

**The Recursive Future And Past Equation Based On The Ananda-Damayanthi Normalized Similarity Measure  
Considered To Exhaustion**

ISSN 1751-3030

*Author:*

**Ramesh Chandra Bagadi**

*Affiliation 1:*

*Data Scientist*  
**International School Of  
Engineering (INSOFE)**  
2nd Floor, Jyothi Imperial,  
Vamsiram Builders, Janardana  
Hills, Above South India  
Shopping Mall, Old Mumbai  
Highway, Gachibowli,  
Hyderabad, TelanganaState,  
500032, India.

Email:  
ramesh.bagadi@insofe.edu.in  
Tel:+91 9440032711

*Affiliation 2:*

*Founder & Owner*  
**texN Consulting  
Private Limited,**  
Gayatrinagar,  
Jilleleguda,  
Hyderabad,  
Telengana State,  
500097, India.

Email:  
rameshcbagadi@  
uwalumni.com  
Tel:+91  
9440032711

*Affiliation 3:*

*Founder & Owner*  
**Ramesh Bagadi  
Consulting LLC**  
(R420752),  
Madison,  
Wisconsin-53715,  
United States Of  
America.

Email:  
rameshcbagadi@  
uwalumni.com

## Abstract

In this research investigation, the author has presented a Recursive Past Equation and a Recursive Future Equation based on the Ananda-Damayanthi Normalized Similarity Measure considered to Exhaustion [1] (please see the addendum of [1] as well).

### The Recursive Future Equation

Given a Time Series  $Y = \{y_1, y_2, y_3, \dots, y_{n-1}, y_n\}$

we can find  $y_{n+1}$  using the following Recursive Future Equation

$$y_{n+1} = \frac{\left\{ \sum_{k=1}^n y_k \left\{ \sum_{j=0}^m \left\{ \frac{S_{kj}}{L_{kj}} \right\} \right\} \right\}}{\sqrt{\sum_{k=1}^n \left\{ \sum_{j=0}^m \left\{ \frac{S_{kj}}{L_{kj}} \right\}^2 \right\}}}$$

where

$S_{kj} = \text{Smaller of The } j^{\text{th}} \text{ Order Difference Residual of } (y_{n+1}, y_k)$

and

$L_{kj} = \text{Larger of The } j^{\text{th}} \text{ Order Difference Residual of } (y_{n+1}, y_k)$

(This will be detailed in the next section)

where  $j = 0 \text{ to } m$  is a Number which makes the Difference Residual  $(L_{kj} - S_{kj})$  tend to Zero.

From the above Recursive Equation, we can solve for  $y_{n+1}$ .

**Proof:**

We consider  $y_k$  and find the Ananda-Damayanthi Similarity [1] between  $y_k$  and  $y_{n+1}$  which we refer as  $\left\{ \frac{S_k}{L_k} \right\} = \left\{ \frac{\text{Smaller of } y_k \text{ and } y_{n+1}}{\text{Larger of } y_k \text{ and } y_{n+1}} \right\}$ . We now consider the lack of similarity part, i.e.,  $(L_k - S_k)$  and again find the Similarity

between  $y_k$  and  $(L_k - S_k)$  (this is the Difference Residual of First Order) which (the aforementioned Similarity) we refer to as

$\left\{ \frac{S_{k1}}{L_{k1}} \right\} = \left\{ \frac{\text{Smaller of } (L_k - S_k) \text{ and } y_{n+1}}{\text{Larger of } (L_k - S_k) \text{ and } y_{n+1}} \right\}$  wherein the Difference Residual of Second Order is  $(L_{k1} - S_{k1})$ . And similarly, we find

$\left\{ \frac{S_{k2}}{L_{k2}} \right\} = \left\{ \frac{\text{Smaller of } (L_{k1} - S_{k1}) \text{ and } y_{n+1}}{\text{Larger of } (L_{k1} - S_{k1}) \text{ and } y_{n+1}} \right\}$ ,  $\left\{ \frac{S_{k3}}{L_{k3}} \right\} = \left\{ \frac{\text{Smaller of } (L_{k2} - S_{k2}) \text{ and } y_{n+1}}{\text{Larger of } (L_{k2} - S_{k2}) \text{ and } y_{n+1}} \right\}$ , .....

$\left\{ \frac{S_{km}}{L_{km}} \right\} = \left\{ \frac{\text{Smaller of } (L_{k(m-1)} - S_{k(m-1)}) \text{ and } y_{n+1}}{\text{Larger of } (L_{k(m-1)} - S_{k(m-1)}) \text{ and } y_{n+1}} \right\}$ . Note that we represent the second index by  $j$  which goes from 0 to  $m$ . We now

add them all. Similarly, we consider such terms for  $k = 1$  to  $n$  and compute such aforementioned quantities and add them all.

We now Normalize ( $L^2$  Norm), i.e., divide each of this value by the quantity  $\sqrt{\sum_{k=1}^n \left\{ \sum_{j=0}^m \left\{ \frac{S_{kj}}{L_{kj}} \right\}^2 \right\}}$ . We equate this value to  $y_{n+1}$  as the

RHS is the Total Normalized Similarity contribution from each element of the Time Series Set  $Y = \{y_1, y_2, y_3, \dots, y_{n-1}, y_n\}$  with respect to  $y_{n+1}$ . Note that the Similarity term corresponding to the Difference Residual of Zeroth Order can be represented as

$\left\{ \frac{S_{k0}}{L_{k0}} \right\}$  which is actually  $\left\{ \frac{S_k}{L_k} \right\}$  itself.

**Defining Error**

We define Error in the following fashion:

*For the Recursive Future Equation:*

### Method 1

Given a Time Series  $Y = \{y_1, y_2, y_3, \dots, y_{n-1}, y_n\}$  we consider only  $Y = \{y_1, y_2, y_3, \dots, y_{n-1}\}$  and use the aforementioned Recursive Future Equation to find the  $n^{\text{th}}$  term. Say this is  ${}^p y_n$  where the  $p$  stands for the ‘predicted’ or ‘forecasted’ value. Then, the Error is defined by

$$\varepsilon_F = \left( \frac{y_n - {}^p y_n}{y_n} \right)$$

### Method 2

Given a Time Series  $Y = \{y_1, y_2, y_3, \dots, y_{n-1}, y_n\}$  we consider it and use the aforementioned Recursive Future Equation to find the  $(n+1)^{\text{th}}$  term. Say this is  ${}^p y_{n+1}$  where the  $p$  stands for the ‘predicted’ or ‘forecasted’ value. We now consider the Time Series Set  $Y = \{y_2, y_3, \dots, y_{n-1}, y_n, {}^p y_{n+1}\}$  and use the aforementioned Recursive Past Equation to generate the term previous to  $y_2$ , i.e.,  ${}^p y_1$ . Then, the Error is defined by

$$\varepsilon_F = \left( \frac{y_1 - {}^p y_1}{y_1} \right)$$

### The Recursive Past Equation

Given a Time Series  $Y = \{y_1, y_2, y_3, \dots, y_{n-1}, y_n\}$

we can find  $y_0$  using the following Recursive Past Equation

$$y_{n+1} = \frac{\left\{ \sum_{k=0}^{n-1} y_k \left\{ \sum_{j=0}^m \left[ \frac{S_{kj}}{L_{kj}} \right] \right\} \right\}}{\sqrt{\sum_{k=0}^{n-1} \left\{ \sum_{j=0}^m \left[ \frac{S_{kj}}{L_{kj}} \right]^2 \right\}}}$$

where

$S_{kj} = \text{Smaller of The } j^{\text{th}} \text{ Order Difference Residual of } (y_n, y_k)$  and

$L_{kj} = \text{Larger of The } j^{\text{th}} \text{ Order Difference Residual of } (y_n, y_k)$

where  $j = 0 \text{ to } m$  is a Number which makes the Difference Residual  $(L_{kj} - S_{kj})$  tend to Zero.

From the above Recursive Equation, we can solve for  $y_0$ .

**Proof:**

We consider  $y_k$  and find the Ananda-Damayanthi Similarity [1] between  $y_k$  and  $y_n$  which turns out to be

$\left\{ \frac{S_k}{L_k} \right\} = \left\{ \frac{\text{Smaller of } y_k \text{ and } y_n}{\text{Larger of } y_k \text{ and } y_n} \right\}$ . We now consider the lack of similarity part, i.e.,  $(L_k - S_k)$  and again find the Similarity between

$y_k$  and  $(L_k - S_k)$  (this is the Difference Residual of First Order) which (the aforementioned Similarity) we refer to as

$\left\{ \frac{S_{k1}}{L_{k1}} \right\} = \left\{ \frac{\text{Smaller of } (L_k - S_k) \text{ and } y_n}{\text{Larger of } (L_k - S_k) \text{ and } y_n} \right\}$  wherein the Difference Residual of Second Order is  $(L_{k1} - S_{k1})$ . And similarly, we find

$\left\{ \frac{S_{k2}}{L_{k2}} \right\} = \left\{ \frac{\text{Smaller of } (L_{k1} - S_{k1}) \text{ and } y_n}{\text{Larger of } (L_{k1} - S_{k1}) \text{ and } y_n} \right\}$ ,  $\left\{ \frac{S_{k3}}{L_{k3}} \right\} = \left\{ \frac{\text{Smaller of } (L_{k2} - S_{k2}) \text{ and } y_n}{\text{Larger of } (L_{k2} - S_{k2}) \text{ and } y_n} \right\}$ , .....,

$\left\{ \frac{S_{km}}{L_{km}} \right\} = \left\{ \frac{\text{Smaller of } (L_{k(m-1)} - S_{k(m-1)}) \text{ and } y_n}{\text{Larger of } (L_{k(m-1)} - S_{k(m-1)}) \text{ and } y_n} \right\}$ . Note that we represent the second index by  $j$  which goes from 0 to  $m$ . We now

add them all. Similarly, we consider such terms for  $k = 0$  to  $n - 1$  and compute such aforementioned quantities and add them

all. We now Normalize ( $L^2$  Norm), i.e., divide each of this value by the quantity  $\sqrt{\sum_{k=0}^{n-1} \left\{ \sum_{j=0}^m \left\{ \frac{S_{kj}}{L_{kj}} \right\}^2 \right\}}$ . We equate this value to  $y_n$  as

the RHS is the Total Normalized Similarity contribution from each element of the Time Series Set  $Y = \{y_0, y_1, y_2, y_3, \dots, y_{n-1}\}$  with

respect to  $y_n$ . Note that the Similarity term corresponding to the Difference Residual of Zeroth Order can be represented as  $\left\{ \frac{S_{k0}}{L_{k0}} \right\}$

which is actually  $\left\{ \frac{S_k}{L_k} \right\}$  itself.

### **Defining Error**

We define Error in the following fashion:

*For the Recursive Past Equation:*

#### *Method 1*

Given a Time Series  $Y = \{y_1, y_2, y_3, \dots, y_{n-1}, y_n\}$  we consider only  $Y = \{y_2, y_3, \dots, y_{n-1}, y_n\}$  and use the aforementioned Recursive Future Past to find the 1<sup>st</sup> term. Say this is  ${}^p y_1$  where the  $p$  stands for the ‘predicted’ or ‘forecasted’ value. Then, the Error is defined by

$$\varepsilon_p = \left( \frac{y_1 - {}^p y_1}{y_1} \right)$$

#### *Method 2*

Given a Time Series  $Y = \{y_1, y_2, y_3, \dots, y_{n-1}, y_n\}$  we consider it and use the aforementioned Recursive Future Equation to find the term previous to  $y_1$ . Say this is  ${}^p y_0$  where the  $p$  stands for the ‘predicted’ or ‘forecasted’ value. We now consider the Time Series Set  $Y = \{{}^p y_0, y_1, y_2, y_3, \dots, y_{n-1}\}$  and use the aforementioned Recursive Future Equation to generate the term next to  $y_{n-1}$ , i.e.,  ${}^p y_n$ . Then, the Error is defined by

$$\varepsilon_F = \left( \frac{y_n - {}^p y_n}{y_n} \right)$$

### Computation Complexity

For the World's fastest Japanese Super-Computer which can compute 1 Quadrillion Computations per second

we can use the equation  $2^{(m+n)} = 10^{15}$  to calculate the Maximum Number of Terms of the Time Series  $n$  for which we wish to predict the  $(n+1)^{th}$  term and  $m$  is the Number Of Difference Residual Terms we wish to consider for each term, to find the  $n$  for a given  $m$  so that the  $(n+1)^{th}$  term is computed in one second.

Furthermore, if we take  $m = 8 \text{ or } 10$  (beyond which the value of the Difference Residuals is near vanishing) and for different amounts of times we can spare for getting the computed answer, the Number of Terms of the Time Series  $n$  that we can consider is given below:

<i>Serial Number</i>	<i>Duration Of Computation</i>	<i>Number of Terms <math>n</math> To Consider</i>
1	1 Second	21.64043 – $m$
2	1 Minute	25.66808 – $m$
3	1 Hour	29.69574 – $m$
4	1 Day	34.2807 – $m$
5	1 Week	37.0886 – $m$
6	1 Month (31 Days)	39.2349 – $m$
7	1 Year	42.79246 – $m$

That is, if the Time Series Set were to contain  $n$  number of terms (as shown in the table for varying values of  $m$ , namely 8 and 10, then the Duration of Computation is tabulated above.

### *For Forecasting Future Element*

We have  $2^{(m+n)}$  number of 6<sup>th</sup> Order Polynomial Equations of the kind as shown in equation A to solve as these account for all the cases of the Time Series Set Elements being greater or lesser than the future  $(n+1)^{th}$  element to be computed, as these equations are being represented by the aforementioned Recursive Future Equation. Only one among them is the correct equation and this can be found by using this thusly computed  $(n+1)^{th}$  value and omitting the first element  $y_1$ , using the Time Series Set  $Y = \{y_2, y_3, \dots, y_{n-1}, y_n, y_{n+1}\}$  we predict the element  $y_1$  using the aforementioned Recursive Past Equation. And one of the  $2^{(m+n)}$  number of 6<sup>th</sup> Order Polynomial Equations of the kind as shown in equation A which gives the best true value of  $y_1$  can be considered as the correct equation and its future element forecast of  $y_{n+1}$  as the correct forecast.

### *For Forecasting Past (to the First) Element*

We have  $2^{(m+n)}$  number of 6<sup>th</sup> Order Polynomial Equations of the kind as shown in equation A to solve as these account for all the cases of the Time Series Set Elements being greater or lesser than the past element  $y_0$  to be computed, as these equations are being represented by the aforementioned Recursive Past Equation. Only one among them is the correct equation and this can be found by using this thusly computed  $y_0$  value and omitting the latest element  $y_n$ , using the Time Series Set  $Y = \{y_0, y_1, y_2, y_3, \dots, y_{n-1}\}$  we predict the element  $y_n$  using the aforementioned Recursive Future Equation. And one of the  $2^{(m+n)}$  number of 6<sup>th</sup> Order Polynomial Equations of the kind as shown in equation A which gives the best true value of  $y_n$  can be considered as the correct equation and its past element forecast of  $y_0$  as the correct forecast.

### **References**

1. Bagadi, R. (2016). Proof Of As To Why The Euclidean Inner Product Is A Good Measure Of Similarity Of Two Vectors. *PHILICA.COM Article number 626*. See the Addendum as well.  
[http://philica.com/display\\_article.php?article\\_id=626](http://philica.com/display_article.php?article_id=626)
2. [http://www.vixra.org/author/ramesh\\_chandra\\_bagadi](http://www.vixra.org/author/ramesh_chandra_bagadi)
3. <http://philica.com/advancedsearch.php?author=12897>