On e-μ-τ Universality and Cabibbo type mixing among leptons

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Using e-μ universality and with the help of the Weinberg type of mixing parameters obtained from e-μ masses, the masses of the charged W_R and the neutral D-boson are shown to be 73.39 GeV and 86.16 GeV respectively. The weak interaction constant of the tau-lepton and its neutrino is shown to be related to the Fermi constant through an angle of 5.6 degrees which also happens to be the angle with Cabibbo type of mixing among leptons.

Keywords: Lepton universality, Fermi constant, Cabibbo Mixing of leptons, Neutrino oscillations.

1. INTRODUCTION:

The weak current is the sum of a leptonic and a hadronic part. The effective action is a product of two such currents. The effective weak action can be broken into three pieces, generating hadronic-hadronic, hadronic-leptonic and leptonic-leptonic interactions,

$$L_{eff} = \frac{G_F}{\sqrt{2}} \sum \left\{ j_{lep} + j_{had} \right\}_{\mu}^{\dagger} \left\{ j_{lep} + j_{had} \right\}^{\mu} \qquad (1)$$

Where we sum over both charged and neutral currents. In Eq. (1) G_F is the Fermi universal weak interaction constant. This constant is supposed to be the same for all lepton-lepton interactions. This uniqueness of G_F is what is known as e-μ-τ universality.

Here we will show that e-mu universality leads to the prediction of W_R and D-boson masses. These bosons are necessary in a gauge model based on the gauge group $SU(2)_L$X $SU(2)_R$ X U(1), in addition to Standard W and Z bosons. On the other-hand e-mu-tau universality is not necessarily valid.

This paper is about that possibility only. In section (2) we will determine the masses of the gauge bosons using e-mu universality. In section (3) we will determine the weak interaction constant for the tau-lepton. In section (4) Cabibbo type of mixing is defined for the leptons. In section 5 neutrino oscillations are considered.

2. e-mu UNIVERSALITY

In gauge theories the Fermi weak interaction constant can be shown to be related to the gauge constant and the mass of the charged boson that mediates the interaction, through the relation,

$$G_F = \frac{g^2}{4\sqrt{2}} \frac{1}{M^2} \qquad (2.1)$$

Where g is the gauge constant and M is the mass of the charged boson that mediates the interaction.

In Ref. [1] it was shown that the electron and muon masses are related by,

$$\frac{2m_e m_\mu}{m_e^2 + m_\mu^2} = \left(\frac{g_V}{g_A}\right)^2_{e\mu} = \left(-1 + 4\sin^2 \vartheta_W\right)^2, \tag{2.2}$$

Where g_v and g_A are the vector and axial vector coupling constants of the charged leptons indicated by the subscripts with the neutral Z-boson [2]. The above equation gives two values for the mixing parameters.

$$x_L = \sin^2 \vartheta_L = \frac{e^2}{g_L^2} = 0.2254, \tag{2.3}$$

$$x_R = \sin^2 \vartheta_R = \frac{e^2}{g_R^2} = 0.2746, \tag{2.4}$$

One of these mixing parameters coincides with the Weinberg mixing parameter of the standard model [5] and the other one arises in a gauge model [3,4] based on the gauge group $SU(2)_L \: X \: SU(2)_R \: X \: U(1)$.

The corresponding gauge constants of this gauge group are g_L, g_R and g_Y respectively. Using Eq. (2.3) in Eq. (2.1) it just follows that,

$$G_F = \frac{e^2}{0.2254}\frac{1}{4\sqrt{2}}\frac{1}{M_{WL}^2}, \tag{2.5}$$

From which the mass of the standard charged W_L boson can be found. Once the W_L boson mass is known, from the electro-weak gauge theory it also follows that,

$$M_Z = \frac{M_{WL}}{\cos\vartheta_L} \tag{2.6}$$

Where M_Z is the mass of the neutral Z-boson of the standard model. But there are two mixing parameters. That means, the electro-weak interactions of the electron, muon and their neutrinos are mediated by the W_L boson, the neutral Z-boson, the charged W_R boson, the neutral D-boson in addition to the photon. The e-μ universality can now be invoked to determine the mass of the charged W_R boson,

$$G_F = \frac{g_L^2}{4\sqrt{2}M_{WL}^2} = \frac{g_R^2}{4\sqrt{2}M_{WR}^2} \tag{2.7}$$

Using Eqn. (2.3) and (2.4) in Eq. (2.7) we observe that,

$$M_{WR} = 0.905997 M_{WL} = 73.3856 GeV. \tag{2.8}$$

From Eq. (2.8) it also follows that,

$$M_D = \frac{M_{WR}}{\cos\vartheta_R} = 86.1632 GeV . \qquad (2.9)$$

Where M_D is mass of the neutral D-boson in the electro-weak gauge model, based on the gauge group $SU(2)_L \ X \ SU(2)_R \ X \ U(1)$ with mixing parameters given by Eqn. (2.3) and (2.4). This gauge model contains the standard electroweak model because $x_L = x_W$ where x_W is the Weinberg mixing parameter.

For this reason only we used the symbol θ_W for the mixing angle in Eq. (2.2).

3. THE TAU-LEPTON

There are three generations of leptons. The charged τ-lepton has no counter partner [2] whereas the electron has a counter partner, the muon. If we wish to find a relation like Eq. (2.2), we should consider the τ-lepton as its own counter partner as in Ref. (2). Hence,

$$\frac{2m_{tau}m_{tau}}{m_{tau}^2 + m_{tau}^2} = \left[-1 + 4\sin^2\vartheta_m\right]^2 \qquad (3.1)$$

Where $\sin^2\theta_m$ is a mixing parameter like the Weinberg mixing parameter, and θ_m is a mixing angle like the Weinberg mixing angle. Eq. (3.1) yields the following values for $\sin^2\theta_m$.

$$\frac{e^2}{g_2^2} = \sin^2\vartheta_m = 0.5 \, or \, 0 . \qquad (3.2)$$

That means there is only one mixing parameter, 0.5, the other one being zero, which implies there is no mixing.

The gauge model of the τ-lepton and its neutrino must then be based on the gauge group $SU(2)_L \ X \ U(1)$, with the corresponding gauge constants g_2 and g_3 respectively. The electroweak interactions of the τ-lepton and its neutrino must therefore be mediated by the charged W_1 boson and Z_1 boson in addition to the photon. In Ref. [2], the masses of these gauge bosons are found using the experimental masses of the τ-lepton and the τ-neutrino:

$$M_{W_1} = 173.5 GeV, \tag{3.3}$$

And,

$$M_{Z_1} = 245.366 GeV. \tag{3.4}$$

If gauge bosons with the above masses are observed in the forthcoming CERN experiments, the e-μ-τ universality is violated. This paper is about that possibility only.

The effective action involving τ-lepton and its neutrino with other leptons must have a coupling constant G'_F and it should be different from the Fermi coupling constant G_F. A natural way is to relate G'_F and G_F through an angle θ where,

$$G'_F = G_F \tan\theta. \tag{3.5}$$

The effective action of τ-lepton and its neutrino with other leptons involves a coupling constant G'_F where,

$$G'_F = \frac{g_2^2}{M_{W_1}^2}\frac{1}{4\sqrt{2}} = \frac{e^2}{(0.5)}\frac{1}{M_{W_1}^2}\frac{}{4\sqrt{2}} . \tag{3.6}$$

From Eqn. (2.5), (3.5) and (3.6), it just follows that,

$$\vartheta = 5.611589^\circ , \tag{3.7}$$

While computing the angle in Eq.(3.7) we took M_{WL}= 81 GeV. This angle is computed using the masses of the mediating charged gauge bosons. It can also be computed in a different way as shown in the next section.

4. CABBIBO TYPE OF MIXING AMONG LEPTONS

Historically the addition of the strange quark to the then known quarks brought in an important complication, the quark mixing angles. In a similar way the addition of the τ-lepton should lead to a lepton mixing angle(s).

$$\nu'_e = \nu_e \cos\vartheta' - \nu_\tau \sin\vartheta' , \tag{4.1}$$

And,

$$\nu'_\tau = \nu_e \sin\vartheta' + \nu_\tau \cos\vartheta' . \qquad (4.2)$$

The angle θ′ is a mixing angle much like the Cabibbo angle in the quark sector and υ_e and υ_τ are mass eigen states [1, 2]. We will follow Fritzsch [6, 7] and Li [8] to compute this angle [9, 10]. Fritzsch-Ansatz for the mass matrix states that only the heaviest one in the generation has a diagonal element and all other lighter masses arise through mixings between neighboring families.

For two generations the mass matrix,

$$M_e = \begin{pmatrix} 0 & \sqrt{m_1 m_e} \\ \sqrt{m_1 m_e} & m_e - m_1 \end{pmatrix} \qquad (4.3)$$

Where m_e is the mass of the electron and m_1 is the mass of the electron neutrino [1]. The above mass matrix is diagonalized by the orthogonal matrix, O_e, where,

$$O_e = \begin{pmatrix} \sqrt{\dfrac{m_e}{m_1 + m_e}} & -\sqrt{\dfrac{m_1}{m_1 + m_e}} \\ \sqrt{\dfrac{m_1}{m_1 + m_e}} & \sqrt{\dfrac{m_e}{m_1 + m_e}} \end{pmatrix} \qquad (4.4)$$

The Fritzsch type of mass matrix M_τ for the τ-lepton and its neutrino is obtained by

$$M_\tau = \begin{pmatrix} 0 & \sqrt{m_2 m_\tau} \\ \sqrt{m_2 m_\tau} & m_\tau - m_2 \end{pmatrix} \tag{4.5}$$

Where m_2 is the mass of the τ-neutrino and m_τ is the mass of the charged τ-lepton. The mass matrix M_τ is diagonalized by the orthogonal matrix O_τ, where

$$O_\tau = \begin{pmatrix} \sqrt{\dfrac{m_\tau}{m_\tau + m_2}} & -\sqrt{\dfrac{m_2}{m_\tau + m_2}} \\ \sqrt{\dfrac{m_2}{m_\tau + m_2}} & \sqrt{\dfrac{m_\tau}{m_\tau + m_2}} \end{pmatrix} \tag{4.6}$$

The Cabibbo-type of lepton mixing matrix is given by the orthogonal matrix V_c, where,

$$V_c = O_{tau} (O_e)^{trans}. \tag{4.7}$$

$$V_c(\theta') = \begin{pmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{pmatrix} \tag{4.8}$$

From Eq. (4.7), (4.4) and (4.5) it just follows that,

$$\vartheta' = \tan^{-1}\sqrt{\frac{m_2}{m_{tau}}} - \tan^{-1}\sqrt{\frac{m_1}{m_e}} \qquad = 5.778816^\circ - 0.204346^\circ = 5.57447^\circ . \qquad (4.9)$$

The angles, θ and θ′are obtained from different numbers. But these are very nearly equal .This is an indication that Cabibbo type of mixing is present among the leptons as well. In the light of what is shown here more theoretical estimates of τ interactions and the τ-lepton life time should be undertaken to confront experiments afresh.

5. NEUTRINO OSCILLATIONS

In view of the mixing of υ_e and υ_τ with the mixing angle θ, the relative phase of υ_e and υ_τ changes because of the mass difference so that a neutrino originating as υ_e' has a non-zero probability of being detected as υ_τ'. If an electron-type neutrino is propagating with momentum P at time t = 0, it will have a probability $P_{V\tau}$ of oscillation into a υ_τ' given by [11],

$$P_{\nu\tau} \approx \sin^2 2\vartheta \sin^2\left[\frac{\Delta m^2 L}{4E_1}\right] \qquad (5.1)$$

Where

$$\Delta m^2 = m_2{}^2 - m_1{}^2 . \qquad (5.2)$$

This is the expression that is usually employed in two flavor analyses of neutrino oscillation experiments. The τ-neutrino mass m_2 is equal to 18.2 MeV. Compared to this mass the mass of the electron neutrino (6.5 eV) is negligible. The energy E_1 of the electron neutrino is given by,

$$E_1 = \sqrt{P^2 + m_1^{\;2}} \approx P + \frac{m_1^{\;2}}{2P} + ... \qquad (5.3)$$

It contains the two theoretical parameters θ and Δm^2 both are now known. Observe that we have written the expression in terms of the path length $L \approx \frac{t}{c}$. In units of a vacuum length, L_V

$$P_{\nu\tau} \approx \sin^2 2\vartheta \sin^2\left[\frac{\Pi L}{L_V}\right] \text{, Where, } \left[L_V = \frac{4\Pi E_1}{\Delta m^2}\right] \qquad (5.4)$$

For $\Delta m^2 = 2.48 eV^2$ and $E_1 = 1MeV$, one has $L_V = 1$ meter .Our analysis is now available for experimental verification. Except the τ-lepton mean life time everything is in conformity.

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