,
$I_{\underline{R}(A)}(x) = \wedge_{y \in [x]R} I_A(y)$,
$F_{\underline{R}(A)}(x) \vee_{y \in [x]R} F_A(y)$
$T_{\overline{R}(A)}(x) = \vee_{y \in [x]R} T_A(y)$,
$I_{\overline{R}(A)}(x) = \vee_{y \in [x]R} I_A(y)$,
$F_{\overline{R}(A)}(x) = \wedge_{y \in [x]R} F_A(y)$
So $0 \le T_{\overline{R}(A)}(x) + I_{\overline{R}(A)}(x) + F_{\overline{R}(A)}(x) \le 3$ and
$0 \le, T_{\underline{R}(A)}(x) + I_{\underline{R}(A)}(x) + F_{\underline{R}(A)}(x) \le 3$ and
$T_{\overline{R}(A)}(x), I_{\overline{R}(A)}(x), F_{\overline{R}(A)}(x)$)
$, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x) : A \to [0,1]$

Where "$\vee$ "and "$\wedge$ " mean "max" and "min " operators respectively, and are the truth, indeterminacy and false values of y with respect to A. It is easy to see that $\underline{R}(A)$ and $\overline{R}(A)$ are two neutrosophic sets in U.

$\underline{R}(A)$ and $\overline{R}(A)$: $A \to A$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $\underline{R}(A)$ and $\overline{R}(A)$ is called the rough neutrosophic set in (U, R). From the above definition, we can see that $\underline{R}(A)$ and $\overline{R}(A)$ have constant membership on the equivalence classes of U.

Example 3.2:
Let U= {S_1, S_2, S_3, S_4, S_5} be the universe of discourse.
Let R be an equivalence relation, where its partition of U is given by
U/R= {{S_1, S_2}, {S_3}, {S_4, S_5}}
A={[S_1,(0.3,0.4,0.5)] [S_2,(0.2,0.4,0.3)] [S_3,(0.5,0.6,0.7)]} be a neutrosophic set of U.
The lower and upper approximations are obtained as
$\overline{R}(A)$ = { [S_1,(0.3,0.4,0.3)] [S_2,(0.3,0.4,0.3)] [S_3,(0.5,0.6,0.7)]}
$\underline{R}(A)$ ={ [S_1,(0.2,0.4,0.5)] [S_4,(0.2,0.4,0.5)] [S_3,(0.5,0.6,0.7)]}

Another neutrosophic set can be defined as
B = {[S_1,(0.2,0.3,0.4)] [S_4,(0.3,0.5,0.4)] [S_5,(0.4,0.6,0.2)]}

The lower and upper approximations are obtained as
$\overline{R}(B)$ = {[S_1,(0.3,0.5,0.4)] [S_4,(0.2,0.3,0.4)] [S_2,(0.4,0.6,0.2)] [S_5,(0.4,0.6,0.2)]}
$\underline{R}(B)$ ={ [S_1,(0.2,0.3,0.4)] [S_2,(0.2,0.3,0.4)]]}

Definition 3.3: If $R(A)=(\underline{R}(A),\overline{R}(A))$ is a rough fuzzy neutrosophic set in (U,R), the rough fuzzy neutrosophic complement of A^C neutrosophic set denoted by $R(A)^c=(\underline{R}(A)^c,\overline{R}(A)^c)$ where $\underline{R}(A)^c,\overline{R}(A)^c$ are defined as

$\underline{R}(A)^c=\{x,),F_{\underline{R}(A)}(x),1-I_{\underline{R}(A)}(x),T_{\underline{R}(A)}(x)/y\in[x]_R,x\in U\}$

and

$\overline{R}(A)^c=$

$\{x,F_{\overline{R}(A)}(x),1-I_{\overline{R}(A)}(x),T_{\overline{R}(A)}(x))/y\in[x]_R,x\in U\}$

Definition3.4: If A_1 and A_2 are two rough fuzzy neutrosophic set of the neutrosophic sets X_1 and X_2 respectively in then we define the following:

1. $(A_1)=(A_2)$ iff $\underline{R}(A_1)=\underline{R}(A_2)$ and $\overline{R}(A_1)=\overline{R}(A_2)$
2. $A_1\subseteq A_2$ iff $\underline{R}(A_1)\subseteq\underline{R}(A_2)$ and $\overline{R}(A_1)\subseteq\overline{R}(A_2)$
3. $A_1\cup A_2\Rightarrow\underline{R}(A_1)\cup\underline{R}(A_2)$ and $\overline{R}(A_1)\cup\overline{R}(A_2)$
4. $A_1\cap A_2\Rightarrow\underline{R}(A_1)\cap\underline{R}(A_2)$ and $\overline{R}(A_1)\cap\overline{R}(A_2)$
5. $A_1+A_2\ \Rightarrow\ \underline{R}(A_1)+\underline{R}(A_2)$ and $\overline{R}(A_1)+\overline{R}(A_2)$
6. $A_1-A_2\ \Rightarrow\ \underline{R}(A_1)-\underline{R}(A_2)$ and $\overline{R}(A_1)-\overline{R}(A_2)$

Definition 3.5:
Let L be a lattice and $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle/x\in L\}$ be a fuzzy neutrosophic set, then A is called fuzzy neutrosophic sublattice of L, if the following conditions are satisfied

(i) $T_A(x\vee y)\geq\min\{T_A(x),T_A(y)\}$,
$T_A(x\wedge y)\geq\min\{T_A(x),T_A(y)\}$

(ii) $I_A(x\vee y)\geq\min\{I_A(x),I_A(y)\}$,
$I_A(x\wedge y)\geq\min\{I_A(x),I_A(y)\}$

(iii) $F_A(x\vee y)\leq\max\{T_A(x),T_A(y)\}$,
$F_A(x\wedge y)\leq\max\{F_A(x),F_A(y)\}$

The set of all FNLs of L is denoted by FNL(L).

Definition 3.6:
A FNS A of L is called a fuzzy neutrosophic ideal of L, if he following conditions are satisfied.

(i) $T_A(x\vee y)\geq\min\{T_A(x),T_A(y)\}$,
$T_A(x\wedge y)\geq\max\{T_A(x),T_A(y)\}$

(ii) $I_A(x\vee y)\geq\min\{I_A(x),I_A(y)\}$,
$I_A(x\wedge y)\geq\max\{I_A(x),I_A(y)\}$

(iii) $F_A(x\vee y)\leq\max\{T_A(x),T_A(y)\}$,
$F_A(x\wedge y)\leq\min\{F_A(x),F_A(y)\}$

The set of all FNIs of L is denoted as FNI(L).

Definition 3.7:
A FnI A of L is called a fuzzy neutrosophic prime ideal if

$T_A(x\wedge y)\geq\max\{T_A(x),T_A(y)\}$,

$I_A(x\wedge y)\geq\max\{I_A(x),I_A(y)\}$ and

$F_A(x\wedge y)\leq\min\{F_A(x),F_A(y)\}$, $\forall x,y\in L$

Theorem 3.8: If A and B are two FNLs (FNIs) of a lattice L, then $A\cap B$ is a FNL(FNI) of L.

Proof:

Let $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle/x\in X\}$ and $B=\{\langle x,T_B(x),I_B(x),F_B(x)\rangle/x\in X\}$, are two FNS of L. Then $A\cap B=\{\langle x,T_{A\cap B}(x),I_{A\cap B}(x),F_{A\cap B}(x)\rangle x\in X\}$

Where

$T_{A\cap B}(x)=\min\{T_A(x),T_B(x)\}$,

$I_{A\cap B}(x)=\min\{I_A(x),I_B(x)\}$ and

$F_{A\cap B}(x)=\max\{F_A(x),F_B(x)\}$ so that

$$\begin{aligned}T_{A\cap B}(x\vee y)&=\min\{T_A(x\vee y),T_B(x\vee y)\}\\&\geq\min\{\min\{T_A(x),T_A(y)\},\min\{T_B(x),T_B(y)\}\}\\&=\min\{\min\{T_A(x),T_B(y)\},\min\{T_A(x),T_B(y)\}\}\\&=\min\{T_{A\cap B}(x),T_{A\cap B}(y)\}\end{aligned}$$

as A and B are FNLs of L, $T_{A\cap B}(x\vee y)=\min\{T_{A\cap B}(x),T_{A\cap B}(y)\}\forall x,y\in L$

Similarly we get

$T_{A\cap B}(x\wedge y)=\min\{T_{A\cap B}(x),T_{A\cap B}(y)\}\forall x,y\in L$

$$\begin{aligned}I_{A\cap B}(x\vee y)&=\min\{I_A(x\vee y),I_B(x\vee y)\}\\&\geq\min\{\min\{I_A(x),I_A(y)\},\min\{I_B(x),I_B(y)\}\}\\&=\min\{\min\{I_A(x),I_B(y)\},\min\{I_A(x),I_B(y)\}\}\\&=\min\{I_{A\cap B}(x),I_{A\cap B}(y)\}\end{aligned}$$

as A and B are FNLs of L,

$I_{A\cap B}(x\vee y)=\min\{I_{A\cap B}(x),I_{A\cap B}(y)\}\forall x,y\in L$

Siilarly we get

$I_{A\cap B}(x\wedge y)=\min\{I_{A\cap B}(x),I_{A\cap B}(y)\}\forall x,y\in L$

Also

$$\begin{aligned}I_{A\cap B}(x\vee y)&=\max\{F_A(x\vee y),F_B(x\vee y)\}\\&\geq\max\{\max\{F_A(x),F_A(y)\},\min\{F_B(x),F_B(y)\}\}\\&=\max\{\max\{F_A(x),F_B(y)\},\min\{F_A(x),F_B(y)\}\}\\&=\max\{F_{A\cap B}(x),F_{A\cap B}(y)\}\end{aligned}$$

as A and B are FNLs of L,

$F_{A\cap B}(x\vee y)=\max\{F_{A\cap B}(x),F_{A\cap B}(y)\}\forall x,y\in L$

Similarly we get

$F_{A\cap B}(x\wedge y)=\max\{F_{A\cap B}(x),F_{A\cap B}(y)\}\forall x,y\in L$

Hence $A\cap B$ is FNL of L

Proof for FNI is similar.

Proposition 3.9: Let L be a lattice and A is an IFL (IFI) of L. Then $\underline{R}(A)$ and $\overline{R}(A)$ are also FNL's (FNI's) of L.

Proof. We will prove the case of FNL. Proof for FNI is similar.
We have

$$T_{\underline{R}(A)}(x\vee y)=\bigwedge_{x'\vee y'\in[x\vee y]_R} T_A(x'\wedge y')\geq \bigwedge_{x'\vee y'\in[x\vee y]_R}[\min\{T_A(x'),T_A(y')\}], \text{since A is FNL of L.}$$

$$\geq \min\{\bigwedge_{x'\in[x]_R} T(x'), \bigwedge_{y'\in[y]_R} T(y')\}=\min\{T_{\underline{R}(A)}(x),T_{\underline{R}(A)}(y)\}, \text{ie} T_{\underline{R}(A)}(x\vee y)\geq \min\{T_{\underline{R}(A)}(x),T_{\underline{R}(A)}(y)\}$$

$$T_{\underline{R}(A)}(x\wedge y)=\bigwedge_{x'\vee y'\in[x\vee y]_R} T_A(x'\wedge y')\geq \bigwedge_{x'\vee y'\in[x\vee y]_R}[\min\{T_A(x'),T_A(y')\}], \text{since A is FNL of L.}$$

$$\geq \min\{\bigwedge_{x'\in[x]_R} T(x'), \bigwedge_{y'\in[y]_R} T(y')\}=\min\{T_{\underline{R}(A)}(x),T_{\underline{R}(A)}(y)\}, \text{ie} T_{\underline{R}(A)}(x\wedge y)\geq \min\{T_{\underline{R}(A)}(x),T_{\underline{R}(A)}(y)\}$$

$$I_{\underline{R}(A)}(x\vee y)=\bigwedge_{x'\vee y'\in[x\vee y]_R} I_A(x'\wedge y')\geq \bigwedge_{x'\vee y'\in[x\vee y]_R}[\min\{I_A(x'),I_A(y')\}], \text{since A is FNL of L.}$$

$$\geq \min\{\bigwedge_{x'\in[x]_R} I(x'), \bigwedge_{y'\in[y]_R} I(y')\}=\min I_{\underline{R}(A)}(x),I_{\underline{R}(A)}(y)\}, \text{ie} I_{\underline{R}(A)}(x\vee y)\geq \min\{I_{\underline{R}(A)}(x),I_{\underline{R}(A)}(y)\}$$

$$I_{\underline{R}(A)}(x\wedge y)=\bigwedge_{x'\vee y'\in[x\vee y]_R} I_A(x'\wedge y')\geq \bigwedge_{x'\vee y'\in[x\vee y]_R}[\min\{I_A(x'),I_A(y')\}], \text{since A is FNL of L.}$$

$$\geq \min\{\bigwedge_{x'\in[x]_R} I(x'), \bigwedge_{y'\in[y]_R} I(y')\}=\min\{I_{\underline{R}(A)}(x),I_{\underline{R}(A)}(y)\}, \text{ie} I_{\underline{R}(A)}(x\wedge y)\geq \min\{I_{\underline{R}(A)}(x),I_{\underline{R}(A)}(y)\}$$

$$F_{\underline{R}(A)}(x\vee y)=\bigvee_{x'\vee y'\in[x\vee y]_R} F_A(x'\vee y')\geq \bigvee_{x'\vee y'\in[x\vee y]_R}[\max\{F_A(x'),F_A(y')\}], \text{since A is FNL of L.}$$

$$\geq \max\{\bigvee_{x'\in[x]_R} F(x'), \bigvee_{y'\in[y]_R} F(y')\}=\max\{F_{\underline{R}(A)}(x),F_{\underline{R}(A)}(y)\}, \text{ie} F_{\underline{R}(A)}(x\vee y)\geq \max\{F_{\underline{R}(A)}(x),F_{\underline{R}(A)}(y)\}$$

$$F_{\underline{R}(A)}(x\wedge y)=\bigvee_{x'\vee y'\in[x\vee y]_R} F_A(x'\wedge y')\geq \bigvee_{x'\vee y'\in[x\vee y]_R}[\max\{F_A(x'),F_A(y')\}], \text{since A is FNL of L.}$$

$$\geq \max\{\bigvee_{x'\in[x]_R} F(x'), \bigvee_{y'\in[y]_R} F(y')\}=\max\{F_{\underline{R}(A)}(x),F_{\underline{R}(A)}(y)\}, \text{ie} F_{\underline{R}(A)}(x\wedge y)\geq \max\{F_{\underline{R}(A)}(x),F_{\underline{R}(A)}(y)\}$$

Hence $\underline{R}(A)$ is a FNL of L.

$$T_{\overline{R}(A)}(x\vee y)=\bigvee_{x'\vee y'\in[x\vee y]_R} T_A(x'\vee y')\geq \bigvee_{x'\vee y'\in[x\vee y]_R}[\min\{T_A(x'),T_A(y')\}], \text{since A is FNL of L.}$$

$$\geq \min\{\bigvee_{x'\in[x]_R} T(x'), \bigvee_{y'\in[y]_R} T(y')\}=\min\{T_{\overline{R}(A)}(x),T_{\overline{R}(A)}(y)\}, \text{ie } T_{\overline{R}(A)}(x\vee y)\geq \min\{T_{\overline{R}(A)}(x),T_{\overline{R}(A)}(y)\}$$

similarly, $T_{\overline{R}(A)}(x\wedge y)\geq \min\{T_{\overline{R}(A)}(x),T_{\overline{R}(A)}(y)\}$

$$I_{\overline{R}(A)}(x\vee y)=\bigvee_{x'\vee y'\in[x\vee y]_R} I_A(x'\vee y')\geq \bigvee_{x'\vee y'\in[x\vee y]_R}[\min\{I_A(x'),I_A(y')\}], \text{since A is FNL of L.}$$

$$\geq \min\{\bigvee_{x'\in[x]_R} I(x'), \bigvee_{y'\in[y]_R} I(y')\}=\min\{I_{\overline{R}(A)}(x),I_{\overline{R}(A)}(y)\}, \text{ie } I_{\overline{R}(A)}(x\vee y)\geq \min\{I_{\overline{R}(A)}(x),I_{\overline{R}(A)}(y)\}$$

similarly, $I_{\overline{R}(A)}(x\wedge y)\geq \min\{I_{\overline{R}(A)}(x),I_{\overline{R}(A)}(y)\}$

$$F_{\overline{R}(A)}(x\vee y)=\bigvee_{x'\vee y'\in[x\vee y]_R} F_A(x'\vee y')\leq \bigwedge_{x'\vee y'\in[x\vee y]_R}[\max\{F_A(x'),F_A(y')\}], \text{since A is FNL of L.}$$

$$\geq \max\{\bigwedge_{x'\in[x]_R} F(x'), \bigwedge_{y'\in[y]_R} F(y')\}=\max\{F_{\overline{R}(A)}(x),F_{\overline{R}(A)}(y)\}, \text{ie} F_{\overline{R}(A)}(x\vee y)\leq \max\{F_{\overline{R}(A)}(x),F_{\overline{R}(A)}(y)\}$$

similarly, $F_{\overline{R}(A)}(x\wedge y)\leq \max\{F_{\overline{R}(A)}(x),F_{\overline{R}(A)}(y)\}$. Hence $\overline{R}(A)$ is a FNL of L.

Definition 3.10: A rough fuzzy neutrosophic set A of L is called a rough fuzzy neutrosophic lattice (RIFL) [rough neutrosophic fuzzy ideal (RIFI)] if both $\underline{R}(A)$ and $\overline{R}(A)$) are FNL's (FNI's) of L.

Theorem 3.11: If A is an FNL (FNI) of L then A is a RIFL (RIFI) of L. Proof. Follow from Proposition 3.4.

Theorem 3.12: If R (A) and R (B) are RFNL's (RIFI's), then R (A) ∩ R (B) is also a RFLN (RFNI).
Proof.

We have
$R(A)\cap R(B)=(\underline{R}(A)\cap\underline{R}(B),\overline{R}(A)\cap\overline{R}(B))$.
Since $R(A)$ and $R(B)$ are RFNL's (RFNI's) we have $\overline{R}(A)$, $\overline{R}(B)$, $\underline{R}(A)$ and $\underline{R}(B)$ are FNL's (FNI's).Then $\underline{R}(A)\cap\underline{R}(B)$ and $\overline{R}(A)\cap\overline{R}(B)$ are FNL's (FNI's) by Theorem 2.5.
So $R(A)\cap R(B)$ is a RFNL (RFNI) by Def 3.5

Remark 3.13: The union of two RFNI's need not be a RFNI. Consider the lattice L= {1, 2, 3, 4, 6, 12} of divisors of 12. Let R ={1,2),(3,6),(4),(12)}be the equivalence class . We define $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle / x\in L\}$ by

$A=\{\langle 1,0.7,0.8,0.6\rangle,\langle 2,0.3,0.7.0.1\rangle,\langle 3,0.4,0.7,0.8\rangle,\langle 4,0.9,0.4,0.5\rangle,\langle 6,0.7,0.1,0.3\rangle,\langle 12,0.6,0.1,0.4\rangle\}$

and $B=\{\langle x,T_B(x),I_B(x),F_B(x)\rangle / x\in L\}$

$B=\{\langle 1,0.7,0.7,0.5\rangle,\langle 2,0.6,0.5.0.4\rangle,\langle 3,0.5,0.5,0.3\rangle,\langle 4,0.7,0.8,0.1\rangle,\langle 6,0.5,0.4,0.3\rangle,\langle 12,0.5,0.3,0.2\rangle\}$

Here A and B are IFI's of L.

Now $R(A)=(\underline{R}(A),\underline{R}(A))$

where $\underline{R}(B)=\{\langle x,T_{\underline{R}(B)}(x),I_{\underline{R}(B)}(x),F_{\underline{R}(B)}(x)\rangle\}$ is

$\underline{R}(A)=\{\langle 1,0.3,0.7,0.6\rangle,\langle 2,0.3,0.7.0.6\rangle,\langle 3,0.4,0.1,0.8\rangle,\langle 4,0.9,0.4,0.5\rangle,\langle 6,0.4,0.1,0.8\rangle,\langle 12,0.6,0.1,0.4\rangle\}$

$\overline{R}(A)=\{\langle 1,0.7,0.8,0.1\rangle,\langle 2,0.7,0.8.0.1\rangle,\langle 3,0.7,0.7,0.3\rangle,\langle 4,0.9,0.4,0.5\rangle,\langle 6,0.7,0.7,0.3\rangle,\langle 12,0.6,0.1,0.4\rangle\}$ Also

$R(B)=(\underline{R}(B),\underline{R}(B))$

where $\underline{R}(B)=\{\langle x,T_{\underline{R}(A)}(x),I_{\underline{R}(A)}(x),F_{\underline{R}(A)}(x)\rangle\}$ is

$\underline{R}(B)=\{\langle 1,0.6,0.5,0.5\rangle,\langle 2,0.6,0.5.0.5\rangle,\langle 3,0.5,0.4,0.3\rangle,\langle 4,0.7,0.8,0.1\rangle,\langle 6,0.5,0.4,0.3\rangle,\langle 12,0.5,0.3,0.2\rangle\}$

$\overline{R}(B)=\{\langle 1,0.7,0.7,0.4\rangle,\langle 2,0.7,0.7.0.4\rangle,\langle 3,0.5,0.5,0.3\rangle,\langle 4,0.7,0.8,0.1\rangle,\langle 6,0.5,0.5,0.3\rangle,\langle 12,0.5,0.3,0.2\rangle\}$

clearly, $\underline{R}(A)$ and $\underline{R}(B)$ are RFNI's.

$A\cup B \Rightarrow$ ($\underline{R}(A)\cup\underline{R}(B)$, $\overline{R}(A)\cup\overline{R}(B)$) and ($\underline{R}(A)\cup\underline{R}(B)$, $\overline{R}(A)\cup\overline{R}(B)$)

$=\{\langle 1,0.6,0.7,0.5\rangle,\langle 2,0.7,0.7.0.4\rangle,\langle 3,0.5,0.5,0.3\rangle,\langle 4,0.9,0.8,0.1\rangle,\langle 6,0.5,0.4,0.3\rangle,\langle 12,0.6,0.3,0.2\rangle\}$

$T_{\underline{R}(A)\cup\underline{R}(B)}(3\vee 4)=T_{\underline{R}(A)\cup\underline{R}(B)}(12)=0.6\ngeq\min\{T_{\underline{R}(A)\cup\underline{R}(B)}(3),T_{\underline{R}(A)\cup\underline{R}(B)}(4)\}=0.7$

Hence $A\cup B$ is not an RIFI.

Remark 3.14: Every RIFI is a RIFL. But the converse is not true.
Consider the lattice and the equivalence relation given in the Result 3.8.Let

$B=\{\langle 1,0.2,0.7,0.2\rangle,\langle 2,0.4,0.4,0.7\rangle,\langle 3,0.2,0.5,0.5\rangle,\langle 4,0.3,0.6,0.2\rangle,\langle 6,0.5,0.5,0.3\rangle,\langle 12,0.3,0.3,0.5\rangle\}$

$\underline{R}(B)=\{\langle 1,0.4,0.4,0.2\rangle,\langle 2,0.4,0.4,0.2\rangle,\langle 3,0.5,0.5,0.3\rangle,\langle 4,0.3,0.6,0.2\rangle,\langle 6,0.5,0.5,0.3\rangle,\langle 12,0.3,0.3,0.5\rangle\}$

$\overline{R}(B)=\{\langle 1,0.2,0.4,0.7\rangle,\langle 2,0.2,0.4,0.7\rangle,\langle 3,0.2,0.5,0.5\rangle,\langle 4,0.3,0.6,0.2\rangle,\langle 6,0.2,0.5,0.5\rangle,\langle 12,0.3,0.3,0.5\rangle\}$ It can be easily verified that R(B) is RFNL, but RFNL because

$\underline{R}(B)(4\wedge 6)=\underline{R}(B)=12=0.3\ngeq\max\{T_{\underline{R}(B)}(4),T_{\underline{R}(B)}(6)=(0.5,0.3)=0.5$

4. Fuzzy Neutrosophic Rough Set (FNRS)

In this section we introduce Fuzzy Neutrosophic rough sublattices and ideals, and define certain characterization of Fuzzy Neutrosophic rough sublattice (ideal) in terms of level rough set.

Definition 4.1

Let X be a rough set and $R(A)=(\underline{R}(A),\overline{R}(A))$, is a FNRS in X. Then we can define an interval valued fuzzy neutrosophic rough set $A=\{\langle x,[\overline{T}_{\underline{R}(A)},T_{\underline{R}(A)}][\overline{I}_{\underline{R}(A)},I_{\underline{R}(A)}],[\overline{F}_{\underline{R}(A)},F_{\underline{R}(A)}]\rangle\}$

Where, $$\overline{T}_{\underline{R}(A)}(x)=T_{\underline{R}(A)}(x) \text{ if } x\in\underline{R}(X) \\ =0 \text{ if } x\in\overline{\overline{R}}(X)$$,

$$\overline{I}_{\underline{R}(A)}(x)=I_{\underline{R}(A)}(x) \text{ if } x\in\underline{R}(X) \\ =0 \text{ if } x\in\overline{\overline{R}}(X)$$

and $$\overline{F}_{\underline{R}(A)}(x)=F_{\underline{R}(A)}(x) \text{ if } x\in\underline{R}(X) \\ =1 \text{ if } x\in\overline{\overline{R}}(X)$$.

Where $\overline{\overline{R}}(X)=\overline{R}(X)-\underline{R}(X)$ and we denote $\vec{T}_A(x)=[\overline{T}_{\underline{R}(A)}(x),\overline{T}_{\overline{R}(A)}(x)]$ $\vec{I}_A(x)=[\overline{I}_{\underline{R}(A)}(x),\overline{I}_{\overline{R}(A)}(x)]$ and $\vec{F}_A(x)=[\overline{F}_{\underline{R}(A)}(x),\overline{F}_{\overline{R}(A)}(x)]$

Definition 4.2:

Let R(X) be a rough lattice and $R(A)=(\underline{R}(A),\overline{R}(A))$ is a FNRS in R(X). Then R(A) is called a fuzzy neutrosophic rough sublattice (FNRI) if for every $x\in\overline{R}(X)$ the following hold.

$(i)\ \overrightarrow{T_A}(x\vee y)\geq\min\{\vec{T}_A(x),\vec{T}_A(y)\}$ $(ii)\ \overrightarrow{T_A}(x\wedge y)\geq\min\{\vec{T}_A(x),\vec{T}_A(y)\}$

$(iii)\ \overrightarrow{I_A}(x\vee y)\geq\min\{\vec{I}_A(x),\vec{I}_A(y)\}$ $(iv)\ \overrightarrow{I_A}(x\wedge y)\geq\min\{\vec{I}_A(x),\vec{I}_A(y)\}$

$(v)\ \overrightarrow{F_A}(x\vee y)\geq\max\{\vec{F}_A(x),\vec{F}_A(y)\}$ $(v)\ \overrightarrow{F_A}(x\wedge y)\geq\max\{\vec{F}_A(x),\vec{F}_A(y)\}\ \forall x,y\in L$,

If conditions replaced by $\overrightarrow{T_A}(x\wedge y)\geq\max\{\vec{T}_A(x),\vec{T}_A(y)\}$, $\overrightarrow{I_A}(x\wedge y)\geq\max\{\vec{I}_A(x),\vec{I}_A(y)\}$ and $\overrightarrow{F_A}(x\wedge y)\geq\min\{\vec{F}_A(x),\vec{F}_A(y)\}$. Then R(A) is called a fuzzy neutrosophic rough ideal (FNRI).

Definition 4.3: Let R(X) be a rough lattice and $R(A)=(\underline{R}(A),\overline{R}(A))$ a FNRS in R(X). Then we define and

$\underline{A}_{(\alpha,\beta,\gamma)},\overline{A}_{(\alpha,\beta,\gamma)})=\underline{A}_{(\alpha,\beta,\gamma)}=\{x\in\underline{R}(X)/T_{\underline{R}(A)}(x)\geq\alpha,I_{\underline{R}(A)}(x)\geq\beta,F_{\underline{R}(A)}(x)\geq\gamma\}$

$\overline{A}_{(\alpha,\beta,\gamma)}=\{x\in\overline{R}(X)/T_{\overline{R}(A)}(x)\geq\alpha,I_{\overline{R}(A)}(x)\geq\beta,F_{\overline{R}(A)}(x)\geq\gamma\}$, then is called L-FNRS.

Theorem 4.4: Let R(X) be a rough lattice and $R(A)=(\underline{R}(A),\overline{R}(A))$ is a FNRS in R(X). Then R(A) is a FNRL iff $(\underline{R}(A)_{(\alpha,\beta,\gamma)},\overline{R}(A)_{(\alpha,\beta,\gamma)})$ is a rough sublattice of R(X).

Proof: First assume that $(\underline{R}(A)_{(\alpha,\beta,\gamma)},\overline{R}(A)_{(\alpha,\beta,\gamma)})$ is a rough sublattice in R(X). We have to prove that R(A) is a FNRL of R(X) . Set $\min\{\vec{T}_A(x),\vec{T}_A(y)\}=[\alpha_0,\alpha_1]$, $\min\{\vec{I}_A(x),\vec{I}_A(y)\}=[\beta_0,\beta_1]$ and $\max\{\vec{F}_A(x),\vec{F}_A(y)\}=[\gamma_0,\gamma_1]$.

Then $\min\{\overline{T}_{\underline{R}(A)}(x),\overline{T}_{\underline{R}(A)}(y)\}=\alpha_0$, $\min\{\overline{T}_{\overline{R}(A)}(x),\overline{T}_{\overline{R}(A)}(y)\}=\alpha_1$

$\min\{\overline{I}_{\underline{R}(A)}(x),\overline{I}_{\underline{R}(A)}(y)\}=\beta_0$, $\min\{\overline{I}_{\overline{R}(A)}(x),\overline{I}_{\overline{R}(A)}(y)\}=\beta_1$

$\max\{\overline{F}_{\underline{R}(A)}(x),\overline{F}_{\underline{R}(A)}(y)\}=\gamma_0$ $\max\{\overline{F}_{\overline{R}(A)}(x),F_{\overline{R}(A)}(y)\}=\gamma_1$

Then $T_{\overline{R}(A)}(x)\geq\alpha_1$, $T_{\overline{R}(A)}(y)\geq\alpha_1$, $I_{\overline{R}(A)}(x)\geq\beta_1$ $I_{\overline{R}(A)}(y)\geq\beta_1$ $F_{\overline{R}(A)}(x)\leq\gamma_0$ $F_{\overline{R}(A)}(y)\leq\gamma_0$,

Hence x, y $x,y\in\overline{A}_{(\alpha_1,\beta_1,\gamma_0)}\Rightarrow x\vee y,x\wedge y\in\overline{A}_{(\alpha_1,\beta_1,\gamma_0)}$, so $T_{\overline{R}(A)}(x\vee y)\geq\alpha_1$, $T_{\overline{R}(A)}(x\wedge y)\geq\alpha_1$

$I_{\overline{R}(A)}(x\vee y)\geq\beta_1$, $I_{\overline{R}(A)}(x\wedge y)\geq\beta_1$, $F_{\overline{R}(A)}(x\wedge y)\leq\gamma_0$, $F_{\overline{R}(A)}(x\vee y)\leq\gamma_0$.

Let $x,y\in\overline{R}(X)\Rightarrow$ either x or $y\in\vec{R}(X)$

If x or $y\in\vec{R}(X)$ then $\alpha_0=0$ and $\beta_1=1$.

So that $\overline{T}_{\underline{R}(A)}(x\vee y)\geq 0=\alpha_0$, $\overline{T}_{\underline{R}(A)}(x\wedge y)\geq 0=\alpha_0$

$\overline{I}_{\underline{R}(A)}(x\vee y)\geq 0=\beta_0$, $\overline{I}_{\underline{R}(A)}(x\wedge y)\geq 0=\beta_0$, $\overline{F}_{\underline{R}(A)}(x\vee y)\leq 1=\gamma_1$ and $\overline{F}_{\underline{R}(A)}(x\wedge y)\leq 1=\gamma_1$

If x and $y\notin\vec{R}(X)$,then $\overline{T}_{\underline{R}(A)}(x)=T_{\underline{R}(A)}(x),\overline{T}_{\underline{R}(A)}(y)=T_{\underline{R}(A)}(y)$, $\overline{I}_{\underline{R}(A)}(x)=I_{\underline{R}(A)}(x),\overline{I}_{\underline{R}(A)}(y)=I_{\underline{R}(A)}(y)$ and $\overline{F}_{\underline{R}(A)}(x)=F_{\underline{R}(A)}(x),\overline{F}_{\underline{R}(A)}(y)=F_{\underline{R}(A)}(y)$, so

$\min\{T_{\underline{R}(A)}(x),T_{\underline{R}(A)}(y)\}=\alpha_0$, $\min\{I_{\underline{R}(A)}(x),I_{\underline{R}(A)}(y)\}=\beta_0$ and $\max\{F_{\underline{R}(A)}(x),F_{\underline{R}(A)}(y)\}=\gamma_1$

$\Rightarrow T_{\underline{R}(A)}(x)\geq\alpha_0,T_{\underline{R}(A)}(y)\geq\alpha_0,I_{\underline{R}(A)}(x)\geq\beta_0,I_{\underline{R}(A)}(y)\geq\beta_0$ and $F_{\underline{R}(A)}(x)\geq\beta_0,F_{\underline{R}(A)}(y)\leq\gamma_1$

$\Rightarrow x,y\in\overline{A}_{(\alpha_0,\beta_0,\gamma_1)}\Rightarrow x\vee y,x\wedge y\in\overline{A}_{(\alpha_0,\beta_0,\gamma_1)}$, since $\overline{A}_{(\alpha_0,\beta_0,\gamma_1)}$ is a sublattice.

So $\overline{T}_{\underline{R}(A)}(x\vee y)\geq\alpha_0$ $\overline{T}_{\underline{R}(A)}(x\wedge y)\geq\alpha_0$ $\overline{I}_{\underline{R}(A)}(x\vee y)\geq\beta_0$ $\overline{I}_{\underline{R}(A)}(x\wedge y)\geq\beta_0$

$\overline{F}_{\underline{R}(A)}(x\vee y)\leq\gamma_1$ and $\overline{F}_{\underline{R}(A)}(x\wedge y)\leq\gamma_1$.

Hence $\overrightarrow{T_A}(x\vee y)=[\overline{T}_{\underline{R}(A)}(x\vee y),\overline{T}_{\overline{R}(A)}(x\vee y)]\geq[\alpha_0,\alpha_1]=\min\{\vec{T}_A(x),\vec{T}_A(y)\}$

$\overrightarrow{T_A}(x\wedge y)=[\overline{T}_{\underline{R}(A)}(x\wedge y),\overline{T}_{\overline{R}(A)}(x\wedge y)]\geq[\alpha_0,\alpha_1]=\min\{\vec{T}_A(x),\vec{T}_A(y)\}$

$\overrightarrow{I_A}(x\vee y)=[\overline{I}_{\underline{R}(A)}(x\vee y),\overline{I}_{\overline{R}(A)}(x\vee y)]\geq[\beta_0,\beta_1]=\min\{\vec{I}_A(x),\vec{I}_A(y)\}$

$\overrightarrow{I_A}(x\wedge y)=[\overline{I}_{\underline{R}(A)}(x\wedge y),\overline{I}_{\overline{R}(A)}(x\wedge y)]\geq[\alpha_0,\alpha_1]=\min\{\vec{I}_A(x),\vec{I}_A(y)\}$

$\overrightarrow{F_A}(x \vee y) = [\overline{F}_{\underline{R}(A)}(x \vee y), \overline{F}_{\overline{R}(A)}(x \vee y)] \leq [\gamma_0, \gamma_1] = \max\{\vec{F}_A(x), F_A(y)\}$

$\overrightarrow{F_A}(x \wedge y) = [\overline{F}_{\underline{R}(A)}(x \wedge y), \overline{F}_{\overline{R}(A)}(x \wedge y)] \leq [\gamma_0, \gamma_1] = \max\{\vec{F}_A(x), F_A(y)\}$

So R(A) is a FNRL

Conversely, assume that $R(A)$ is a FNRL of R(X). We have to prove that $\underline{A}_{(\alpha,\beta,\gamma)}$ and $\overline{A}_{(\alpha,\beta,\gamma)}$ are sublattices of L. Let $x, y \in \underline{A}_{(\alpha,\beta,\gamma)}$, then $T_{\underline{R}(A)}(x) \geq \alpha, T_{\underline{R}(A)}(y) \geq \alpha$ $I_{\underline{R}(A)}(x) \geq \beta, I_{\underline{R}(A)}(y) \geq \beta$, $F_{\underline{R}(A)}(x \leq \gamma, F_{\underline{R}(A)}(y) \leq \gamma$.

So $\min\{\vec{T}_A(x), \vec{T}_A(y)\} \geq [\alpha, \min\{T_{\overline{R}(A)}(x), T_{\overline{R}(A)}(y)\}]$, $\min\{\vec{I}_A(x), \vec{I}_A(y)\} \geq [\beta, \min\{I_{\overline{R}(A)}(x), I_{\overline{R}(A)}(y)\}]$ and $\max\{\vec{F}_A(x), \vec{F}_A(y)\} \leq [\gamma, \max\{F_{\overline{R}(A)}(x), F_{\overline{R}(A)}(y)\}]$. Hence

$\overrightarrow{T_A}(x \vee y) \geq [\alpha, \min\{T_{\overline{R}(A)}(x), T_{\overline{R}(A)}(y)\}], \overrightarrow{T_A}(x \wedge y) \geq [\alpha, \min\{T_{\overline{R}(A)}(x), T_{\overline{R}(A)}(y)\}]$

$\overrightarrow{I_A}(x \vee y) \geq [\beta, \min\{I_{\overline{R}(A)}(x), I_{\overline{R}(A)}(y)\}], \overrightarrow{I_A}(x \wedge y) \geq [\beta, \min\{I_{\overline{R}(A)}(x), I_{\overline{R}(A)}(y)\}]$

$\overrightarrow{F_A}(x \vee y) \leq [\max\{F_{\overline{R}(A)}(x), F_{\overline{R}(A)}(y)\}, \gamma], \overrightarrow{F_A}(x \wedge y) \leq [\max\{F_{\overline{R}(A)}(x), F_{\overline{R}(A)}(y)\}, \gamma]$

From these inequalities we get $\overline{T}_{\underline{R}(A)}(x \vee y) \geq \alpha, \overline{T}_{\underline{R}(A)}(x \wedge y) \geq \alpha$, $\overline{I}_{\underline{R}(A)}(x \vee y) \geq \beta, \overline{I}_{\underline{R}(A)}(x \wedge y) \geq \beta$, $\overline{F}_{\underline{R}(A)}(x \vee y) \leq \gamma, \overline{F}_{\underline{R}(A)}(x \wedge y) \leq \gamma$.

Since $x \vee y, x \wedge y \in \underline{R}(X)$

We have

$\overline{T}_{\underline{R}(A)}(x \vee y) = T_{\underline{R}(A)}(x \vee y), \overline{T}_{\underline{R}(A)}(x \wedge y) = \overline{T}_{\underline{R}(A)}(x \wedge y)$, $\overline{I}_{\underline{R}(A)}(x \vee y) = I_{\underline{R}(A)}(x \vee y), \overline{I}_{\underline{R}(A)}(x \wedge y) =$ $\overline{I}_{\underline{R}(A)}(x \wedge y)$, $\overline{F}_{\underline{R}(A)}(x \vee y) = F_{\underline{R}(A)}(x \vee y), \overline{F}_{\underline{R}(A)}(x \wedge y) =. \overline{F}_{\underline{R}(A)}(x \wedge y)$.

Hence $x \vee y, x \wedge y \in \underline{A}_{(\alpha,\beta,\gamma)}$.

Thus $\underline{A}_{(\alpha,\beta,\gamma)}$ is a sublattice. Similarly if $x, y \in \overline{A}_{(\alpha,\beta,\gamma)}$ then $T_{\overline{R}(A)}(x) \geq \alpha$, $T_{\overline{R}(A)}(y) \geq \alpha$ $I_{\overline{R}(A)}(x) \geq \beta$ $I_{\overline{R}(A)}(y) \geq \beta$ and $F_{\overline{R}(A)}(x) \leq \gamma$, $F_{\overline{R}(A)}(y) \leq \gamma$.

Hence $\vec{T}_A(x \vee y) \geq [0, \alpha], \vec{T}_A(x \wedge y) \geq [0, \alpha]$, $\vec{I}_A(x \vee y) \geq [0, \beta], \vec{I}_A(x \wedge y) \geq [0, \beta]$, $\vec{F}_A(x \vee y) \geq [\gamma, 1], \vec{F}_A(x \wedge y) \geq [\gamma, 1]$.

Hence $I_{\overline{R}(A)}(x \vee y) \geq \beta, I_{\overline{R}(A)}(x \wedge y) \geq \beta$, $F_{\overline{R}(A)}(x \vee y) \leq \gamma, F_{\overline{R}(A)}(x \wedge y) \leq \gamma$, so $x \vee y, x \wedge y \in \overline{A}_{(\alpha,\beta,\gamma)}$ Thus $\overline{A}_{(\alpha,\beta,\gamma)}$ is a sublattice

5. References

1. Antony Crispin Sweety C, Arockiarani I. Rough sets in Fuzzy Neutrosophic approximation space" [accepted]
2. Antony Crispin Sweety C, Arockiarani I. Fuzzy neutrosophic rough sets JGRMA, 2014; 2(3):54-59.
3. Dubios D, Prade H. Rough fuzzy sets and fuzzy rough sets, Int J Gen Syst. 1990; 17:191-208.
4. Gong Z, Sun B, Chen D. Rough set theory for the interval-valued fuzzy information systems, Inf Sci, 2008; 178:1968-1985.
5. Mi J.-S, Leung Y, Zhao H.-Y, Feng T. Generalized Fuzzy Rough Sets determined by a triangular norm, Inf Sci, 2008; 178:3203-3213.
6. Pawlak Z. Rough Sets, Int J Comput Inform Sci. 1982; 11:341-356.
7. Smarandache F. A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability,
8. Set and Logic. Rehoboth: American Research Press. 1999.
9. Smarandache F. Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics University of New Mexico, Gallup, NM 87301, USA (2002).
10. Sun B, Gong Z, Chen D. Fuzzy rough set theory for the interval-valued fuzzy information systems, Inf. Sci. 2008; 178:2794-2815.
11. Vasantha Kadasamy and Florentin Smarandache ~ Neutrosophic Lattices Neutrosophic Lattices Neutrosophic Sets and Systems, 2013; 2:42-50.
12. Wu WZ, Mi J.-S, Zhang W.-X. Generalized Fuzzy Rough Sets," Inf. Sci., 2003; 151:263-282.
13. Zhang Z. On interval type-2 rough fuzzy sets, Knowledge Based Syst., 2012; 35:1-13.