Cosmographic Time and Distance

John R. Berryhill

Abstract

The calculation of lookback time and particle horizon in the ΛCDM model is simplified by use of an explicit formula for the cosmic expansion scale factor $S(t)$.

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In describing the universal expansion $S(t)$,[1] we distinguish the physical position $\boldsymbol{r}(t)$ of an object from its permanent ("comoving") coordinate $\boldsymbol{x}$ through the relation[2]

$$\boldsymbol{r}=S(t)\boldsymbol{x}.$$

We postulate that light travels on null geodesics. At the infinitessimal level, this means

$$ds^2=0=\boldsymbol{dr}^2-c^2\,dt^2,$$

where the increment ds is relativistically invariant. We also have $\boldsymbol{dr}=S(t)\boldsymbol{dx}$, at a fixed time t. Therefore,

$$S(t)^2\boldsymbol{dx}^2=c^2\,dt^2$$

and

$$\frac{c\,dt}{S(t)}=\boldsymbol{dx}.$$

So $c\,dt/S(t)$ is an increment of coordinate distance, and we can track a light signal from one given $\boldsymbol{x}$ to another by integrating $S(t)^{-1}$ between the times of emission and reception:

$$\Delta\boldsymbol{x}=c\int S(t)^{-1}\,dt,$$

where the limits of integration are t_E and t_R. If the signal is both emitted and received a tiny bit later, the limits of integration are from $t_E+\delta t_E$ to $t_R+\delta t_R$. But for $\Delta\boldsymbol{x}$ to remain the same, what we add at time t_R must precisely equal what we subtract at t_E, *i.e.*,

$$c\,\delta t_R/S(t_R)=c\,\delta t_E/S(t_E).$$

If we think of $c\,\delta t$ as the wavelength of the light, we have

$$\frac{\lambda_R}{\lambda_E}=\frac{S(t_R)}{S(t_E)}\equiv z+1.$$

This is the standard definition[3] of the cosmological redshift z. It is an effect of the relentless expansion, rather than a velocity effect.

If we extend the limits of integration so that $t_E=0$ and $t_R=t_0$ (the age of the cosmos), and if the integral can actually be carried out, the result will determine the maximum possible distance a photon could ever have traveled, and thus an absolute limit on the greatest observable distance D_H:

$$D_H=S(t_0)\int c\,dt/S(t).$$

We have an explicit formula[1] for $S(t)$,

$$S^3(t)=\frac{4}{3}\pi G\frac{\rho_0}{\lambda}\left[\cosh(3t\sqrt{\lambda})-1\right],$$

and we can exploit this to perform the integration numerically. In doing so, we recognize that the denominator $S(t)$ in the integral becomes vanishingly small as $t\rightarrow 0$. Therefore, we work backwards from t_0 in terms of lookback time t_0-t, and we decrement time logarithmically.

Fig. 1 plots lookback time (red) and present physical distance (blue) versus redshift z, out as far as $z=12$. For the lowest redshifts, where most of the observational data lie, time and distance are well determined by z. The curves then flatten and become less definitive. Still, for a z value of 12, the graph estimates a lookback time near 13 Gyr, and a present distance of 32 Glyr. Nothing travels faster than light, but while the light was traveling, the distance from emitter to receiver was increasing.

The earliest t graphed in Fig. 1 is still almost a billion years after the beginning of expansion. To go earlier than this, we must proceed very carefully. Fig. 2 extends the graph of redshift versus time to much higher redshifts and much earlier times, but now employs a logarithmic scale for both variables.

The blue curve in Fig. 2 plots $S(t_0)/S(t)$, *i.e.*, z+1. The entire range of Fig. 1 fits into the lower-left corner of Fig. 2, where t/t_0 < ~.027.[4] Beyond that point, the curve is indistinguishable from a straight line. I have terminated the graph at z = ~1100, the redshift attributed to the Cosmic Background Radiation, the earliest observational data, at t = 485,000 yrs.

The slope of the straight line in Fig. 2 is precisely (*i.e.,* asymptotically) -2/3, although the y-intercept is not precisely 0. Nevertheless,

$$S(t_0)/S(t)=(t/t_0)^{-2/3}$$

is a useful approximation sometimes seen in the literature. It provides the estimate

$$D_H=c\,t_0\int\frac{d\tau}{\tau^{2/3}},$$

where $\tau=t/t_0$, and yields the value $D_H=3c\,t_0$, about 42 billion light-years. We can improve upon this somewhat by using the actual curve of Fig. 1 (for z < 12) rather than this approximation. We remove $3c\,t_0[(1)^{1/3}-(.027)^{1/3}]$ = 29.5 Glyr, and we replace it with the 32 Glyr of Fig. 1, for a grand total of 44.5 Glyr.

Fig. 2 employs the exact expression for $S(t_0)/S(t)$ and demonstrates that $(t/t_0)^{-2/3}$ is an excellent approximation in early times. Moreover, and most importantly, this approximation yields a finite integral, *i.e.*, it shows that a horizon exists. Of course, the horizon recedes as time goes on. We also note that the realm z < 12 encompasses more than 90% of lookback time as well as 70% of the present distance to the horizon.

One should bear in mind that the comoving coordinate $\boldsymbol{x}$ is conceptually important but arbitrary, if not altogether fictitious. Together with its companion density[5] ρ_0, it serves as a placeholder during development of the theory, which ultimately confronts observable z and $\rho(t_0)$. There is no way to determine an $\boldsymbol{x}$ or ρ_0. Nevertheless, in books and articles, one often sees the comoving coordinate discussed as if it were a real thing.

Similarly, we called the formula for $S^3(t)$ "explicit," although it contains the arbitrary ρ_0. But it gives an unambiguous prediction of the ratio $S(t_1)/S(t_2)$ at two different times, *i.e.*, redshift. As a practical matter, $S(t)/S(t_0)$ easily normalizes $S(t)$ to unity in our present era.

The explication in this and the previous article[1] is based on a few general assumptions. The universe is isotropic and homogeneous. Therefore (surprisingly) there exists a universal time.[6] Three-space is orthogonal to that time and *could* have curvature ±1, but is, in fact, "flat" (k=0).[7] The large-scale motion we observe is described as a universal expansion $S(t)$. All these principles precede, and are independent of, General Relativity.[8]

GR allows a more comprehensive dynamics, which adds effects (such as radiation pressure) that are, however, irrelevant in our present era. And it comes with a burden of dark verbiage that creates an intimidating mystique but does not help to clarify the subject at hand – the observable universe.

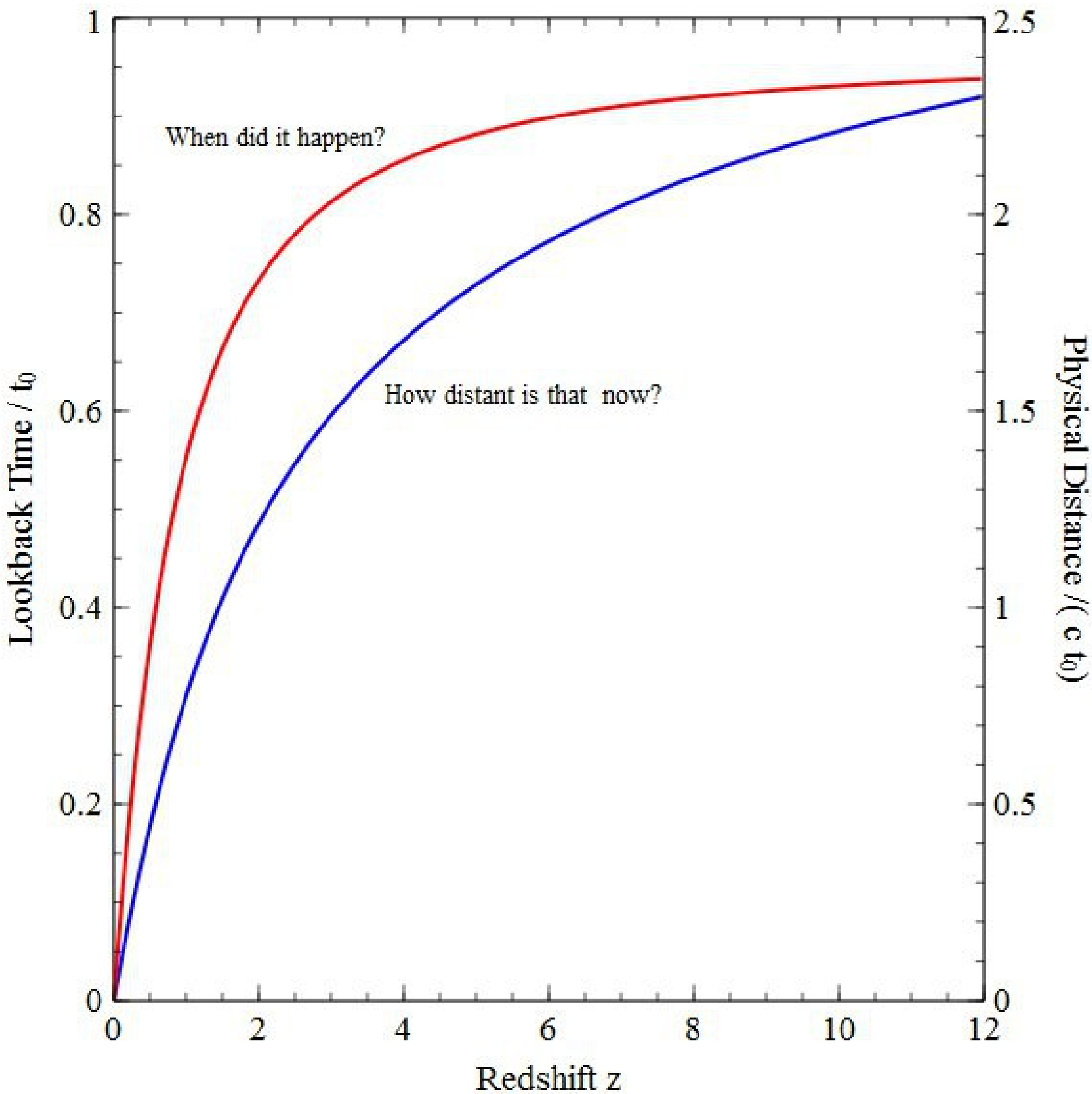

Fig. 1: Based on $S(t)$, lookback time $t_0 - t$ (red) and present actual distance (blue) are plotted versus redshift. The age of the universe t_0 here is taken as 13.9 Gyr.

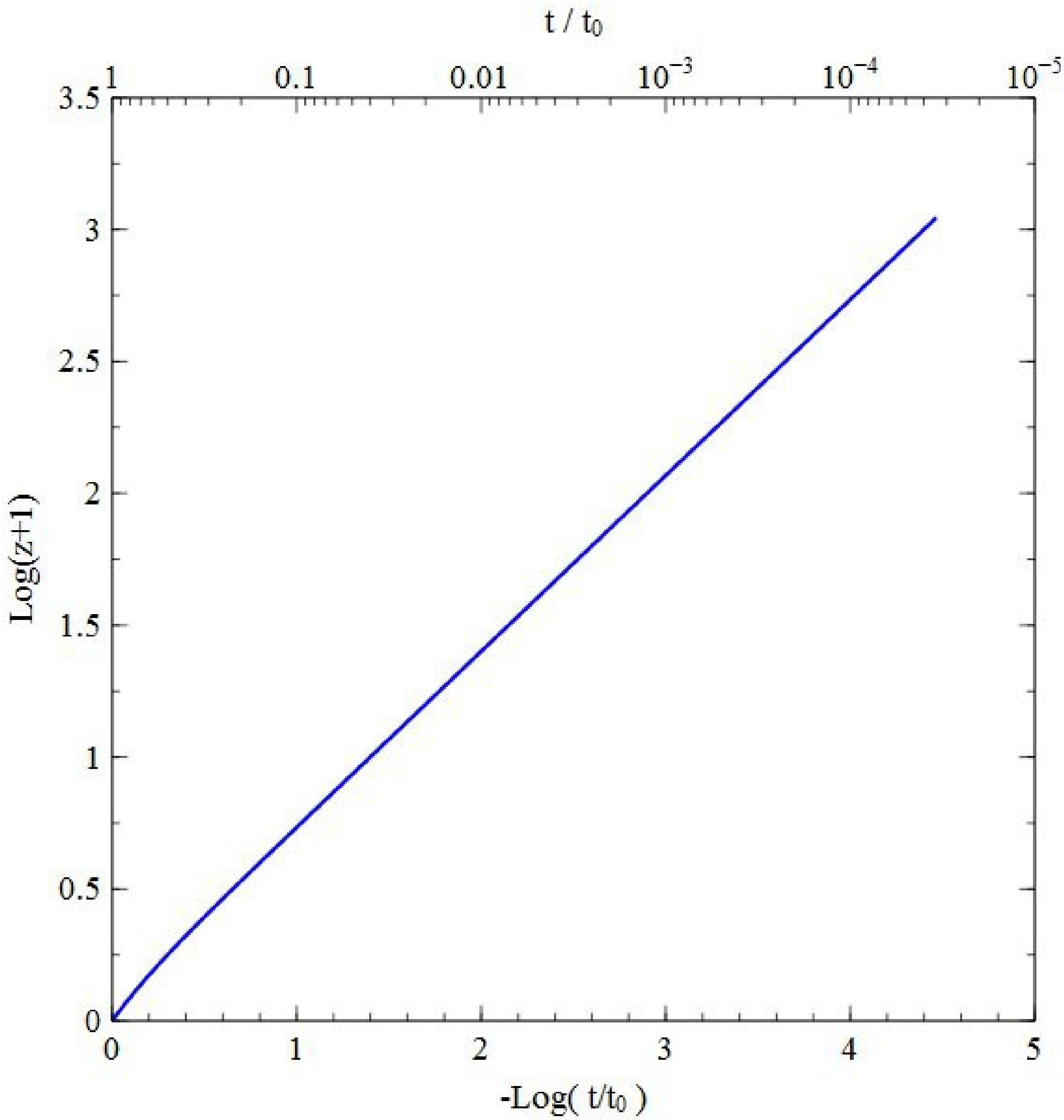

Fig. 2: Based on $S(t)$, a log-log plot of redshift versus $\tau = t/t_0$, terminated at CMB time. Asymptotically, a straight line with slope (-) 2/3.

Notes

1. See “Cosmology on the Back of an Envelope,” viXra 1704.0303.

2. This $\boldsymbol{r}$ should be thought of as a straight line connecting us with the distant object. In the expansion paradigm, $\boldsymbol{r}$ is required to be zero when $t = 0$, but not $\boldsymbol{x}$.

3. For example, Weinberg, *Cosmology*, 2008, p.10.

4. At the point where $t/t_0 = .027$, $\log\tau = -1.57$.

5. Some authors mean $\rho(t_0)$ when they write ρ_0.

6. See Weyl's Postulate.

7. See Robertson-Walker line element.

8. It needs to be said that the “Newtonian” approach to cosmology was discovered by E. A. Milne and W. H. McCrea in the face of considerable backlash from the GR cult.