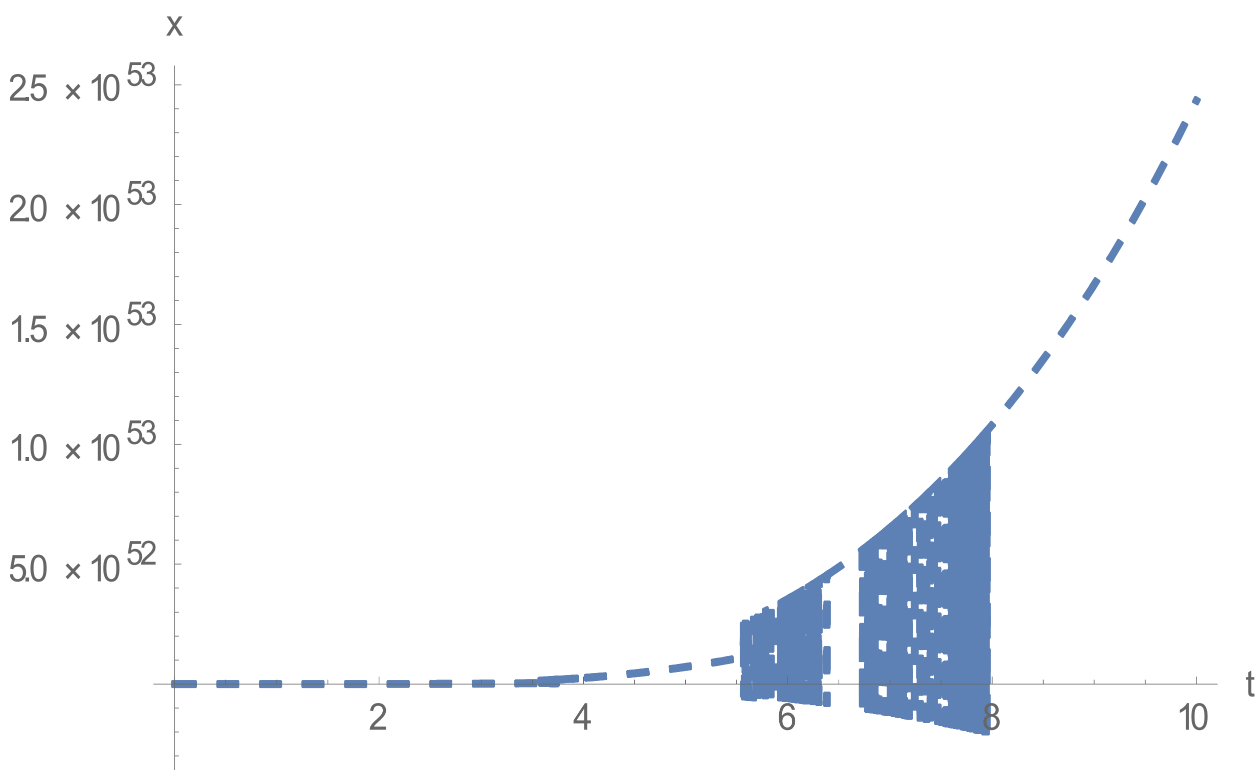

Six Easy Pieces in Computational Physics

EXPLORATION WITH MATHEMATICA

Victor Christianto | Founder & CEO of Ketindo.com | May 17, 2017
URL: http://researchgate.net/profile/Victor_Christianto

1 *"The heavens declare the glory of God; and the firmament sheweth his handywork.*

2 *Day unto day uttereth speech, and night unto night sheweth knowledge.*

3 *There is no speech nor language, where their **voice** is not heard. "*

(Psalm 19:1-3, KJV)

Preface

The present book consists of 6 papers that I and some colleagues developed throughout the last 3-4 years. The subjects discussed cover wireless energy transmission, soliton model of DNA, cosmology, and also solutions of Navier-Stokes equations both in 2D and 3D.

Some additional graphical plots for solution of 3D Navier-Stokes equations are also given. Hopefully the readers will find these papers at least interesting to ponder.

The author wishes to express his sincere and deep gratitude to Our Father in Heaven, Jesus Christ and Holy Spirit who have enabled him and helped him throughout so many troubles and desperate time. Jesus Christ is the Good Shepherd.

Soli Deo Gloria!

Indonesia, 17 May 2017

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Short Biography

Ir. Victor Christianto, MTh., DDiv. was born in East Java, Indonesia, and then he studied engineering in a state university in East Java from 1987-1992. Then he worked until 2008. In December 2008 he was granted a scholarship to study gravitation and cosmology at *Institute of Gravitation and Cosmology* in Peoples' Friendship University of Russia (PFUR) in Moscow until June 2009. Since October 2009, he works and dedicates his life for Jesus Christ. Then in 2011 he went to study theology. He graduated as Master in Theology in September 2014. He published more than 50 papers and 14 books. Since August 2015, he holds Doctor of Divinity and administers www.Sci4God.com.

Biografi Ringkas

Ir. Victor Christianto, MTh., DDiv. saat ini adalah staf pengajar di Institut Pertanian Malang sekaligus lulusan program pascasarjana dari Sekolah Tinggi Teologi Satyabhakti, Malang bulan September 2014 (www.sttsati.org). Selanjutnya ia menempuh program doktoral melalui sistem dissertation only di Jerusalem Christian Bible College, Malawi, yang diselesaikannya pada bulan Agustus 2015.

Dan juga pernah bekerja sebagai editor proyek Alkitab Yang Terbuka (AYT) di Yayasan Lembaga Sabda, hingga Oktober 2015.

Ia pernah menempuh studi gravitasi dan kosmologi di *Institute of Gravitation and Cosmology* di Peoples's Friendship University of Russia, Moscow (Desember 2008-Juni 2009), sebelum beralih fokus ke teologi.

Bidang minatnya meliputi antara lain: teologi, sejarah Kekristenan Perdana, Naskah-naskah Laut Mati, energi terbarukan dan kosmologi. Ia telah menerbitkan 3 buku spiritual, yaitu: *Grace for you: 44 Guides for Living Inspired by Jesus Christ* (2013), *A.L.I.C.E. with Jesus: A LIfe-Changing Experience with Jesus* (2014), dan *Drink the New Wine* (2014). Ketiga buku tersebut telah diterbitkan oleh Blessed Hope Publishing, Germany, dan dapat diperoleh di http://www.morebooks.de.

Telah menerbitkan lebih dari 45 paper di berbagai jurnal, dan lebih dari 17 buku baik sendiri maupun bersama rekan-rekan ilmuwan mancanegara. Beberapa paper terbaru bisa diakses di http://www.prespacetime.com, http://www.sciGod.com dan http://www.scipress.com.

Aktivitas saat ini di antaranya adalah aktivitas gerejawi, menulis buku dan mengelola blog untuk menyongsong kedatangan Kristus kali kedua, www.sci4God.com. Selain itu dia juga mengelola situs untuk pengembangan energi terbarukan di Indonesia dan Asia: http://www.ketindo.com

Other books

Other books by this author and his colleagues:

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An Exact Solution of modified KdV (mKdV) Equation as a reduction of Self-Dual Yang-Mills theory

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http://www.researchgate.net/profile/Victor_Christianto

Abstract. It is known for quite a long time that Self-Dual Yang Mills (SDYM) theory reduce to Korteweg- DeVries equation, but recently Shehata and Alzaidy have proved that SDYM reduces to modified KdV equation. Therefore, this paper discusses an exact solution of modified Korteweg-DeVries equation with Mathematica. An implication of the proposed solution is that it is possible to consider hadrons as (a set of) KdV soliton.

Introduction

It is known for quite a long time that Self-Dual Yang Mills (SDYM) theory reduce to Korteweg- DeVries equation [1][2], but recently Shehata and Alzaidy have proved that SDYM reduces to modified KdV equation [3]. Therefore, this paper discusses an exact solution of modified Korteweg- DeVries equation with Mathematica. I use Mathematica rel. 9.0. An implication of the proposed solution is that it is possible to consider hadrons as (a set of) KdV soliton.

Self-Dual Yang Mills theory and its canonical reduction to Korteweg-DeVries equation

It has been shown since 1990s that many, and possibly all, integrable systems can be obtained by dimensional reduction of self-dual Yang Mills.[1] Moreover, according to Schiff [1] a remarkable piece of evidence for this was produced a few years ago by Mason and Sparling, who showed how to obtain the Korteweg-de Vries (KdV) and Nonlinear Schrodinger (NLS) equations from SDYM. Schiff also showed how the reduction method of Mason and Sparling could be extended to obtain certain three dimensional versions of the KdV and NLS equations from SDYM.[1] But it seems no one tries to reduce SDYM to mKDV, see also [2].
In this regard, it seems very interesting that A.R. Shehata and J.F. Alzaidy were able to reduce SDYM to mKdV equation in their 2011 paper.[3] The following is a summary of their canonical reduction of SDYM:

The SDYM equations can be written in compact form as follows [3, p.148]:

$$P_t + [P, R] = 0, \tag{1}$$

$$R_x - Q_t - [Q, R] = 0. \tag{2}$$

Let P take the canonical form

$$P = \begin{pmatrix} 0 & k \\ -k & 0 \end{pmatrix} \tag{3}$$

$$R = \begin{pmatrix} 0 & -u_{xx} - 2u^3 \\ u_{xx} + 2u^3 & 0 \end{pmatrix}, \tag{4}$$

$$Q = \begin{pmatrix} 0 & u \\ -u & 0 \end{pmatrix}, \qquad (5)$$

From eq. (5) then they obtain the mKdV equation:

$$u_t + 6u_x u^2 + u_{xxx} = 0 \qquad (6)$$

Solution of KdVequation with Mathematica

The KdV equation can be written as follows [6]:

$$u_t + 6u_x u + u_{xxx} = 0 \qquad (7)$$

Meanwhile the non-dimensional KdV equation and its solution are given by:

$$\text{DSolve}[D[u[t,x],t] + D[u[t,x],\{x,3\}] - 6u[t,x]D[u[t,x],x] == 0, u[t,x], \{t,x\}]$$

$$\{\{u[t,x] \to \frac{1}{6C[2]}(C[1] - 8C[2]^3 + 12C[2]^3\text{Tanh}[tC[1] + xC[2] + C[3]]^2)\}\}$$

$$\{\{u[t,x] \to \frac{1}{6C[2]}(C[1] - 8C[2]^3 + 12C[2]^3\text{Tanh}[tC[1] + xC[2] + C[3]]^2)\}\} [[1,1,2]]$$

$$\frac{C[1] - 8C[2]^3 + 12C[2]^3\text{Tanh}[tC[1] + xC[2] + C[3]]^2}{6C[2]}$$

Solution of modified KdV equation with Mathematica

Shehata and Alzaidy obtained mKdV equation [3]:

$$u_t + 6u_x u^2 + u_{xxx} = 0 \qquad (8)$$

Its exact solution is given by:

$$\text{DSolve}[D[u[t,x],t] + D[u[t,x],\{x,3\}] - D[u[t,x],x]6u[t,x]\text{^}2 == 0, u[t,x], \{t,x\}]$$

$$\{\{u[t,x] \to -C[2]\text{Tanh}[xC[2] + 2tC[2]^3 + C[3]]\}, \{u[t,x] \to C[2]\text{Tanh}[xC[2] + 2tC[2]^3 + C[3]]\}\}$$

An implication of the proposed solution is that it is possible to consider hadrons as (a set of) KdV soliton, see for example [8]. This proposition apparently deserves further investigations.
It seems worth mentioning here that there are also other approaches to find solutions of KdV/mKdV equations for example using Backlund transformation [3][4], and also using numerical programming [9].

Concluding remarks

It is known for quite a long time that Self-Dual Yang Mills (SDYM) theory reduce to Korteweg-DeVries equation, but recently Shehata and Alzaidy have proved that SDYM reduces to modified KdV equation. Therefore, this paper discusses an exact solution of modified Korteweg-DeVries equation with Mathematica. An implication of the proposed solution is that it is possible to consider hadrons as (a set of) KdV soliton. This proposition apparently deserves further investigations.

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10.1007/978-1-4612-0211-0

Report

A Graphic Plot for a Soliton Solution of Sine-Gordon model of DNA

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ABSTRACT

There are many models of DNA, both the linear ones and the nonlinear ones. One interesting model in this regard is the sine-Gordon model of DNA as proposed by Salerno. It belongs to nonlinear model of DNA which is close to realistic model. Here we discuss a graphical plot of soliton solution of such a sine-Gordon model of DNA.

Key Words: soliton solution, sine-Gordon, DNA, graphic.

Introduction

There are many models of DNA, both the linear ones and the nonlinear ones [1]. One interesting model in this regard is the sine-Gordon model of DNA as proposed by Salerno [2], see also Daniel and Vasumathi [3]. It belongs to nonlinear model of DNA which is close to realistic model. A review of physical significance of such a sine-Gordon model was given in [6].

Here we discuss a graphical plot of soliton solution of such a sine-Gordon model of DNA.

Soliton solution of a sine-Gordon model of DNA

Assuming the wavefunction Ψ to be a function of x and t, then the sine-Gordon model of DNA can be written as follows: [3, p.7]

$$\Psi_{tt} - \Psi_{zz} + \sin(\Psi) = 0 \tag{1}$$

or in Mathematica expression:

```
Ψ=U[x-c t];
pde=D[Ψ,x,x]-D[Ψ,t,t]-sin[Ψ]?0
```

Now we will use Mathematica 9.0 to simplify and give graphical plot [3, p.443].To simplify with Mathematica:

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URL: http://researchgate.net/profile/Victor_Christianto. Email: victorchristianto@gmail.com

$$-\sin[U[z]] + U''[z] - c^2U''[z] = 0 \tag{2}$$

The result is known as kink soliton wave: [3, p.444]

$$\Phi = 4\text{ArcTan}[c\text{Sinh}[x/\text{Sqrt}[1 - c^\wedge 2]]/\text{Cosh}[ct/\text{Sqrt}[1 - c^\wedge 2]]] \tag{3}$$

or in Mathematica:

$$4\text{ArcTan}\left[c\text{Sech}\left[\frac{ct}{\sqrt{1-c^2}}\right]\text{Sinh}\left[\frac{x}{\sqrt{1-c^2}}\right]\right]$$

Differentiating for t, it yields:

$$\partial_t\left(4\text{ArcTan}\left[c\text{Sech}\left[\frac{ct}{\sqrt{1-c^2}}\right]\text{Sinh}\left[\frac{x}{\sqrt{1-c^2}}\right]\right]\right)$$

$$-\frac{4c^2\text{Sech}[\frac{ct}{\sqrt{1-c^2}}]\text{Sinh}[\frac{x}{\sqrt{1-c^2}}]\text{Tanh}[\frac{ct}{\sqrt{1-c^2}}]}{\sqrt{1-c^2}(1+c^2\text{Sech}[\frac{ct}{\sqrt{1-c^2}}]^2\text{Sinh}[\frac{x}{\sqrt{1-c^2}}]^2)}$$

Simplifying the above result, it yields:

$$\text{Simplify}\left[-\frac{4c^2\text{Sech}\left[\frac{ct}{\sqrt{1-c^2}}\right]\text{Sinh}\left[\frac{x}{\sqrt{1-c^2}}\right]\text{Tanh}\left[\frac{ct}{\sqrt{1-c^2}}\right]}{\sqrt{1-c^2}\left(1+c^2\text{Sech}\left[\frac{ct}{\sqrt{1-c^2}}\right]^2\text{Sinh}\left[\frac{x}{\sqrt{1-c^2}}\right]^2\right)}\right]$$

$$-\frac{8c^2\text{Sinh}[\frac{ct}{\sqrt{1-c^2}}]\text{Sinh}[\frac{x}{\sqrt{1-c^2}}]}{\sqrt{1-c^2}(1-c^2+\text{Cosh}[\frac{2ct}{\sqrt{1-c^2}}]+c^2\text{Cosh}[\frac{2x}{\sqrt{1-c^2}}])}$$

The 3D plot is given below for c= 0.72

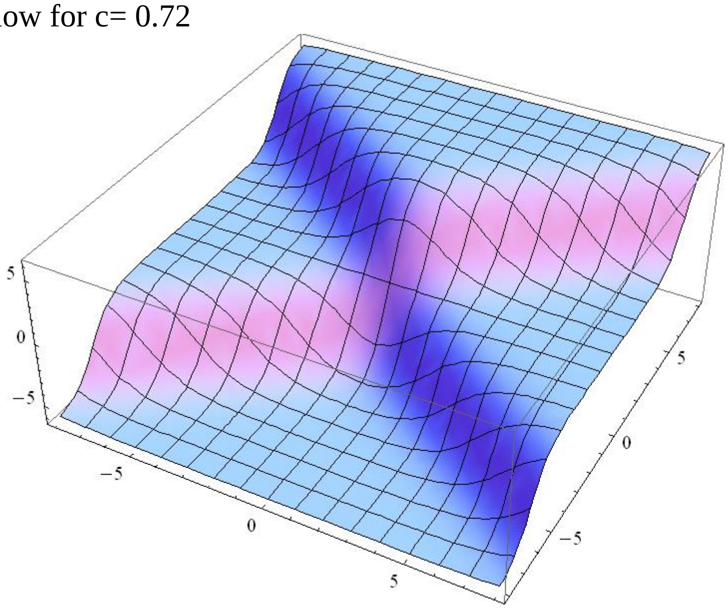

Figure 1. Mathematica plot of soliton solution on sine-Gordon equation for c=0.72

Perturbed Sine-Gordon Equation (SGE)

Perturbed SGE come in a variety of forms. One common form is a damped and driven SGE [7, p.17]:

$$\Psi_{tt} + \Phi\Psi_t - \Psi_{zz} + \sin(\Psi) = F \tag{4}$$

In addition, the following two versions of the perturbed SGE have been studied in the literature, including:

a. Directly forced SGE: [7, p.19]

$$\Psi_{tt} - \Psi_{zz} + \sin(\Psi) = Mf(\omega t) \tag{5}$$

b. Damped and drived SGE:

$$\Psi_{tt} - \Psi_{zz} + \sin(\Psi) = Mf(\omega t) - \alpha\Psi_t + \eta \tag{6}$$

In the meantime, (2+1)D SGE with additional spatial coordinate (y) is defined as [7,p.21]:

$$\Psi_{tt} = \Psi_{xx} + \Psi_{yy} - \sin(\Psi) \tag{7}$$

In their in-depth review of SGE, Ivancevic and Ivancevic [7] discuss potential applications of SGE solitons in DNA, protein folding, microtubules, neural impulse conduction and muscular contraction soliton. New insights may be expected in the near future in these biological fields, based on sine-Gordon equation soliton.

Conclusion

There are many models of DNA, both the linear ones and the nonlinear ones [1]. One interesting model in this regard is the sine-Gordon model of DNA as proposed by Salerno [2]. It belongs to nonlinear model of DNA which is close to realistic model. Here we have discussed a graphical plot of soliton solution of such a sine-Gordon model of DNA.

Considering that sine-Gordon equation has been used extensively by particle physicists, it would be interesting to study possibility to improve or alter DNA using electromagnetic field/pulse such as laser. This may be considered as a DNA enhancement method. New insights may be expected in the near future in these biological fields, based on sine-Gordon equation soliton.

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Report

An Exact Solution of a Coupled ODE for Wireless Energy Transmission via Magnetic Resonance

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ABSTRACT
In the present paper we argue that it is possible to find an exact solution of coupled magnetic resonance equation for describing wireless energy transmission. We also make an analogy between the graphical plot of this problem with the spiral galaxies.

Key Words: coupled ODE, wireless, energy transmission, magnetic resonance.

Introduction

There are some interests in the literature on possible methods to transmit energy wirelessly. While it has been known for quite a long time that this method is allowed theoretically (since Maxwell and Hertz), until recently there is slow progress in this direction.

For instance, Karalis et al. [1] and Kurs et al. [2] have presented their experiments with coupled magnetic resonance, and they reported that the efficiency rate of this method remains low. However, we do believe with that progress in material science research will someday bring new applications to the proposed concept.

In the present paper we argue that it is possible to find an exact solution of coupled magnetic resonance equation for describing wireless energy transmission, as discussed by Karalis [1] and Kurs et al.[2]. We also make an analogy between the graphical plots of this problem with the spiral galaxies.

This paper is a follow up paper of our 2008 paper [3].

A matrix model of coupled magnetic resonance

Kurs et al. [2] argue that it is possible to represent the physical system behind wireless energy transmit using coupled-mode theory. The simplified version of the system of two resonant objects is given by Karalis et al. [1, p.2] as follows:

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$$\frac{da_1}{dt} = -i(\omega_1 - i\Gamma_1)a_1 + i\kappa a_2, \tag{1}$$

and

$$\frac{da_2}{dt} = -i(\omega_2 - i\Gamma_2)a_2 + i\kappa a_1 \tag{2}$$

Therefore we can write the above two equations in matrix coupled ODE as follows:

$$[\dot{a}] = [C][a_i], \tag{3}$$

where:

$$[\dot{a}] = \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \end{bmatrix}, \tag{4}$$

$$[a_i] = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \tag{5}$$

$$[C] = \begin{bmatrix} -i\alpha & i\kappa \\ i\kappa & -i\beta \end{bmatrix}, \tag{6}$$

and

$$\alpha = (\omega_1 - i\Gamma_1), \tag{7}$$

and

$$\beta = (\omega_2 - i\Gamma_2). \tag{8}$$

or in Mathematica expression, the above matrix ODE (3)-(8) can be expressed as follows:

```
A={{-iα,iκ},{iκ,-iβ}};
B={0,0};
Eigenvalues[A]
X[t_] = {x[t], y[t]};
system=X'[t]==A.X[t]+B;
sol=DSolve[system, {x,y},t]
particularsols=Partition[Flatten[Table[{x[t], y[t]}/.sol/.{C[1] →
1/i, C[2]→ 1/j}, {i,-20,20, 6}, {j,-20,20, 6}]], 2];
```

The solution is given by:

$$\left\{\frac{1}{2}i\left(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2}\right), \frac{1}{2}i\left(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2}\right)\right\}$$

$$\begin{aligned}\{\{x \to \text{Function}[\{t\}, ((&e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\alpha - e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\alpha \\ &- e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\beta + e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\beta \\ &+ e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2} \\ &+ e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})C[1]) \\ &/(2\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2}) \\ &- \frac{(e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})} - e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})})\kappa C[2]}{\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2}}],\end{aligned}$$

$$y \to \text{Function}[\{t\}, -\frac{(e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})} - e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})})\kappa C[1]}{\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2}}$$
$$+ ((-e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\alpha + e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\alpha$$
$$+ e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\beta - e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\beta$$
$$+ e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2}$$
$$+ e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})C[2])$$
$$/(2\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})]\}\}$$

Comparison with other similar problem of coupled ODE

Now we would like to compare the above problem with a coupled ODE of the form:

$$[\dot{a}] = [C][a_i], \tag{9}$$

Where:

$$[\dot{a}] = \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \end{bmatrix}, \tag{10}$$

$$[a_i] = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \tag{11}$$

$$[C] = \begin{bmatrix} 7 & -8 \\ 5 & -5 \end{bmatrix}, \tag{12}$$

The solution is given by Mathematica as follows:

```
A={{7,-8},{5,-5}};
B={0,0};
Eigenvalues[A]
X[t_] = {x[t], y[t]};
system=X'[t]==A.X[t]+B;
sol=DSolve[system, {x,y},t]
particularsols=Partition[Flatten[Table[{x[t],  y[t]}/.sol/.{C[1]  →
1/i, C[2]→ 1/j}, {i,-20,20, 6}, {j,-20,20, 6}]], 2];
     ParametricPlot[Evaluate[particularsols], {t,-35,35}, PlotRange
-> All,  PlotPoints→ 70, Method->{Compiled → False}]
```

$$\{1+2i, 1-2i\}$$

$$\{\{x \to \text{Function}[\{t\}, -4e^t C[2]\text{Sin}[2t] + e^t C[1](\text{Cos}[2t] + 3\text{Sin}[2t])], y \to \text{Function}[\{t\}, e^t C[2](\text{Cos}[2t] - 3\text{Sin}[2t]) + \frac{5}{2}e^t C[1]\text{Sin}[2t]]\}\}$$

The result can be plotted graphically as follows:

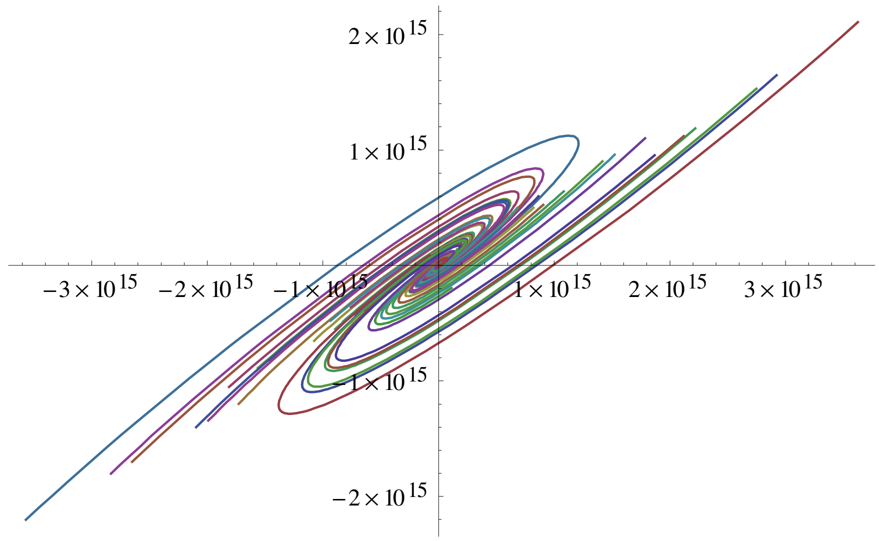

Figure 1. Graphical plot of solution of coupled ODE with Mathematica

It is interesting to remark here that the graphical plot seems to be analogous to spiral arms of spiral galaxies. Provided that both equations of coupled ODE (6) and (12) have similar values, then it may be possible to suppose that the spiral galaxies can be modeled as a coupled-magnetic problem. This possibility may be worth exploring further, both numerically and also as physical model.

Conclusion

There are some interests in the literature on possible methods to transmit energy wirelessly. While it has been known for quite a long time that this method is allowed theoretically (since Maxwell and Hertz), until recently there is slow progress in this direction.

In the present paper we argue that it is possible to find an exact solution of coupled magnetic resonance equation for describing wireless energy transmission, as discussed by Karalis [1] and Kurs et al.[2]. We also make an analogy between the graphical plot of this problem with the spiral galaxies.

It is interesting to remark here that the graphical plot of a coupled ODE seems to be analogous to spiral arms of spiral galaxies. Provided the both equations of coupled ODE (6) and (12) have similar values, then it may be possible to suppose that the spiral galaxies can be modeled as a coupled-magnetic problem.

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Report

An Exact Solution of Riccati Form of Navier-Stokes Equations with Mathematica

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ABSTRACT

There are many obtained solutions for 2D Navier-Stokes equations. This paper discusses an exact analytical solution of Riccati form of Navier-Stokes equations with Mathematica. To our best knowledge, this solution has never been presented elsewhere before, and it is faster compared to solution of Riccati equation using Differential Transform Method (DTM).

Key Words: exact solution, Riccati form, Navier-Stokes equestion, Methematica.

Introduction

Many solutions for 2D Navier-Stokes equations have been obtained, see for example [3][7]. This paper discusses an exact analytical solution of Riccati form of Navier-Stokes equations using Mathematica. I use Mathematica ver. 9.0.

To our best knowledge, this solution has never been presented elsewhere before, and it is faster compared to solution of Riccati equation using Differential Transform Method (DTM), see [6].

Standard solution of Riccati equation

Based on Mathematica software, the standard solution of Riccati equation is obtained as follows:[4, p.178]

```
Clear["Global'*"]
ode=y'[x]+a y[x]^2+y[x]/x+1/a□0
sol=DSolve[ode,y,x]
```

$$\frac{1}{a}+\frac{y[x]}{x}+ay[x]^2+y'[x]==0 \tag{1}$$

Applying DSolve, we get:

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$$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{-\text{BesselY}[1,x] - \text{BesselJ}[1,x]C[1]}{a(\text{BesselY}[0,x] + \text{BesselJ}[0,x]C[1])}\right]\right\}\right\}$$

And

y=y[x]/.sol[[1]]

$$\frac{-\text{BesselY}[1,x] - \text{BesselJ}[1,x]C[1]}{a(\text{BesselY}[0,x] + \text{BesselJ}[0,x]C[1])}$$

Solution of Riccati form of Navier-Stokes equations

Argentini obtained a general exact solution of ODE version of 2D Navier-Stokes equations in Riccati form as follows [1][2]:

$$\dot{u}_1 - \alpha.u_1^2 + \beta = 0, \tag{2}$$

where:

$$\alpha = \frac{1}{2\upsilon},$$

and

$$\beta = -\frac{1}{\upsilon}(\frac{\dot{q}}{\rho} - f_1)s - \frac{c}{\upsilon}.$$

The solution of Riccati equation is notoriously difficult to find, so this author decides to use Mathematica software in order to get an exact analytical solution.

The Mathematica code to solve this problem is not quite straightforward. First we express equation (2) as follows:

```
Clear["Global'*"]
de = y'[x] == αy[x]^2 − β;
soln=DSolve[de,y[x],x];
soln=y[x]/.(soln/.{(x^4)^(1/2)->x^2})//First
```

The result is given below:

$$-\frac{\sqrt{\beta}\text{Tanh}[x\sqrt{\alpha}\sqrt{\beta} + \sqrt{\alpha}\sqrt{\beta}C[1]]}{\sqrt{\alpha}}$$

To “get rid of” the Bessel functions of order ±5/4 we need to apply the reduction rules:

$$\text{rule} = \text{BesselJ}[p+s,x] \to (2p/x)\text{BesselJ}[p,x] - \text{BesselJ}[p-s,x];$$

$$\text{rule1} = \text{rule}/.\{p \to 1/4, s \to 1, x \to x\text{^}2/2\}$$
$$\text{rule2} = \text{rule}/.\{p \to -(1/4), s \to -1, x \to x\text{^}2/2\}$$

Then the results are as follows:

$$\text{BesselJ}[\frac{5}{4}, \frac{x^2}{2}] \to -\text{BesselJ}[-\frac{3}{4}, \frac{x^2}{2}] + \frac{\text{BesselJ}[\frac{1}{4}, \frac{x^2}{2}]}{x^2}$$

$$\text{BesselJ}[-\frac{5}{4}, \frac{x^2}{2}] \to -\frac{\text{BesselJ}[-\frac{1}{4}, \frac{x^2}{2}]}{x^2} - \text{BesselJ}[\frac{3}{4}, \frac{x^2}{2}]$$

When we make these substitutions and adjust the arbitrary constant notation, we get the following simple form:

$$\text{soln} = \text{soln}/.\{\text{rule1}, \text{rule2}, C[1] \to 1/c\}//\text{Simplify}$$

Then we get the following solution of equation (2):

$$-\frac{\sqrt{\beta}\text{Tanh}\,[\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}]}{\sqrt{\alpha}}$$

which is an exact analytical solution of Riccati expression of 2D Navier Stokes equations. To our best knowledge, this solution has never been presented elsewhere before, and it is faster compared to solution of Riccati equation using Differential Transform Method (DTM), see [6].

The next step is to transform the solution from trigonometric function to exponential form:

$$\text{TrigToExp}[-\frac{\sqrt{\beta}\text{Tanh}[\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}]}{\sqrt{\alpha}}]$$

$$-\frac{(-e^{-\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}} + e^{\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}})\sqrt{\beta}}{(e^{-\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}} + e^{\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}})\sqrt{\alpha}}$$

and then we can make a graphical plot:

$$\text{Manipulate}[\text{Plot}[-\frac{(-e^{-\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}} + e^{\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}})\sqrt{\beta}}{(e^{-\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}} + e^{\frac{(1+cx)\sqrt{\alpha}\sqrt{\beta}}{c}})\sqrt{\alpha}}, \{x, -8,8\}], \{c, -8,8\}, \{\alpha, -2,2\}, \{\beta, -2,2\}]$$

The plot is shown below for certain values of c, alpha and beta:

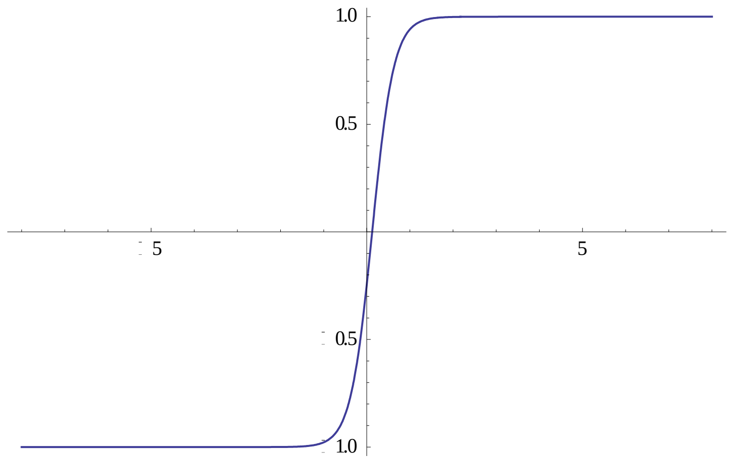

Figure 1. Graphical plot of solution of Riccati expression of NS equation

Concluding remarks

This paper discusses an exact analytical solution of Riccati form of Navier-Stokes equations using Mathematica. I use Mathematica ver. 9.0.

To our best knowledge, this solution has never been presented elsewhere before, and it seems to be faster compared to solution of Riccati equation using Differential Transform Method (DTM).

The next question is how to generalize the obtained result for 3D case of NS equations.

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Two Applications of Riccati ODE in Nonlinear Physics and Their Computer Algebra Solutions

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Abstract

In this paper, we will solve 2 Riccati ODEs using Maxima computer algebra package with applications in: (a) generalized Gross-Pitaevskii equation, (b) cosmology problem. The results presented below deserve further investigations in particular for comparison with existing analytical solutions.

1. Introduction

The Riccati equation, named after the Italian mathematician Jacopo Francesco Riccati, is a basic first-order nonlinear ordinary differential equation (ODE) that arises in different fields of mathematics and physics.[4]

Riccati differential equations are known to have many applications in nonlinear physics [1]. In this paper, we will explore only 4 of possible applications of Riccati ODE in literature, i.e. (a) generalized Gross-Pitaevskii equation, and (b) cosmology problem. Instead of using standard solution method to solve Riccati ODE, we will use Maxima computer algebra package.

We hope that our results may stimulate further serious investigation on finding numerical solutions of Riccati ODE in various domains of nonlinear physics, number theory, and cosmology.

2. **Problem 1: Generalized Gross-Pitaevskii equation (GPE)**

The authors in [4] presented the generalized GPE in (3+1)D for the BEC wave function u(x,y,z,t) with distributed time-dependent coefficients: [4]

$$i\partial_t u + \frac{\beta(t)}{2}\Delta u + \chi(t)|u|^2 u + \alpha(t) r^2 u = i\gamma(t)u, \tag{1}$$

Which can be transformed easily into a Riccati ODE form as follows:

$$\frac{da}{dt} + 2\beta(t)a^2 - \alpha(t) = 0 \tag{2}$$

The above Riccati ODE (2) can be rewritten as follows:[3]

$$a(t)' + 2.b(t) \cdot a(t)^2 - c(t) = 0. \tag{3}$$

Maxima expression of Riccati ODE (3) is as follows:[2]

‘diff(a(t),t)+2*b(t)*a(t)^2-c(t)=0 (4)

The Maxima result for this problem is as shown below:

(%i14) 'diff(y,x)+2*b*y^2-c=0;

(%o14) $\frac{d}{d\,x}\,y + 2\,b\,y^2 - c = 0$

(%i16) ode2(%,y,x);

Is b c positive or negative? negative;

(%o16) $-\frac{\operatorname{atan}\left(\frac{\sqrt{2}\,b\,y}{\sqrt{-b\,c}}\right)}{\sqrt{2}\,\sqrt{-b\,c}} = x + \%c$

3. **Problem 2: Cosmology problem**

It can be shown that in Friedmann-Robertson-Walker spacetime the set of Einstein’s equations with the cosmological constant set to zero reduce to differential equations for scale factor a(t), which is a function of comoving time t.[5] Choosing the equation of state to be barotropic and after some transformation and introducing conformal time, the equation reduces to a Riccati equation as follows:[5]

$$u' + cu^2 + kc = 0, \tag{5}$$

The above equation of cosmological Riccati equation has been obtained previously by Faraoni, see [5].

Equation (5) can be rewritten for Maxima as follows:

‘diff(a(t),t)+c*a(t)^2+k*c=0 (4)

The result is given below:

(a) Option 1: k=negative constant

(%i24) 'diff(y,x)+c*(y^2+k)=0;

(%o24) $\frac{d}{d\,x}\,y+c\,(y^2+k)=0$

(%i25) ode2(%,y,x);

*Is k positive or negative?*negative;

(%o25) $-\frac{\log\left(-\frac{\sqrt{-k}-y}{y+\sqrt{-k}}\right)}{2\,c\,\sqrt{-k}}=x+\%c$

(b) Option 2: k=positive constant

(c) (%i27) 'diff(y,x)+c*(y^2+k)=0;

(%o27) $\frac{d}{d\,x}\,y+c\,(y^2+k)=0$

(%i28) ode2(%,y,x);

*Is k positive or negative?*positive;

(%o28) $-\frac{\operatorname{atan}\left(\frac{y}{\sqrt{k}}\right)}{c\,\sqrt{k}}=x+\%c$

4. Concluding remarks

In this paper, we solve 2 Riccati ODEs using Maxima computer algebra package with applications in: (a) generalized Gross-Pitaevskii equation, (b) cosmology problem. The results as presented below deserve further investigations in particular for comparison with existing analytical solutions.

It is highly recommended to verify these results with other computer algebra packages, such as Maple or Mathematica.

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VC, SE, FS, YU

Solving Numerically a System of Coupled Riccati ODEs for Incompressible Non-Stationary 3D Navier-Stokes Equations

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Abstract. In two recent papers, Sergey Ershkov derived a system of two coupled Riccati ODEs as solution of non-stationary incompressible 3D Navier-Stokes equations. Now in this paper, we solve these coupled Riccati ODEs using computer algebra packages, i.e.: a) Maxima and b) Mathematica 11. The result seems to deserve further investigation in particular in comparison with rigid body motion, which will be discussed elsewhere.

INTRODUCTION

The Riccati equation, named after the Italian mathematician Jacopo Francesco Riccati, is a basic first-order nonlinear ordinary differential equation (ODE) that arises in different fields of mathematics and physics.[4]
In fluid mechanics, there is an essential deficiency of the analytical solutions of Navier–Stokes equations for 3D case of non-stationary flow. The Navier-Stokes system of equations for incompressible flow of Newtonian fluids should be presented in the Cartesian coordinates as below (under the proper initial conditions):[1]

$$\nabla . \vec{u} = 0, \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{\nabla p}{\rho} + \nu \cdot \nabla^2 \vec{u} + \vec{F}, \tag{2}$$

Where u is the flow velocity, a vector field; ρ is the fluid density, p is the pressure, v is the kinematic viscosity, and F represents external force (per unit mass of volume) acting on the fluid.[1]
In ref. [1], Ershkov explores the ansatz of derivation of non-stationary solution for the Navier–Stokes equations in the case of incompressible flow, which was suggested earlier. In general case, such a solution should be obtained from the mixed system of 2 coupled Riccati ordinary differential equations (in regard to the time-parameter t). But instead of solving the problem analytically, we will try to find a numerical solution.
The coupled Riccati ODEs read as follows:[1]

$$a' = \frac{w_y}{2} \cdot a^2 - (w_x \cdot b) \cdot a - \frac{w_y}{2}(b^2 - 1) + w_z \cdot b, \tag{3}$$

$$b' = -\frac{w_x}{2} \cdot b^2 - (w_y \cdot a) \cdot b - \frac{w_x}{2}(a^2 - 1) + w_z \cdot a \tag{4}$$

We are going to rewrite the above coupled equations in Maxima language, and then in Mathematica code.

COMPUTER ALGEBRA SOLUTION

The above coupled Riccati ODEs (1) and (2) can be rewritten as follows:[3]

$$a(t)' = \frac{v}{2} \cdot a(t)^2 - (u \cdot b(t)) \cdot a(t) - \frac{v}{2}(b(t)^2 - 1) + w \cdot b(t), \tag{5}$$

$$b(t)' = -\frac{u}{2} \cdot b(t)^2 - (v \cdot a(t)) \cdot b(t) - \frac{u}{2}(a(t)^2 - 1) + w \cdot a(t) \tag{6}$$

We will find out the solution of the above coupled ODE equations using two computer algebra packages: Maxima, then Mathematica.

A. Solving with Maxima

Maxima expression of coupled Riccati ODEs (5) and (6) are as follows:[3]

‘diff(a(t),t)=v/2*a(t)^2-(u*b(t))*a(t)-v/2*(b(t)^2-1)+w*b(t), (7)

‘diff(b(t),t)=-u/2*b(t)^2-(v*a(t))*b(t)-u/2*(a(t)^2-1)+w*a(t). (8)

The Maxima results are as shown below:

(%i3) 'diff(a(t),t)=v/2*a(t)^2-(u*b(t))*a(t)-v/2*(b(t)^2-1)+w*b(t);

$$\text{(\%o3)} \quad \frac{d}{d\,t}\,a(t) = b(t)\,w - \frac{(b(t)^2-1)\,v}{2} + \frac{a(t)^2\,v}{2} - a(t)\,b(t)\,u$$

(%i4) 'diff(b(t),t)=-u/2*b(t)^2-(v*a(t))*b(t)-u/2*(a(t)^2-1)+w*a(t);

$$\text{(\%o4)} \quad \frac{d}{d\,t}\,b(t) = a(t)\,w - a(t)\,b(t)\,v - \frac{b(t)^2\,u}{2} - \frac{(a(t)^2-1)\,u}{2}$$

(%i5) desolve([%o3,%o4],[a(t),b(t)]);

(%o5) [a(t)=ilt(-(

((laplace(b(t)^2,t,g34120)-laplace(a(t)^2,t,g34120))*v+2*laplace(a(t)*b(t),t,g34120)*u-2*a(0))*

g34120^2+(

(2*laplace(a(t)*b(t),t,g34120)*v+(laplace(b(t)^2,t,g34120)+laplace(a(t)^2,t,g34120))*u-2*b(0))*

w-v)*g34120-u*w)/(2*g34120^3-2*w^2*g34120),g34120,t),b(t)=ilt(-(

(2*laplace(a(t)*b(t),t,g34120)*v+(laplace(b(t)^2,t,g34120)+laplace(a(t)^2,t,g34120))*u-2*b(0))*

g34120^2+(

((laplace(b(t)^2,t,g34120)-laplace(a(t)^2,t,g34120))*v+2*laplace(a(t)*b(t),t,g34120)*u-2*a(0))* w-u)*g34120-v*w)/(2*g34120^3-2*w^2*g34120),g34120,t)]

It is clear that the above result is undecipherable, so now we will try to compute the solution using NDSolve function in Mathematica.

B. Solving with Mathematica 11

First, equations (5) and (6) can be rewritten in the form as follows:

$$x(t)'=\frac{v}{2}\cdot x(t)^2-(u\cdot y(t))\cdot x(t)-\frac{v}{2}(y(t)^2-1)+w\cdot y(t), \tag{9}$$

$$y(t)'=-\frac{u}{2}\cdot y(t)^2-(v\cdot x(t))\cdot y(t)-\frac{u}{2}(x(t)^2-1)+w\cdot x(t) \tag{10}$$

Then we can put the above equations into Mathematica expression:

```
v=1;
u=1;
w=1;
{xans6[t_], vans6[t_]}=
 {x[t],y[t]}/.Flatten[NDSolve[{x'[t]==(v/2)*x[t]^2-(u*y[t])*x[t]-(v/2)*(y[t]^2-1)+w*y[t], y'[t]==-(u/2)*y[t]^2-
(v*x[t])*y[t]-(u/2)*(x[t]^2-1)+w*x[t], x[0]==1,y[0]==0}, {x[t],y[t]},{t,0,10}]]
graphx6 = Plot[xans6[t],{t,0,10}, AxesLabel->{"t","x"},PlotStyle->Dashing[{0.02,0.02}]];
Show[graphx6,graphx6]
```

The result is as shown below:

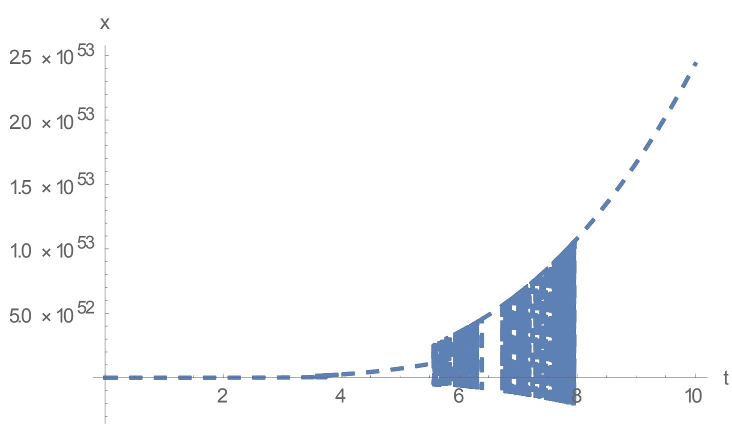

FIGURE 1. Graphical plot of solution for case v=u=w=1

The graphical plot of Mathematica solution for equations (5) and (6) shows some interesting features, such as areas shown in shades and also areas showing blank points. These features need to be explored and analysed, but it is beyond the scope of this paper.

SUMMARY AND CONCLUDING REMARKS

Using Maxima package we solve the two coupled Riccati ODEs as solution of non-stationary 3D Navier-Stokes equations.[1][2]

However, we admit that the obtained computer solution is not easily plotted graphically using Maxima, therefore we decided to verify this result with other computer algebra package, i.e. Mathematica 11.
The solution obtained here opens up new ways to interpret existing solutions of known Navier-Stokes problem in physics and engineering fields, especially those associated with nonlinear hydrodynamics and turbulence modelling. The result seems quite interesting to compare with solution of rigid body motion which is to be discussed later on by Ershkov.

ACKNOWLEDGMENTS

The authors (VC & SE) would like to express their sincere and deep gratitude to Jesus Christ and Holy Spirit who have empowered them to solve this notoriously very difficult problem. *Soli Deo Gloria!*

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In[223]:=
```
v = 1;
u = 1;
w = 1;
{xans6[t_], vans6[t_]} =
 {x[t], y[t]} /. Flatten[
   NDSolve[{x'[t] == (v/2) * x[t]^2 - (u * y[t]) * x[t] - (v/2) * (y[t]^2 - 1) + w * y[t],
     y'[t] == -(u/2) * y[t]^2 - (v * x[t]) * y[t] - (u/2) * (x[t]^2 - 1) + w * x[t],
     x[0] == 1, y[0] == 0}, {x[t], y[t]}, {t, 0, 10}]]
```

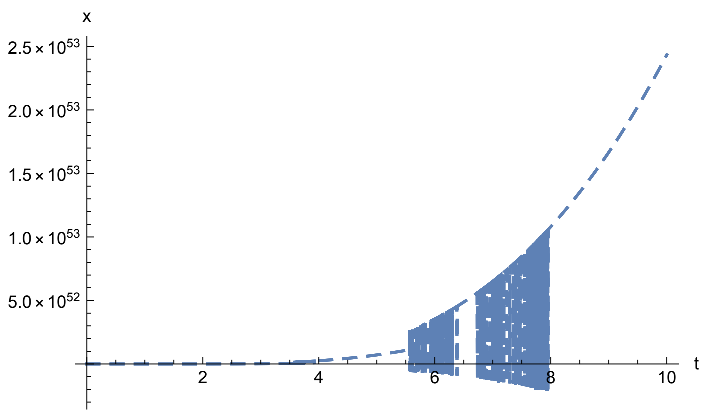
NDSolve: At t == 1.5642693104281447`, step size is effectively zero; singularity or stiff system suspected.

Out[226]=
```
{InterpolatingFunction[ Domain: {{0., 1.56}} Output: scalar ][t],

 InterpolatingFunction[ Domain: {{0., 1.56}} Output: scalar ][t]}
```

In[229]:=
```
graphx6 =
  Plot[xans6[t], {t, 0, 10}, AxesLabel → {"t", "x"}, PlotStyle → Dashing[{0.02, 0.02}]];
Show[graphx6, graphx6]
```

Out[230]=



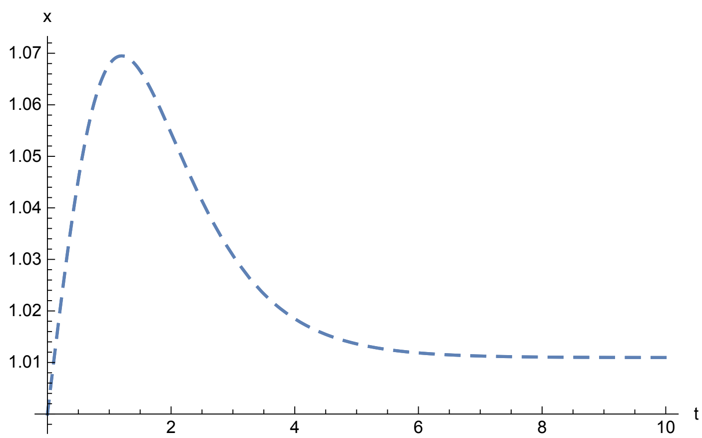
In[1]:=
```
v = 0.1;
u = 1;
w = 1;
{xans6[t_], vans6[t_]} =
 {x[t], y[t]} /. Flatten[
   NDSolve[{x'[t] == (v/2) * x[t]^2 - (u * y[t]) * x[t] - (v/2) * (y[t]^2 - 1) + w * y[t],
     y'[t] == -(u/2) * y[t]^2 - (v * x[t]) * y[t] - (u/2) * (x[t]^2 - 1) + w * x[t],
     x[0] == 1, y[0] == 0}, {x[t], y[t]}, {t, 0, 10}]]
```

Out[4]=
```
{InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][t],
 InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][t]}
```

In[5]:=

In[6]:=
```
graphx6 =
  Plot[xans6[t], {t, 0, 10}, AxesLabel → {"t", "x"}, PlotStyle → Dashing[{0.02, 0.02}]];
Show[graphx6, graphx6]
```

Out[7]=



In[8]:=
```
v = 1;
u = 0.1;
w = 1;
{xans6[t_], vans6[t_]} =
 {x[t], y[t]} /. Flatten[
   NDSolve[{x'[t] == (v/2) * x[t]^2 - (u * y[t]) * x[t] - (v/2) * (y[t]^2 - 1) + w * y[t],
     y'[t] == -(u/2) * y[t]^2 - (v * x[t]) * y[t] - (u/2) * (x[t]^2 - 1) + w * x[t],
     x[0] == 1, y[0] == 0}, {x[t], y[t]}, {t, 0, 10}]]
```

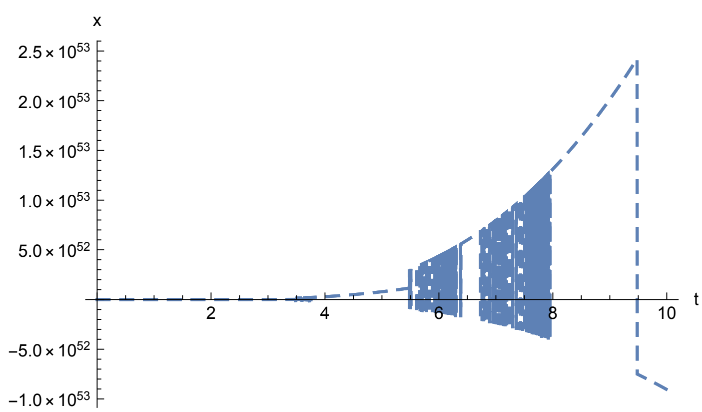
NDSolve: At t == 1.4785356140966346`, step size is effectively zero; singularity or stiff system suspected.

Out[11]=
```
{InterpolatingFunction[ Domain: {{0., 1.48}} Output: scalar ][t],
 InterpolatingFunction[ Domain: {{0., 1.48}} Output: scalar ][t]}
```

In[12]:=

In[13]:=
```
graphx6 =
  Plot[xans6[t], {t, 0, 10}, AxesLabel → {"t", "x"}, PlotStyle → Dashing[{0.02, 0.02}]];
Show[graphx6, graphx6]
```

Out[14]=



In[15]:=
```
v = 1;
u = 1;
w = 0.1;
{xans6[t_], vans6[t_]} =
 {x[t], y[t]} /. Flatten[
   NDSolve[{x'[t] == (v/2) * x[t]^2 - (u * y[t]) * x[t] - (v/2) * (y[t]^2 - 1) + w * y[t],
     y'[t] == - (u/2) * y[t]^2 - (v * x[t]) * y[t] - (u/2) * (x[t]^2 - 1) + w * x[t],
     x[0] == 1, y[0] == 0}, {x[t], y[t]}, {t, 0, 10}]]
```

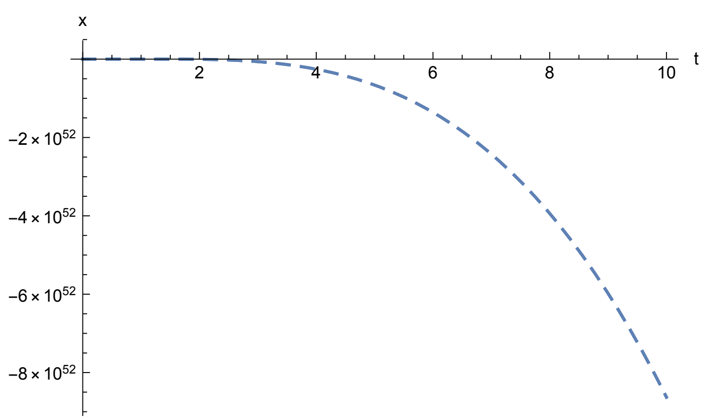
NDSolve: At t == 1.3078834392369034`, step size is effectively zero; singularity or stiff system suspected.

Out[18]=
```
{InterpolatingFunction[ Domain: {{0., 1.31}} Output: scalar ][t],
 InterpolatingFunction[ Domain: {{0., 1.31}} Output: scalar ][t]}
```

In[19]:=

In[20]:=
```
graphx6 =
  Plot[xans6[t], {t, 0, 10}, AxesLabel → {"t", "x"}, PlotStyle → Dashing[{0.02, 0.02}]];
Show[graphx6, graphx6]
```

Out[21]=



In[29]:=
```
v = 0.5;
u = 0.5;
w = 0.5;
{xans6[t_], vans6[t_]} =
 {x[t], y[t]} /. Flatten[
   NDSolve[{x'[t] == (v/2) * x[t]^2 - (u * y[t]) * x[t] - (v/2) * (y[t]^2 - 1) + w * y[t],
     y'[t] == -(u/2) * y[t]^2 - (v * x[t]) * y[t] - (u/2) * (x[t]^2 - 1) + w * x[t],
     x[0] == 1, y[0] == 0}, {x[t], y[t]}, {t, 0, 10}]]
```

NDSolve: At t == 3.1285386230391614`, step size is effectively zero; singularity or stiff system suspected.

Out[32]=
```
{InterpolatingFunction[ Domain: {{0., 3.13}} Output: scalar ][t],
 InterpolatingFunction[ Domain: {{0., 3.13}} Output: scalar ][t]}
```

In[33]:=

In[34]:=
```
graphx6 =
  Plot[xans6[t], {t, 0, 10}, AxesLabel → {"t", "x"}, PlotStyle → Dashing[{0.02, 0.02}]];
Show[graphx6, graphx6]
```

Out[35]=