Nonlinearity, Entropy and Chaos in Music

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This article explores the nonlinear aspects underlying music , particularly focusing on melody. By using the concept of scale as the basis, the article explores ways to formulate and study the features and 'feature richness' of a given melody or Raga, and to do this, the Raga scale is represented as a 1-Dimensional array. The Signature graph of a Raga plotted as Interval as a function of Note position, established a graphic visualization of the Raga. The progression and trend of intervals was computed using the Second Level Interval Array. This trend graph reveals the complexity in a Raga structure, through looping, crowded and intricate curves in the graph. Next, the concept of chaos in the context of melody is explored, fundamentally by performing a sensitivity test, which analyzes that given a Raga, and a particular evolution path, how starting at two nearby Swaras results in two entirely different ending Swaras, when sampled after a certain period in time. As a measure of the complexity in a Raga, the entropy, a measure of uncertainty is proposed, and computed using the interval arrays as bases for an occurrence array yielding empirical probabilities. The entropy is seen as a measure of richness, a measure of variety of inter-Swara intervals that a given Raga possesses. One notes that Ragas with high entropy, on account of their interval richness, usually fall under the category of pleasant, appealing and melodious Ragas. These are also the Ragas one finds being employed in film music, clearly owing to their pleasant feel.

Introduction

The fact that music is the most powerful and captivating of all the art forms, probably needs no introduction. From adding and conveying subtle messages in other art forms such as theatre, to using music as a way of prayer and communion as well as using it as a main or supplementary approach to therapy and treatment, the vastness of applications of music is a testimony to its versatility.

So too, a lot of research has gone into understanding why music is this pleasant and appealing, and these researches involve anything from theoretical constructs, acoustic based physics and derivations, to neuroscientific and psychological valence-arousal based research, to fMRI based experimental analysis of musical affect and impact, finally to database based heuristic and survey-based analysis of music. By and large, such methods have focused on melody principally, among all the limbs and features of music, since it is the tune based aspect, called Shruthi that contributes most to the pleasant feelings music is famously known to evoke.

In Indian music, Shruthi is given much importance, and is viewed as the Mother, as much as Laya or rhythm is viewed as the Father aspect of music. All songs and musical pieces performed either in the established (Kalpitha) or extempore (Kalpana – Manodharma) revolve around a set melody called Raaga, which is established by a certain variant selection and ordering of the seven notes (Swaras) in an octave in ascent and descent, called Arohana and Avarohana respectively.

The ordering of Arohana and Avarohana entails among others, the following:

1. Each of the seven notes, with the exception of Sa (C/Do) and Pa (G/Sol) accept of multiple variants, such as two for Ma (F/Fa), and three for Ri (D/Re), Ga (E/Mi), Dha (A/La) and Ni (B/Ti). Suffixes of -A,-I, and optionally -U, such as RA, RI, and RU are used to distinguish between two or three variants as appropriate. Capital letters such as S,R,G etc are used in all other cases of notations.
2. Of the Swaras admitting three variants, one is always a Vivaadhi or 'dissonant' Swara, and is characterized by the fact that the neighbouring note is played as such a Vivaadhi variant of the concerned swara. For example, Vivaadhi Ri and Vivaadhi Dha (RU and DU) are played in nearly the same pitch as GI and NI, whereas Vivaadhi Ga and Ni (GA and NA) are played

in nearly the same pitch as RI and DI. Thus, a Raga having notes SANUDUPAMA... sounds very similar to SANUNIPAMA...

3. RA, GI, MI, DA and NI are the minor or Komal notes, which are black keys as seen in a standard piano, whereas RI, GU, MA, DI and NU, along with the stationary SA and PA are the major or Theevra notes, the white keys. As a general rule of thumb these two classes of notes evoke melancholic and joyful feelings respectively.
4. Apart from choosing from one (or rarely more) variants of each Swara, each Raga also entails a certain selection of the notes to be played. An ascending or descending Scale in which all seven notes appear in their natural order (SRGMPDNS or SNDPMGRS), are called Sampoorna. Ragas with both Sampoorna scales are called Sampoorna Ragas, with Maayamaalavagaula denoted by SARAGUMAPADANUSA-SANUDAPAMAGURASA.
5. Occassionally a Raga may eschew notes in either scale, called “Varja”, and depending on whether the scale has five or six notes it is called Audava or Shaadava. As an example is Bilahari – SARIGUPADISA-SANUDIPAMAGURISA - an Audava Sampoorna Raga.
6. Ragas may also entail certain ordering of Swaras, that deviate from the linear ordering of the seven notes. For example is Poornachandrika – SARIGUMAPADIPASA-SANUPAMARIGUMARISA. Such nonlinear ordering, called 'Vakra' insist that certain swaras may be approaced only in certain ways, as well as introducing certain patterns that become a recurring motif in rendition, such as the RGMRS in the above example.
7. In addition to these aspects, sometimes a Raga might also entail multiple variants in the same note, either varying in ascent and descent, or even at times placed side by side in a chromatic fashion. Typical example is Bhairavi – SAGIRIGIMAPADINISA-SANIDAPAMAGIRISA, involving two variants of D. Such Ragas are called Bhashanga Ragas.

A good combination of these various factors, reflected in the Arohana and Avarohana of a raga, establishes its unique signature, setting it apart from all other melodies however close they may seem to each other. This imparts a unique flavour to a certain Raga, and thus its unique impact and affect upon rendition.

Thus, by using the Raga Arohana-Avarohana Scale (RAAS) as the base platform, the present article explores the nonlinearity and its significance in melody, and the central quantitative measure employed in the entropy.

Quantitative Depiction of the RAAS

The starting step is to depict the RAAS defining each Raga as a 1-Dimensional Array denoted by R. To achieve this, the base Swara or SA is set to the number 1, indicating that this is the first and starting note. Subsequently, in the lower tetrachord reaching until PA, the notes RA,RI,GI,GU,MA,MI and PA are assigned the values 2,3,4,5,6,7 and 8 respectively, whereas the upper tetrachord from PA until the higher SA having the notes DA,DI,NI and NU are assigned 9,10,11 and 12, with the higher SA assigned 13. The difference between the two SA's is 12, which is the number of notes within one octave. Vivaadhi Swaras RU,GA,DU and NA share their values 4,3,11 and 10 with their neighbouring notes. The ascending and descending scale are depicted in tandem with the higher SA at 13 being the common Swara.

For example, Maayamaalavagaula SARAGUMAPADANUSA-SANUDAPAMAGURASA is represented by the array R_mayamalavagaula = [1 2 5 6 8 9 12 13 12 9 8 6 5 2 1].

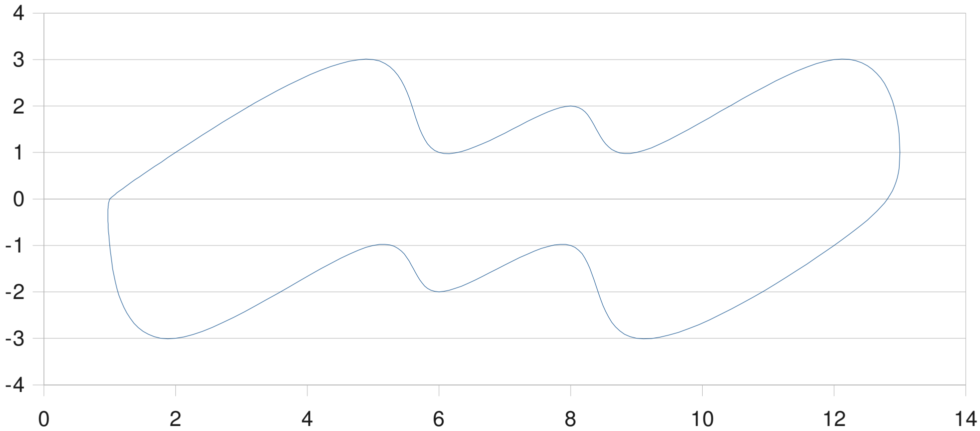
Interval Array and the Signature Graph

A key component in analysing the various features of melody is the way in which each Swara in Raga structure transits to the next. To achieve this, the concept of Interval Array I is introduced. For every element i in a given array R, the corresponding element in I denotes the difference between the current and previous element, i.e. I(i) = R(i) – R(i-1), with the only exceptions – I(1)and an additional element at the last always set to 0.

Thus, continuing from the above example, the interval array for Maayamaalavagaula is obtained as I_mayamalavagaula = [0 1 3 1 2 1 3 1 -1 -3 -1 -2 -1 -3 -1 0].

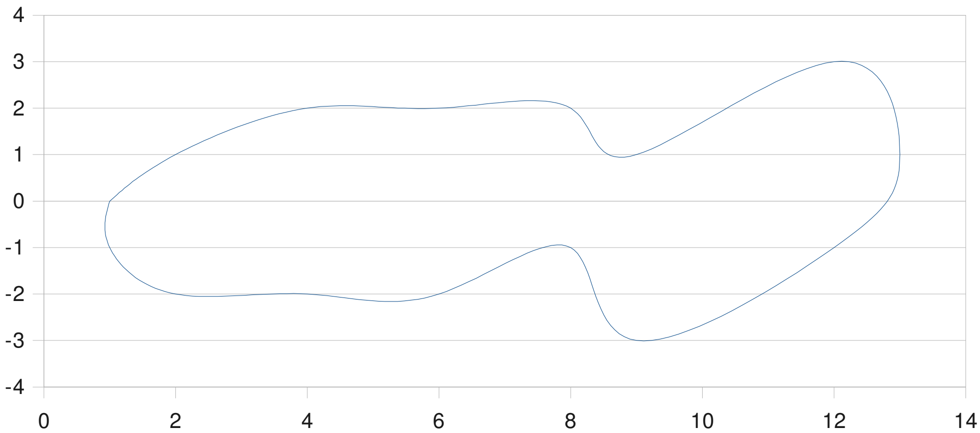
As the next step, a graph, called the Signature Graph, is plotted as a lined XY Plot between R and I arrays. In essence, this graph denotes the various Vakra, Varja and thus the jumping features of a Raga in various notes, and thus in a way established a Raga's unique graphic visualization, thus earning it the name Signature Graph or Sgraph.

The Sgraph of Maayamaalavagaula(SARAGUMAPADANUSA-SANUDAPAMAGURASA) is as follows:



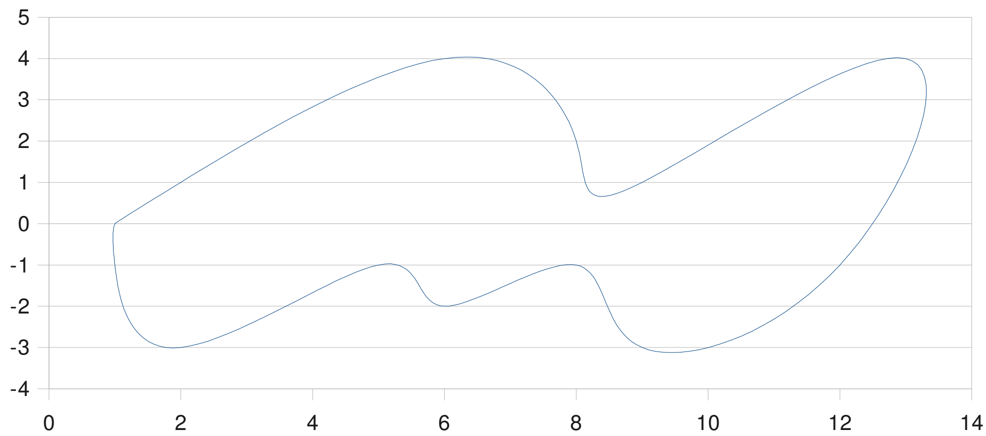
The graph establishes the 1-3-1-2-1-3-1 pattern of the interval array I. The X-axis regions of 1-8 and 8-13 establish the upper and lower tetrachords, whereas Y-axis regions above and below 0 establish the Arohana and Avarohana respectively.

However, when one note GU is changed to GI yielding another Raga Dhenuka, one finds its Sgraph as follows:

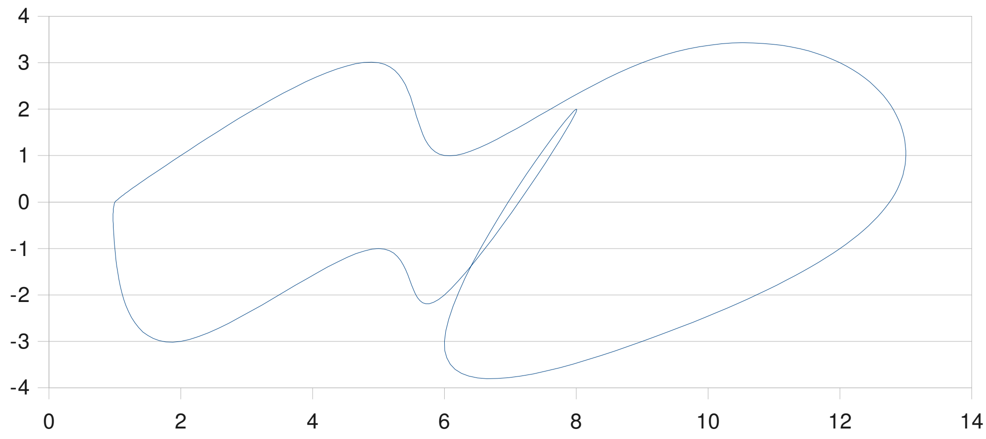


One can see that this graph shows considerable difference in the lower tetrachord, which contains the changed Swara G.

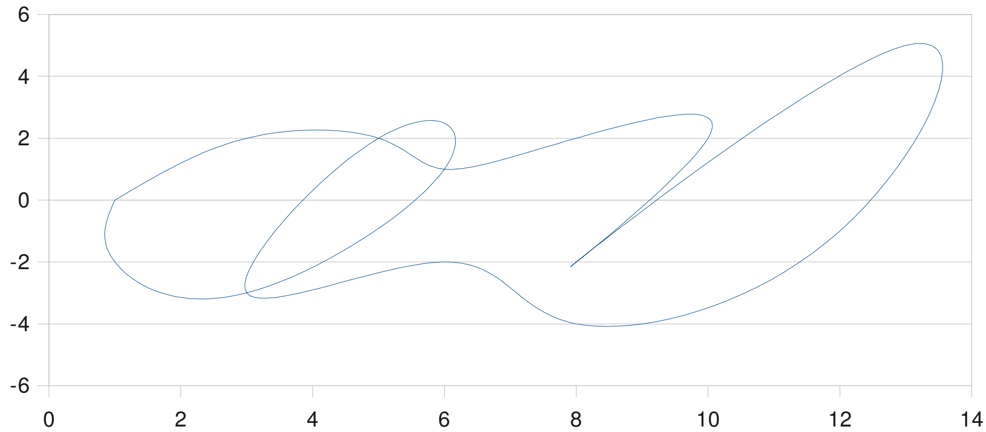
Also, with the Maayamaalavagaula scale, removing two notes in the ascent to give Saaveri – SARAMAPADASA-SANUDAPAMAGURASA yields this Sgraph:



Thus, one can see the effect of note changes and removals in the signature. Similarly nonlinear ordering of the scale or Vakratva also reflects in the graph. For example, LalithaPanchamam which is SARAGUMADANUSA-SANUDAMAPAMAGURASA yields this Sgraph:



Similarly, another Raga Poornachandrika SARIGUMAPADIPASA-SANUPAMARIGUMARISA yields this Sgraph:



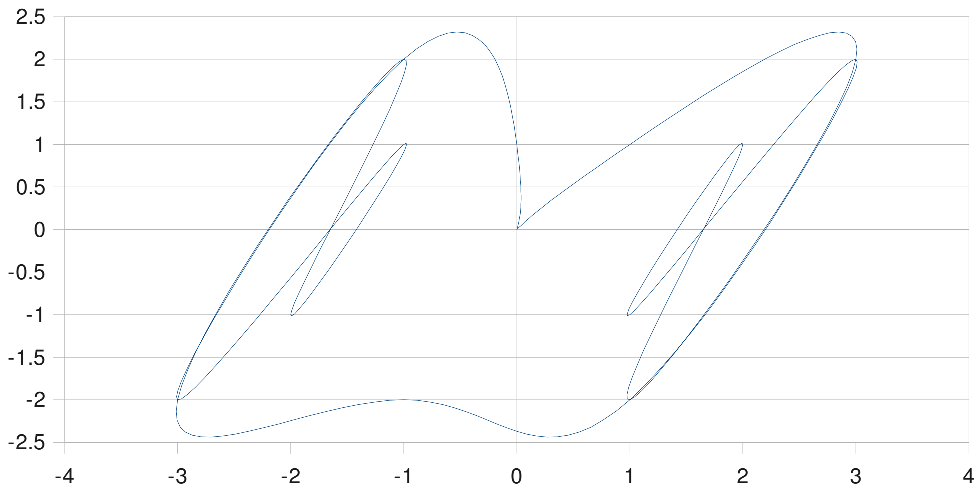
In general, one can observe Vakratva of Ragas in the Sgraph by the presence of loops and spikes. Thus, while giving a visualization of the trend of Ragas, the Sgraph also establishes in shape and pattern the unique signature of each Raga, which will seldom be seen for any other Raga.

The Trend Graph

Clearly from the signature graphs, one understands that the interval array establishes the unique identity of a Raga, and thus might hold the key to understanding the features of each Raga and hence its impact in evoking various emotions. Thus, holding I as the principal variable, another array J is computed as the difference of intervals in I. Thus, J(i) = I(i) – I(i-1).

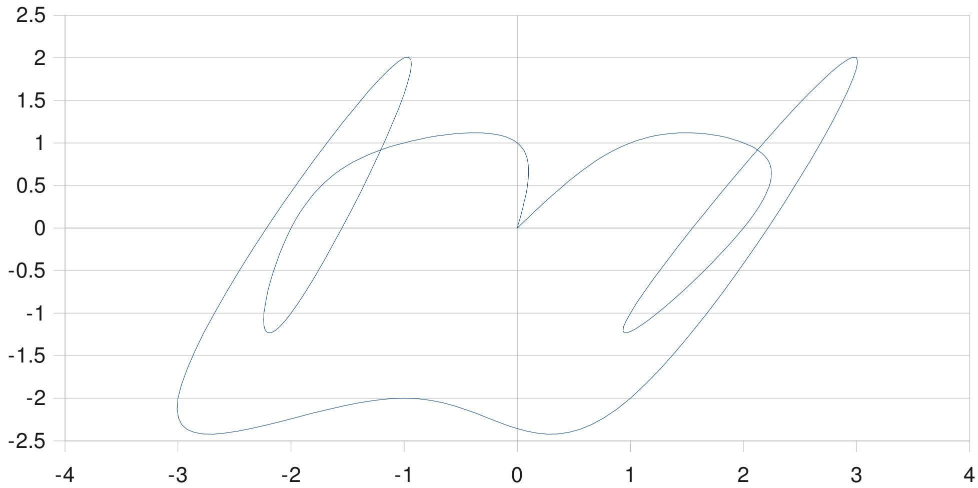
Again, continuing from the above examples, the Second Level Interval Array (SLIA) J for Maayamaalavagaula is J_mayamalavagaula = [0 1 2 -2 1 -1 2 -2 -2 -2 2 -1 1 -2 2 1].

A graph plotted between the I and J arrays reveals how the intervals between Swaras vary as one proceeds through the ascending and descending scales of the Raga. Thus, with the interval I as the primary variable, this graph of J as a function of I, depicts an visualization of the interval values vis-a-vis the interval differences, making it similar to the phase portrait plots usually plotted in chaos theory. Since the graph shows the nature of change of intervals in a Raga, the graph is called the Trend Graph or Tgraph, and is as follows for Maayamaalavagaula.

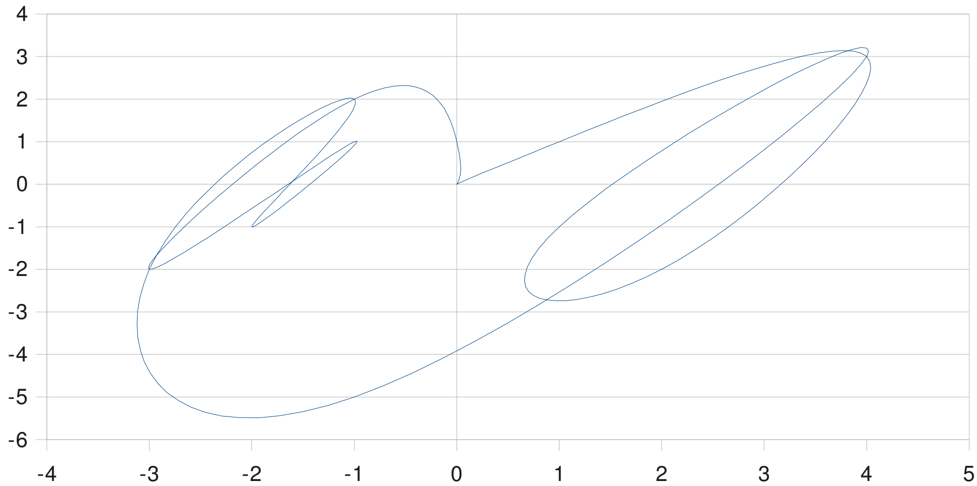


Symmetry about the Y-axis in the Tgraph indicates a symetry in the Arohana-Avarohana.

The Tgraph for the G note changed Raga Dhenuka is as follows:

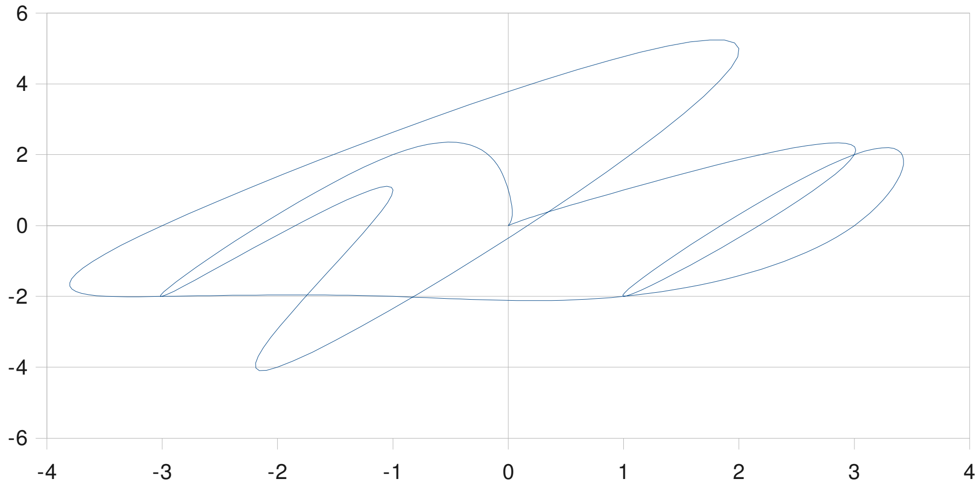


One sees that a change in a single note G changes not only the RG and GM intervals but also the procession in which the intervals themselves change. Similarly, for the Varja Audava-Sampoorna Saaveri derived from Maayamaalavagaula, the Tgraph is as follows:

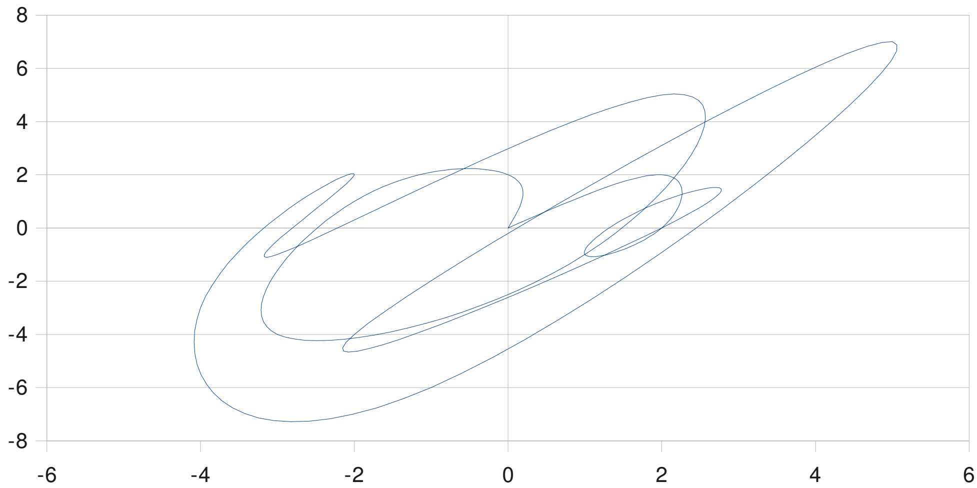


The most noticeable fact about this Tgraph is clearly its asymmetry, since the notes GU and NU have been eschewed in the Arohana alone. Furthermore, similar to te concept of phase portrait the presence of loops and intricacies in the Tgraph of a Raga are clues to its underlying nonlinearity.

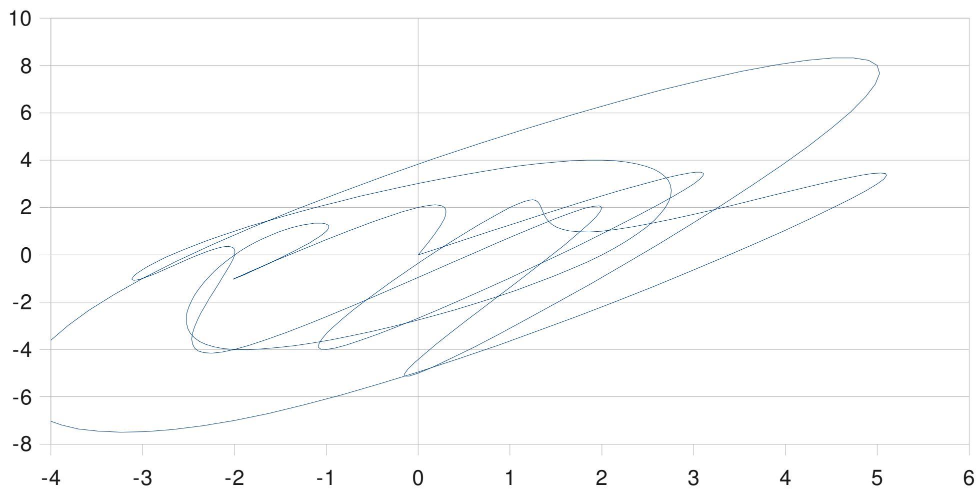
The Tgraph for the Vakra Raga Lalitha panchamam is as shown:



Finally, this is the Tgraph for Poornachandrika



In general, from the Tgraphs one observes that as the scale becomes more and more nonlinear by employing Varja and Vakra patterns, the Tgraph becomes much more asymmetrical, and patterns become more loopy, crowded and intricate, all of which point toward an increase in the complexity of the structure. This is best seen in highly nonlinear patterned Ragas such as Reethigaula – SAGIRIGIMANIDAMANINISA-SANIDAMAGIMAPAMAGIRISA.



Entropy and Chaos

From the Tgraph and Sgraph, one understands that most of the feature analysis reflects in the intricacies and loops and crowdedness of the graph, which in turn reflects in its complexity. Related concept in nonlinear studies is the concept of chaos, which is best understood by the concept of sensitivity. That is, taking the rendition of Raga as a system, two closely spaced starting points resulting in largely different outcomes over evolution of time is an indication of sensitivity.

That is, in a Raga such as Kathanakuthuhalam, given by SARIMADINUGUPASA-SANUDIPAMAGURISA, with a certain evolution criterion such as ascending through three notes and subsequently descending through three notes, starting a rendition with G takes one through GPSGRSN, whereas starting a rendition with M takes one through MDNGRSN phrases. At the end of exactly seven Swaras, one sees that the differences in end points lower N and higher N is a whole octave (12), whereas differences between the corresponding starting points G and M is only 1.

On the contrary, if the same test is applied to Shankarabharanam, given by SARIGUMAPADINUSA-SANUDIPAMAGURISA, one observes the 3 ascent – 3 descent evolution pattern starting from G and M runs as GMPDPMG and MPDNDPM results in ending notes G and

M with difference of 1, which is the same as the starting difference 1 between G and M.

Thus, from the above examples, one sees that Kathanakuthuhulam is highly sensitive to the starting Swara, in that if two renditions are started even from neighbouring Swaras, the resulting Swaras after certain evolution of time are spaced far apart. This in essence is the principle of sensitivity and chaos.

It is clear that the factors giving rise to this chaos is the complexity of ascending and descending scales, which in turn is caused by the presence of Vakra, Varja and Bhashanga in these scales, and these are the aspects best captured in the interval based analysis using the Sgraph and Tgraph.

From another viewpoint, one observes that the chaotic nature of a Raga most likely coincides with the degree of unpredictability and uncertainty of the notes sampled at random intervals, since, lack of knowledge of the starting note exactly means that erroneously assuming even the neighbouring note as starting note, one's predictions of a note rendered at a given instant of time will go horribly wrong. This is due to lack of a simple and linear structure in the chaotic Raga scales. This 'disorder' and 'uncertainty' is best described in thermodynamics and information theory by one quantitative measure – Entropy.

To compute the entropy of a given Raga, the following procedure is adopted, based on the I array. Noting that intervals between two Swaras can occur anywhere between 1 (neighbouring notes) and 12 (an entire octave apart), a 12 element occurrence array O is created. The ith element of this array is simply the number of times the interval i occurs in the array I, considering the absolute value of i, i.e. Interval of i in both ascent and descent are treated the same. Thus, for Maayamaalavagaula, whose interval array I= [0 1 3 1 2 1 3 1 -1 -3 -1 -2 -1 -3 -1 0], one finds the occurrence array O = [8 2 4 0 0 0 0 0 0 0 0 0].

Next, normalization of O is done, by dividing each element in O by the sum of all ellements in O (14 in the above example). This is done to produce each element of O as a proportion of the whole. This array of decimals in O gives the percentage of which each interval occurs in the Raga scale. For Maayamaalavagaula, the normalized O values for intervals 1, 2 and 3 are 0.32, 0.28 and 0.36. In other words, the values in the normalized O array can be seen as an empirical probability of intervals for any given Raga.

With this information, the natural logarithm of each element in O is computed as ln(O(i)), and the entropy E is given as $E = -\sum O(i) \ln(O(i))$.

The entropy of Maayamaalavagaula is obtained as 0.96. However, for a Varja Raga such as Saaveri, the normalized O = [0.35 0.3 0.3 0.3 0 0 0 0 0 0 0 0], and correspondingly, E = 1.24. The higher value seen is mainly due to the presence of 4 non-zero interval probabilities, as opposed to 3 in Maayamaalavagaula.

For the significantly chaotic Raga Kathanakuthuhulam seen earlier, the normalized Occurrence O = [0.28 0.35 0.28 0.19 0.19 0 0.19 0 0 0 0 0], and E is a high 1.47.

From the above discussion, it would benefit if entropy is seen as a measure of the 'richness' of a Raga, since more number of non-zero values in O implies that the Raga possesses a larger variety of intervals, and subsequently the E value is higher. From theory, one already knows that the highest E is obtained for Ragas having equiprobable case – same value in all elements of O. Such a Raga is also one that would have every Swara interval imaginable and it would be close to impossible in predicting the Raga note rendered at any given instant.

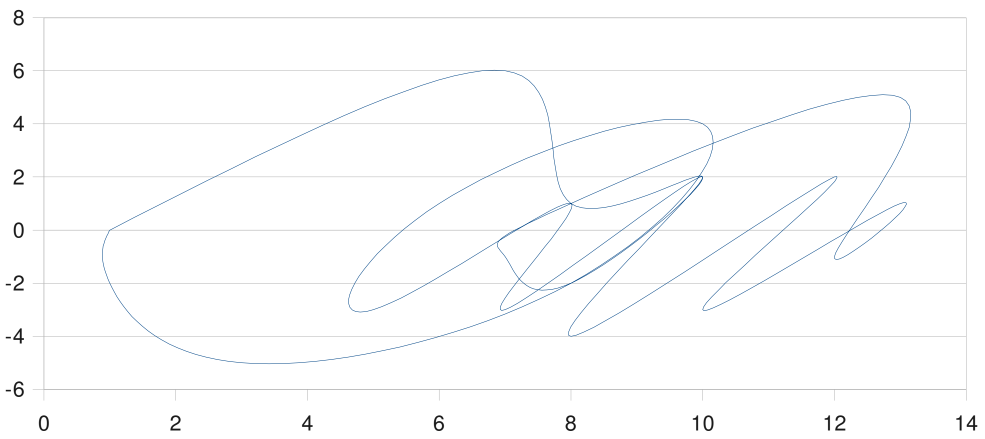
Few Example Analyses

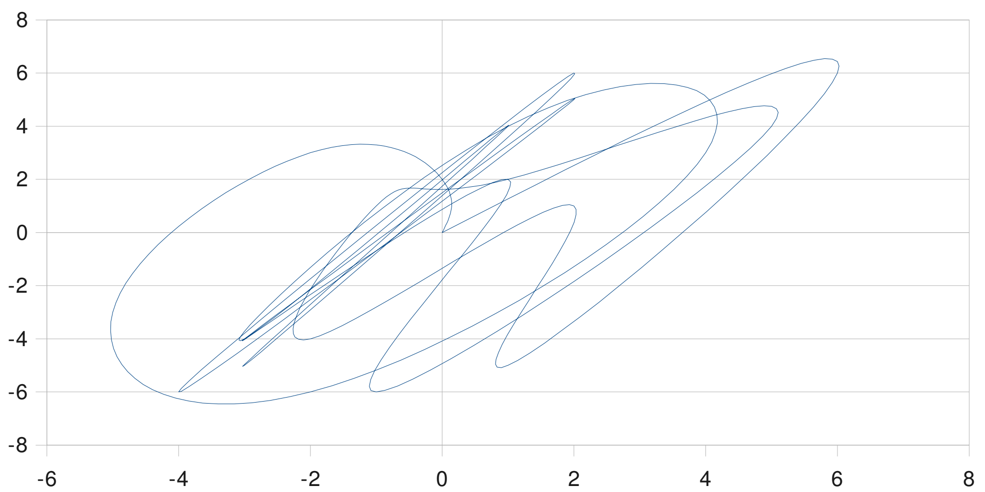
The following gives few examples of the entropies obtained for various Ragas, sorted by type of scale and structure. These results are based out of a set of 530 Ragas in vogue, analysed using Sgraphs, Tgraphs and Entropies.

Raga	Scale	Entropy
SAMPOORNA SCALES (VIVAADHI AND NON-VIVAADHI)		
Hanumathodi	SARAGIMAPADANISANIDAPAMAGIRASA	0.6
Bhavapriya	SARAGIMIPADANISANIDAPAMIGIRASA	1.0
Kanakaangi	SARAGAMAPADANASANADAPAMAGARASA	0.96
Gaayakapriya	SARAGUMAPADANASA-SANADAPAMAGURASA	0.96
Jhaalavaraali	SARAGAMIPADANUSANUDAPAMIGARASA	0.8
COMBINATIONS WITH AUDAVA AND SHADAVA SCALES		
Mohanam	SARIGUPADISADIPAGURISA	0.67
Shivaranjani	SARIGIPADISADIPAGIRISA	1.33
Hamsadhwani	SARIGUPANUSANUPAGURISA	1.33
Amrithavarshini	SAGUMIPANUSANUPAMIGUSA	1.05
Bhoopalam	SARAGIPADASADAPAGIRASA	1.05
Devagandharam	SAGIMAPANISANIDIPAMAGIRISA	0.87
Naata	SARUGUMAPADUNUSANUPAMARUSA	1.27
Bhauli	SARAGUPADASANUDAPAMAGURASA	0.93
Malahari	SARAMAPADASADAPAMAGURASA	1.24
NON-BHASHANGA VAKRA RAGAS (VARJA AND NON-VARJA)		
Maalavi	SARIGUMAPANIMADINISANIDINIPAMAGUMARISA	1.35
Reethigaula	SAGIRIGIMANIDAMANINISANIDAMAGIMAPAMAGI RISA	1.14
Kunthalavarali	SAMAPADINIDISANIDIPAMASA	1.24
Gaula	SARAMAPANUSANUPAMARAGUMARASA	1.22
Salanganaata	SARASAMAPADASANUSADAPAMAGURASA	1.25
BHASHANGA VAKRA RAGAS (VARJA AND NON-VARJA)		
Saaranga	SARASAPAMIPADINUSADIPAMIRIGUMARISA	1.33
Punnaagavaraali	SANISARIGIMAPADANIDAPAMAGIRASANISA	0.56
Anandhabhairavi	SAGIRIGIMAPADIPASANIDIPADAPAMAGIRISA	1.13
Bhairavi	SAGIRIGIMAPADINISANIDAPAMAGIRISA	0.85
SOME RAGAS WITH HIGH ENTROPIES		
Suraranjani	SAGUPARIMADISANUDIPAMAGURISA	1.48
Shambhukriya	SAGURIMAPANUSANUPANUMAGURISA	1.46

Maand	SAGUMADINUSANUSADIPANUDIGUMARISA	1.47
Anandhi	SAMIPADIPAMIPASANUSADINUPADIMIPAGUMADIPARISA	1.60
Amrithabehag	SAMIGUPANUDISANUDIMIGUSA	1.47
Bangaala	SARIGUMAPAMARIPASANUPAMAGUMARISA	1.44
Bhaageshree	SAGIMADINIDISANIDIMAPADIGIMARISA	1.46
Kaapi	SARIMAPANUSANIDINIPAGUMANIPAGIRISA	1.40
Kallolam	SAPAMADUNUSADUNUPAMAGUSA	1.47
Takka	SADARASAGUMAPAMADANUSANUDAMAGURASA	1.42
Kathanakuthuhulam	SARIMADINUGUPASANUDIPAMAGURISA	1.47
Malhaar	SARIPAMAPANIDINUSADINIPAGIMARISA	1.40
Gaud Saarang	SAGURIMAGUPAMIDIPASANUDIPAMAGUPARISA	1.47

Among all the 530 Ragas analysed, the highest entropy is found for the Raga named Anandhi, at 1.6, with the scale SAMIPADIPAMIPASA-SANUSADINUPADIMIPAGUMADIPARISA. The Sgraph and Tgraph for this Raga is as shown below:





As discussed earlier, both the graphs show an increasing degree of complexity and intricate, crowded patterns, testifying to the high Entropy obtained, as well as to the high amount of nolinearity and chaos in the Raga.

Conclusion

In this article, the melody based aspect of music has been explored, in light of various aspects such as Sampoorna, Varja, Vakra, Bhashanga and Vivaadhi in scales. By formulating an interval array, an analysis of the intervals and their trends and progressions in the ascent and descent of Ragas was explored.

The Signature graph of a Raga was plotted as Interval as a function of Note position, and this graph established a graphic visualization of the uniqueness in each Raga, with nonlinearities in scales appearing in the form of loops.

Furthermore, the progression and trend of intervals was computed using the Second Level Interval Array. This trend graph, known as the Tgraph, is seen to be akin to the phase portrait of chaotic analysis, and reveals the complexity in a Raga structure, through looping, crowded and intricate curves in the graph.

After understanding the graphs, the concept of chaos in the context of melody is explored, fundamentally by performing a sensitivity test, which analyzes that given a Raga, and a particular evolution path, how starting at two nearby Swaras results in two entirely different ending Swaras, when sampled after a certain period in time. A visual correlation between the chaos and complexity in the Raga Structure is understood.

As a measure of the complexity in a Raga, the entropy, a measure of uncertainty is proposed, and computed using the interval arrays as bases for an occurrence array yielding empirical probabilities. The entropy is seen as a measure of richness, a measure of variety of inter-Swara intervals that a given Raga possesses.

Upon examining the formulation and the tabulation of Ragas as above, one understands a few points. First, high entropy indicates a good balance of closely spaced and well separated Swaras, since such a configuration adds to the richness and unpredictability of a Raga. Second, a high entropy with such variation in intervals is most likely possible only with a good combination of Varja and Vakra in the scales. Third, the entropy almost always occurs in Ragas with complex scales that introduce certain repititive patterns while rendering, and for this reason, these Ragas are often described as those with little scope for extempore elaboration as a Raga Alapana.

However, beyond all these details, one notes that Ragas with high entropy, on account of their interval richness, usually fall under the category of pleasant, appealing and melodious Ragas, with some of the very famous ones – Kaapi, Bhaageshree, Maand, Kathanakuthuhulam, Anandhi, Malhar and Gaud Sarang falling within this category. These are also the Ragas one finds being employed in film music, clearly owing to their pleasant feel.

There might be a good reason to observe a general correlation between 'most enjoyable Ragas' and high entropy Ragas, and such an observation owes to the fact that the human system always enjoys a little bit of uncertainty thrown in by richness and variety, for as they say “variety is the spice of life”!