A Review of Two Derivations of Maxwell-Dirac Isomorphism and a Few Plausible Extensions

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Abstract

The problem of the formal connection between electrodynamics and wave mechanics has attracted the attention of a number of authors, especially there are some existing proofs on Maxwell-Dirac isomorphism. Here the author will review two derivations of Maxwell-Dirac isomorphism i.e. by Hans Sallhofer and Volodimir Simulik. A few plausible extensions will be discussed too.

Introduction

There are some papers in literature which concerned with the formal connection between classical electrodynamics and wave mechanics, especially there are some existing proofs on Maxwell-Dirac isomorphism. Here the author will review two derivations of Maxwell-Dirac isomorphism i.e. by Hans Sallhofer and Volodimir Simulik.

While we are aware that the papers of those mentioned authors are quite old, we will discuss some recent papers which seem to point out to new development in classical electrodynamics, for instance the use of quaternion algebra and also the notion of longitudinal wave solution of Maxwell equations.

Hans Sallhofer's method

Summing up from one of Sallhofer's papers[1], he says that under the sufficiently general assumption of periodic time dependence the following connection exists between source-free electrodynamics and wave mechanics:

$$\sigma \cdot \begin{bmatrix} rotE + \frac{\mu}{c}\frac{\partial}{\partial t}H = 0 \\ rotH - \frac{\varepsilon}{c}\frac{\partial}{\partial t}H = 0 \\ div\,\varepsilon E = 0 \\ div\,\mu H = 0 \end{bmatrix}_{divE=0} \equiv \left[(\gamma \cdot \nabla + \gamma^{(4)}\partial_4)\Psi = 0\right] \qquad (1)$$

In words: Multiplication of source-free electrodynamics by the Pauli-vector yields wave mechanics.[1] In simple terms, this result can be written as follows:

$$P \cdot M = D, \qquad (2)$$

Where:

P = Pauli vector,

M = Maxwell equations,

D = Dirac equations.

We can also say: Wave mechanics is a solution-transform of electrodynamics. Here one has to bear in mind that the well-known circulatory structure of the wave functions, manifest in Dirac's hydrogen solution, is not introduced just by the Pauli-vector.[1]

Volodimir Simulik's method

Simulik described another derivation of Maxwell-Dirac isomorphism. In one of his papers[2], he wrote a theorem suggesting that the Maxwell equations of source-free electrodynamics which can be written as follows:

$$
\begin{aligned}
&rotE + \frac{\mu}{c}\frac{\partial}{\partial t}H = 0 \\
&rotH - \frac{\varepsilon}{c}\frac{\partial}{\partial t}H = 0 \\
&\quad divE = 0 \\
&\quad divH = 0
\end{aligned} \tag{3}
$$

Are equivalent to the Dirac-like equation [2]:

$$
\left[\gamma.\nabla - \begin{pmatrix} \varepsilon 1 & 0 \\ 0 & \mu 1 \end{pmatrix} \frac{1}{c}\frac{\partial}{\partial t}\right]\Psi^{c1} = 1, \tag{4}
$$

Where in the usual representation

$$
\gamma = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \tag{5}
$$

And σ are the well-known Pauli matrices.

A few plausible extensions of Maxwell-Dirac isomorphism

a. It is known that the original Maxwell equations were expressed in quaternion algebra, instead of vector language, so there is a kind of revival from time to time to recover the original quaternionic Maxwell equations. With the help of Gersten's decomposition method, we were able to derive Maxwell equations in Quaternion space starting from quaternionic Dirac equations.[4]

   We started with a basic assumption of quaternionic square root as follows [4]:

$$
k = \left(E_{qk} + i\vec{p}_{qk}\right)q_k \tag{6}
$$

   Then we proceed with Gersten's decomposition method to re-derive Maxwell from quaternionic Dirac equation. This approach seems quite worthy for further investigations.

b. Further improvement may be expected, for example to alter slightly the Maxwell equations by using gradient magnetic field. This has been explored by Simulik recently[4], and he was able to prove the existence of longitudinal wave. More specifically, he considers the following set of equations:

$$\partial_0 \vec{E} - curl\vec{H} = -gradE^0, \tag{7}$$

$$\partial_0 \vec{H} - curl\vec{E} = -gradH^0, \tag{8}$$

$$div\vec{E} = -\partial_0 E^0,$$

(9)

$$div\vec{H} = -\partial_0 H^0. \tag{10}$$

Concluding remarks

The problem of the formal connection between electrodynamics and wave mechanics has attracted the attention of a number of authors, especially there are some existing proofs on Maxwell-Dirac isomorphism. Here the author reviews two derivations of Maxwell-Dirac isomorphism i.e. by Hans Sallhofer and Volodimir Simulik. A few plausible extensions are discussed too, for example the use of quaternion algebra and also an extension to include longitudinal wave.

This paper was inspired by an old question: Is there a consistent and realistic description of wave function, both classically and quantum mechanically?

It can be expected that the above discussions will shed some lights on such an old problem especially in the context of physical meaning of quantum wave function. This is reserved for further investigations.

Acknowledgement

Special thanks to Ms. Elsa Qin from MDPI. The author (VC) also would like to express his gratitude to Jesus Christ who always encouraged and empowered him in many occasions. He is the Good Sheoherd. *Soli Deo Gloria!*

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Document history:

version 1.0: May 24th, 2017, pk. 22.35

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