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Shortest Path Problem under Trapezoidal Neutrosophic Information

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***Abstract*—In this study, we propose an approach to determine the shortest path length between a pair of specified nodes s and t on a network whose edge weights are represented by trapezoidal neutrosophic numbers. Finally, an illustrative example is provided to show the applicability and effectiveness of the proposed approach.**

Keywords—trapezoidal fuzzy neutrosophic sets; score function; shortest path problem

I. INTRODUCTION

In 1995, Smarandache introduced the concept of Neutrosophy. It is a branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Smarandache [1] introduced the concept of neutrosophic set (NS) and neutrosophic logic as generalization of the concepts of fuzzy sets [3], intuitionistic fuzzy sets [4]. Neutrosophic set has the ability to deal with certain type of uncertain information such as incomplete, indeterminate and inconsistent information, which exist in real world, cannot be dealt with fuzzy sets as well as intuitionistic fuzzy sets. The concept of neutrosophic set is characterized by three independent membership degrees namely truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F).

In order to practice NSs in real –life applications conveniently. Smarandache [1] and Wang et al. [5] introduced a subclass of the neutrosophic sets, called single-valued neutrosophic sets in which the values of the three membership function T, I, F belongs to the unit interval [0, 1]. SVNS was studied deeply by many researchers. The concept of single valued neutrosophic sets has caught attention to the researcher on various topics such as to be such as the decision making problem, medical diagnosis and so on. Additional literature on single valued neutrosophic sets can be found in [6-14]. Also later on, Smarandache extended the neutrosophic set to neutrosophic overset, underset, and offset [15]. However, in uncertain and complex situations, the truth-membership, the indeterminacy-membership and the falsity-membership independently of SVNS cannot be represented with exact real numbers or interval numbers Moreover, triangular fuzzy number can handle effectively fuzzy data rather than interval number. For this purpose, Biswas et al. [16] proposed the concept of triangular fuzzy number neutrosophic sets (TFNNS) by combining triangular fuzzy numbers (TFNs) and single valued neutrosophic set (SVNS) and define some of its operational rules and developed triangular fuzzy neutrosophic number weighted arithmetic averaging and triangular fuzzy neutrosophic number weighted arithmetic geometric averaging operators to solve multi-attribute decision making problem. In TFNNS the truth, indeterminacy and the falsity-membership functions are expressed with triangular fuzzy numbers instead of real numbers. In addition, Ye [17] presented the concept of trapezoidal fuzzy neutrosophic set and developed trapezoidal fuzzy neutrosophic number weighted arithmetic averaging and trapezoidal fuzzy neutrosophic number weighted arithmetic geometric averaging operators to solve multi-attribute decision making problem. Very Recently, Broumi et al. [18-26] proposed the concept of neutrosophic graphs, interval valued neutrosophic graphs and bipolar single valued neutrosophic graphs. Smarandache and Kandasamy [27-29] proposed another variant of neutrosophic graphs based on literal indeterminacy. The shortest path problem is one of the most fundamental problems in graph theory which has many applications diversified field such operation research, computer science, communication network and so on. In a network, the shortest path problem concentrate at finding the path from one source to destination node with minimum weight, where some weight is attached to each edge connecting a pair of nodes. In the literature, many shortest path problems [30-39] have been studied with different types of input data, including fuzzy set, intuitionistic fuzzy sets, trapezoidal intuitionistic fuzzy sets vague set. Till now, few research papers deal with shortest path in neutrosophic environment. Broumi et al. [40] proposed an algorithm for solving neutrosophic shortest path problem based on score function. The same authors in [41-44] proposed an algorithm

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to determine the shortest path in a network where the weight of edges are represented by a neutrosphic numbers. The shortest path problem involves addition and comparison of the edge lengths. Since the addition and comparison are not alike those between two precise real numbers. In this paper, the addition operation and the order relation have been given by Ye.[17]. In this paper a new method is proposed for solving shortest path problems in a network which the edges length are characterized by single valued triangular neutrosophic numbers.

The rest of the paper has been organized in the following way. In Section 2, a brief overview of neutrosophic sets, single valued neutrosophic sets and triangular fuzzy number neutrosophic sets. In section 3, the network terminology is presented. In Section 4, an algorithm is proposed for finding the shortest path and shortest distance in trapezoidal fuzzy neutrosophic graph. In section 5 an illustrative example is provided to find the shortest path and shortest distance between the source node and destination node. Finally, in Section 6 the conclusion and proposal for further research is provided

II. PRELIMINARIES

This section gives a brief overview of concepts of some neutrosophic sets, single valued neutrosophic sets and trapezoidal fuzzy neutrosophic sets [2, 5, 17]

Definition 2.1 [2]. Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X}, where the functions T, I, F: X→]$^-$0,1$^+$[define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element x ∈ X to the set A with the condition:

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of]$^-$0,1$^+$[.

Since it is difficult to apply NSs to practical problems, Wang et al. [5] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [5]. Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1]. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\} \quad (2)$$

And for every x∈X

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3. \quad (3)$$

Definition 2.3 [17]. Assume that X be the finite universe of discourse and F [0, 1] be the set of all trapezoidal fuzzy numbers on [0, 1]. A trapezoidal fuzzy neutrosophic set (TrFNS) $\tilde{A}$ in X is represented

$$\tilde{A} = \{< x: \tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)>, x \in X\} \quad (4)$$

where $\tilde{T}_A(x): X \to F[0,1]$, $\tilde{I}_A(x): X \to F[0,1]$ and $\tilde{F}_A(x): X \to F[0,1]$. The trapezoidal fuzzy numbers

$$\tilde{T}_A(x) = (T_A^1(x), T_A^2(x), T_A^3(x), T_A^4(x)), \quad (5)$$

$$\tilde{I}_A(x) = (I_A^1(x), I_A^2(x), I_A^3(x), I_A^4(x)) \text{ and} \quad (6)$$

$\tilde{F}_A(x)$ = ($F_A^1(x)$, $F_A^2(x)$, $F_A^3(x)$, $F_A^4(x)$) ,respectively, denote the truth-membership, indeterminacy-membership and a falsity-membership degree of x in $\tilde{A}$ and for every x∈ X

$$0 \le T_A^4(x) + I_A^4(x) + F_A^4(x) \le 3. \quad (7)$$

For notational convenience, the trapezoidal fuzzy neutrosophic value (TrFNV) $\tilde{A}$ is denoted by $\tilde{A} = \langle (a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\rangle$ where,

$$(T_A^1(x), T_A^2(x), T_A^3(x), T_A^4(x)) = (a_1,a_2,a_3,a_4), \quad (8)$$

$$(I_A^1(x), I_A^2(x), I_A^3(x), I_A^4(x)) = (b_1,b_2,b_3,b_4), \text{ and} \quad (9)$$

$$(F_A^1(x), F_A^2(x), F_A^3(x), F_A^4(x)) = (c_1,c_2,c_3,c_4) \quad (10)$$

T are the parameters satisfy the following relations $a_1 \le a_2 \le a_3 \le a_4$, $b_1 \le b_2 \le b_3 \le b_4$ and $c_1 \le c_2 \le c_3 \le c_4$

where, the truth membership function is defined as follows

$$\tilde{T}_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{x-a_1}{a_2-a_1} & a_3 \le x \le a_4 \\ 0 & otherwise \end{cases} \quad (11)$$

The indeterminacy membership function is defined as follows:

$$\tilde{I}_A(x) = \begin{cases} \dfrac{x-b_1}{b_2-b_1} & b_1 \le x \le b_2 \\ 1 & b_2 \le x \le b_3 \\ \dfrac{b_4-x}{b_4-b_3} & b_3 \le x \le b_4 \\ 0 & otherwise \end{cases} \quad (12)$$

and the falsity membership function is defined as follows:

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$$\tilde{F}_A(x)=\begin{cases}\dfrac{x-c_1}{c_2-c_1} & c_1\le x\le c_2\\ 1 & c_2\le x\le c_3\\ \dfrac{c_4-x}{c_4-c_3} & c_3\le x\le c_4\\ 0 & otherwise\end{cases} \quad (13)$$

Definition 2.4 [17]. A trapezoidal fuzzy neutrosophic number $\tilde{A}=\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\rangle$ is said to be zero triangular fuzzy number neutrosophic number if and only if

(a_1,a_2,a_3,a_4) =(0, 0, 0, 0), (b_1,b_2,b_3,b_4) =(1, 1, 1, 1) and (c_1,c_2,c_3,c_4) =(1, 1, 1, 1) (14)

Definition 2.5 [17]. Let $\tilde{A}_1=\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\rangle$ and $\tilde{A}_2=\langle(e_1,e_2,e_3,e_4),(f_1,f_2,f_3,f_4),(g_1,g_2,g_3,g_4)\rangle$ be two TrFNVs in the set of real numbers, and $\lambda>0$. Then, the operations rules are defined as follows;

(i) $$\tilde{A}_1\oplus\tilde{A}_2=\left\langle\begin{array}{l}\begin{pmatrix}a_1+e_1-a_1e_1,a_2+e_2-a_2e_2,\\ a_3+e_3-a_3e_3,a_4+e_4-a_4e_4\end{pmatrix},\\ (b_1\ f_1,b_2\ f_2,b_3\ f_3,b_4\ f_4),\\ (c_1g_1,c_2g_2,c_3g_3,c_4g_4)\end{array}\right\rangle \quad (15)$$

(ii) $$\tilde{A}_1\otimes\tilde{A}_2=\left\langle\begin{array}{l}(a_1e_1,a_2e_2,a_3e_3,a_4e_4),\\ \begin{pmatrix}b_1+f_1-b_1f_1,b_2+f_2-b_2f_2,\\ b_3+f_3-b_3f_3,b_4+f_4-b_4f_4\end{pmatrix},\\ \begin{pmatrix}c_1+g_1-c_1g_1,c_2+g_2-c_2g_2,\\ c_3+g_3-c_3g_3,c_4+g_4-c_4g_4\end{pmatrix}\end{array}\right\rangle \quad (16)$$

(iii) $$\lambda\tilde{A}=\left\langle\begin{array}{l}\begin{pmatrix}(1-(1-a_1)^{\lambda},1-(1-a_2)^{\lambda},\\ 1-(1-a_3)^{\lambda}),1-(1-a_4)^{\lambda})\end{pmatrix}\\ (b_1^{\lambda},b_2^{\lambda},b_3^{\lambda},b_4^{\lambda}),(c_1^{\lambda},c_2^{\lambda},c_3^{\lambda},c_4^{\lambda})\end{array}\right\rangle \quad (17)$$

(iv)

$$\tilde{A}_1^{\lambda}=\left\langle\begin{array}{l}(a_1^{\lambda},a_2^{\lambda},a_3^{\lambda},a_4^{\lambda}),\\ \left((1-(1-b_1)^{\lambda},1-(1-b_2)^{\lambda},1-(1-b_3)^{\lambda}),1-(1-b_4)^{\lambda})\right),\\ \left((1-(1-c_1)^{\lambda},1-(1-c_2)^{\lambda},1-(1-c_3)^{\lambda}),1-(1-c_4)^{\lambda})\right)\end{array}\right\rangle$$

where $\lambda>0$ (18)

Ye [17] gave the following definition of score function and accuracy function. The score function S and the accuracy function H are applied to compare the grades of TrFNS. These functions shows that greater is the value, the greater is the TrFNS and by using these concept paths can be ranked.

Definition 2.6. Let $\tilde{A}_1=\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\rangle$ be a TrFNV, then, the score function $S(\tilde{A}_1)$ and an accuracy function $H(\tilde{A}_1)$ of TrFNV are defined as follows:

(i)

$$s(\tilde{A}_1)=\frac{1}{12}\begin{bmatrix}8+(a_1+a_2+a_3+a_4)-(b_1+b_2+b_3+b_4)\\ -(c_1+c_2+c_3+c_4)\end{bmatrix} \quad (19)$$

(ii) $$H(\tilde{A}_1)=\frac{1}{4}\left[(a_1+a_2+a_3+a_4)-(c_1+c_2+c_3+c_4)\right] \quad (20)$$

In order to make a comparisons between two TrFNV, **Ye [17]**, presented the order relations between two TrFNVs.

Definition 2.7 Let $\tilde{A}_1=\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\rangle$ and $\tilde{A}_2=\langle(e_1,e_2,e_3,e_4),(f_1,f_2,f_3,f_4),(g_1,g_2,g_3,g_4)\rangle$ be two TrFNVs in the set of real numbers. Then, we define a ranking method as follows:

1) If $s(\tilde{A}_1)\succ s(\tilde{A}_2)$ *, then* $\tilde{A}_1$ *is greater than* $\tilde{A}_2$ *, that is,* $\tilde{A}_1$ *is superior to* $\tilde{A}_2$ *, denoted by* $\tilde{A}_1\succ\tilde{A}_2$

2) If $s(\tilde{A}_1)=s(\tilde{A}_2)$ *, and* $H(\tilde{A}_1)\succ H(\tilde{A}_2)$ *then* $\tilde{A}_1$ *is greater than* $\tilde{A}_2$ *, that is,* $\tilde{A}_1$ *is superior to* $\tilde{A}_2$ *, denoted by* $\tilde{A}_1\succ\tilde{A}_2$.

III. NETWORK TERMINOLOGY

Consider a directed network G(V, E) consisting of a finite set of nodes V={1, 2,…,n} and a set of m directed edges E$\subseteq$ VxV . Each edge is denoted is denoted by an ordered pair (i, j) where i, j $\in$ V and $i\neq j$. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path P_{ij} = {i =i_1, (i_1,i_2) , i_2 ,…, i_{l-1} , (i_{l-1},i_l) , i_l =j} of alternating nodes and edges. The existence of at least one path P_{si} in G (V, E) is assumed for every i $\in$V-{s}.

d_{ij} denotes trapezoidal fuzzy neutrosophic number associated with the edge (i, j), corresponding to the length necessary to traverse (i, j) from i to j. the neutrosophic distance along the path P is denoted as d(P) is defined as

$$\text{d(P)}=\sum_{(i,\,j\in P)} d_{ij} \quad (21)$$

Remark: A node i is said to be predecessor node of node j if

1) Node i is directly connected to node j.

2) The direction of path connecting node i and j from i to j.

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IV. TRAPEZOIDAL FUZZY NEUTROSOPHIC PATH PROBLEM

In this paper the edge length in a network is considered to be a neutrosophic number, namely, trapezoidal fuzzy neutrosophic number.

The algorithm for the shortest path proceeds in 6 steps.

Step 1 Assume $\tilde{d}_1$ = <(0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)> and label the source node (say node1) as [$\tilde{d}_1$ =<(0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)>

Step 2 Find $\tilde{d}_j$ = minimum{ $\tilde{d}_i \oplus \tilde{d}_{ij}$ }; j = 2,3,…,n.

Step 3 If minimum occurs corresponding to unique value of i i.e., i = r then label node j as [$\tilde{d}_j$, r]. If minimum occurs corresponding to more than one values of i then it represents that there are more than one interval valued neutrosophic path between source node and node j but trapezoidal fuzzy neutrosophic distance along path is $\tilde{d}_j$, so choose any value of i.

Step 4 Let the destination node (node n) be labeled as [$\tilde{d}_n$, *l*], then the trapezoidal fuzzy neutrosophic shortest distance between source node is $\tilde{d}_n$.

Step 5 Since destination node is labeled as [$\tilde{d}_n$, *l*], so, to find the trapezoidal fuzzy neutrosophic shortest path between source node and destination node, check the label of node l. Let it be [$\tilde{d}_l$, p], now check the label of node p and so on. Repeat the same procedure until node 1 is obtained.

Step 6 Now the trapezoidal fuzzy neutrosophic shortest path can be obtained by combining all the nodes obtained by the step 5.

Remark 5.1 Let $\tilde{A}_i$; i =1, 2,…, n be a set of trapezoidal fuzzy neutrosophic numbers, if S($\tilde{A}_k$) < S($\tilde{A}_i$), for all i, the trapezoidal fuzzy neutrosophic number is the minimum of $\tilde{A}_k$

V. ILLUSTRATIVE EXAMPLE

In order to illustrate the above procedure consider a small example network shown in Fig. 2, where each arc length is represented as trapezoidal fuzzy neutrosophic number as shown in Table 2. The problem is to find the shortest distance and shortest path between source node and destination node on the network.

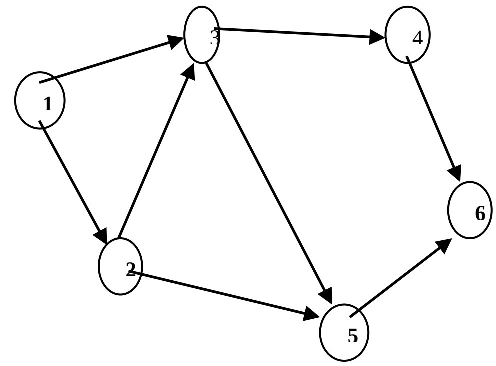

Fig. 1. A network with trapezoidal fuzzy neutrosophic edges

In this network each edge have been assigned to trapezoidal fuzzy neutrosophic number as follows:

TABLE I. WEIGHTS OF THE TRAPEZOIDAL FUZZY NEUTROSOPHIC GRAPHS

Edges	Trapezoidal fuzzy neutrosophic distance
1-2	<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>
1-3	<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>
2-3	<(0.3, 0.4, 0.6, 0.7), (0.1, 0.2, 0.3, 0.5), (0.3, 0.5, 0.7, 0.9)>
2-5	<(0.1, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.7), (0.2, 0.3, 0.6, 0.7)>
3-4	<(0.2, 0.3, 0.5, 0.6), (0.2, 0.5, 0.6, 0.7), (0.4, 0.5, 0.6, 0.8)>
3-5	<(0.3, 0.6, 0.7, 0.8), (0.1, 0.2, 0.3, 0.4), (0.1, 0.4, 0.5, 0.6)>
4-6	<(0.4, 0.6, 0.8, 0.9), (0.2, 0.4, 0.5, 0.6), (0.1, 0.3, 0.4, 0.5)>
5-6	<(0.2, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.6), (0.1, 0,3, 0.5, 0.6)>

Solution since node 6 is the destination node, so n= 6.

assume $\tilde{d}_1$ = <(0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)>and label the source node (say node 1) as [<(0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)> ,-], the value of $\tilde{d}_j$; j= 2, 3, 4, 5 ,6 can be obtained as follows:

Iteration 1 Since only node 1 is the predecessor node of node 2, so putting i = 1 and j = 2 in step 2 of the proposed algorithm, the value of $\tilde{d}_2$ is

$\tilde{d}_2$ = minimum{ $\tilde{d}_1 \oplus \tilde{d}_{12}$ } = minimum{<(0, 0, 0), (1, 1, 1), (1, 1, 1)> ⊕ <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>= <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>

Since minimum occurs corresponding to i = 1, so label node 2 as

[<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>, 1]

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$\tilde{d}_2$ = <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>

Iteration 2 The predecessor node of node 3 are node 1 and node 2, so putting i = 1, 2 and j = 3 in step 2 of the proposed algorithm, the value of $\tilde{d}_3$ is $\tilde{d}_3$ = minimum{ $\tilde{d}_1 \oplus \tilde{d}_{13}, \tilde{d}_2 \oplus \tilde{d}_{23}$ }= minimum{<(0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)> $\oplus$ <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> , <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)> $\oplus$ <(0.3, 0.4, 0.6, 0.7), (0.1, 0.2, 0.3, 0.5), (0.3, 0.5, 0.7, 0.9)>} = minimum{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> , <(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15, 0.3), (0.12, 0.25, 0.42, 0.72)>}

S (<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>) using Eq.19, we have

$$s(\tilde{A}_1) = \frac{1}{12}\begin{bmatrix} 8+(a_1+a_2+a_3+a_4)-(b_1+b_2+b_3+b_4) \\ -(c_1+c_2+c_3+c_4) \end{bmatrix} = 0.54$$

S (<(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15,0.3), (0.12, 0.25, 0.42, 0.72)>) = 0.70

Since S (<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>) < S (<(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15,0.3), (0.12, 0.25, 0.42, 0.72)>)

So, minimum{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> , <(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15, 0.3), (0.12, 0.25, 0.42, 0.72)>}

$\tilde{d}_3$ = <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>

Since minimum occurs corresponding to i = 1, so label node 3 as [(<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>) , 1]

$\tilde{d}_3$= <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>

Iteration 3. The predecessor node of node 4 is node 3, so putting i = 3 and j = 4 in step 2 of the proposed algorithm, the value of $\tilde{d}_4$ is $\tilde{d}_4$ = minimum{ $\tilde{d}_3 \oplus \tilde{d}_{34}$ }= minimum{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3,0.4)> $\oplus$ <(0.2, 0.3, 0.5, 0.6), (0.2, 0.5, 0.6, 0.7), (0.4, 0.5, 0.6, 0.8)>} = <(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>

So minimum{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3,0.4)> $\oplus$ <(0.2, 0.3, 0.5, 0.6), (0.2, 0.5, 0.6, 0.7), (0.4, 0.5, 0.6, 0.8)>}= <(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>

Since minimum occurs corresponding to i = 3, so label node 4 as [<(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)> ,3]

$\tilde{d}_4$ =<(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>

Iteration 4 The predecessor node of node 5 are node 2 and node 3, so putting i= 2, 3and j= 5 in step 2 of the proposed algorithm, the value of $\tilde{d}_5$ is $\tilde{d}_5$ = minimum{ $\tilde{d}_2 \oplus \tilde{d}_{25}, \tilde{d}_3 \oplus \tilde{d}_{35}$ } = minimum{<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)> $\oplus$ <(0.1, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.7), (0.2, 0.3, 0.6, 0.7)>, <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> $\oplus$ <(0.3, 0.6, 0.7, 0.8), (0.1, 0.2, 0.3, 0.4), (0.1, 0.4, 0.5, 0.6)>} =

Minimum{<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)> , <(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>}

S (<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>) = 0.69

S (<(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>) = 0.81

Since S (<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>) S (<(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>)

minimum{<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)> , <(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>}

= <(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>

Since minimum occurs corresponding to i=2, so label node 5 as [<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>, 2]

$\tilde{d}_5$ = <(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25,0.42), (0.02, 0.06, 0.18, 0.56)>

Iteration 5. The predecessor node of node 6 are node 4 and node 5, so putting i = 4, 5and j = 6 in step 2 of the proposed algorithm, the value of $\tilde{d}_6$ is $\tilde{d}_6$ = minimum{ $\tilde{d}_4 \oplus \tilde{d}_{46}, \tilde{d}_5 \oplus \tilde{d}_{56}$ }= minimum{<(0.36, 0.58, 0.75,0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)> $\oplus$ <(0.4, 0.6, 0.8, 0.9), (0.2, 0.4, 0.5, 0.6), (0.1, 0.3, 0.4, 0.5)>, <(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)> $\oplus$ <(0.2, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.6), (0.1, 0.5,0.3, 0.6)>} = minimum{<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>, <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)> }

S (<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>) = 0.87

S (<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>) = 0.81

Since S (<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>) < S (<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>)

minimum{<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>, <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)> }= <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>

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$\tilde{d}_6$ = <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>

Since minimum occurs corresponding to i = 5, so label node 6 as [<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>, 5]

Since node 6 is the destination node of the given network, so the trapezoidal fuzzy neutrosophic shortest distance between node 1 and node 6 is <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>

Now the trapezoidal fuzzy neutrosophic shortest path between node 1 and node 6 can be obtained by using the following procedure:

Since node 6 is labeled by [<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>, 5], which represents that we are coming from node 5. Node 5 is labeled by [<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25,0.42), (0.02, 0.06, 0.18, 0.56)>, 2] , which represents that we are coming from node 2. Node 2 is labeled by [<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>, 1] which represents that we are coming from node 1. Now the trapezoidal fuzzy neutrosophic shortest path between node 1 and node 6 is obtaining by joining all the obtained nodes. Hence the trapezoidal fuzzy neutrosophic shortest path is $1 \to 2 \to 5 \to 6$

The trapezoidal fuzzy neutrosophic shortest distance and the neutrosophic shortest path of all nodes from node 1 is shown in the table 2 and the labeling of each node is shown in figure 4

TABLE II. TABULAR REPRESENTATION OF DIFFERENT TRAPEZOIDAL FUZZY NEUTROSOPHIC DISTANCE AND SHORTEST PATH

Node	$\tilde{d}_i$	trapezoidal fuzzy neutrosophic shortest path between the i-th and 1st node
2	<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>	$1 \to 2$
3	<(0.2, 0.4, 0.5), (0.3, 0.5, 0.6), (0.1, 0.2, 0.3)>	$1 \to 3$
4	<(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>	$1 \to 3 \to 4$
5	<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25,0.42), (0.02, 0.06, 0.18, 0.56)>	$1 \to 2 \to 5$
6	<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>	$1 \to 2 \to 5 \to 6$

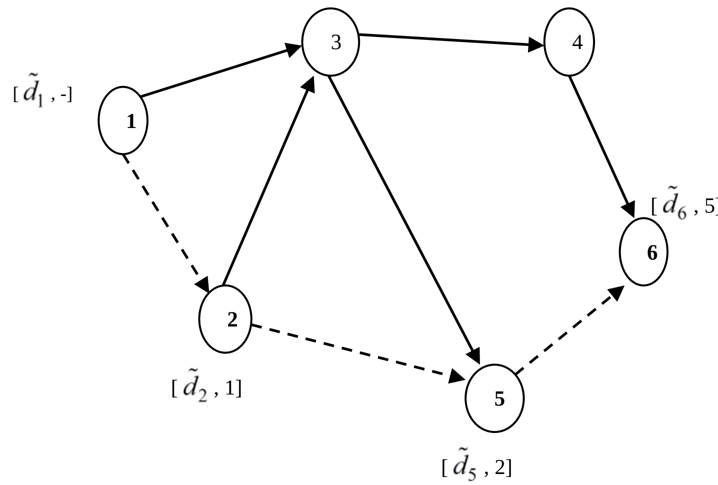

Fig. 2. Network with trapezoidal fuzzy neutrosophic shortest distance of each node from node 1

VI. CONCLUSION

In this paper, an algorithm has been developed for solving shortest path problem on a network where the edges are characterized by trapezoidal fuzzy number neutrosophic. The example of simple network problem illustrate the efficiency of the proposed algorithm. So in future work, we plan to implement this approach practically.

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