Is mass a real physical quantity?

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Abstract

Although mass is a very common and fundamental concept, but the problem of mass is still one of the key problems of modern Physics, up to the present the experts are not able to reach a consensus. In this paper, we have discussed the problems relating to mass, energy and matter, and been aware that mass is neither the amount of matter an object has, nor the measure of inertia and the source of gravitational field, whereas energy is the measure of the inertia of an object and also is the source of gravitational field. As before, Mass being used to measure the inertia and to calculate gravitational force is just an approximate method only for very slowly moving bodies and particles for which the rest energy is much larger than the kinetic energy. Actually the concept of mass is a superfluous artificial concept, just rest energy divided by a constant c^2 (If selecting $c = 1$, the mass is exactly equal to the rest energy), does not have any other meaning. If the concept of mass is completely superseded by energy(or more precisely, rest energy), the physical equations are completely perfect, their meanings are clearer, and the puzzles such as the relationship of mass and energy, nature of matter, the essence of the weak equivalence principle, the physical meaning of Higgs mechanism, etc., can be made clearer.

Key words: mass; energy; matter; Einstein's formula; relativity; Higgs mechanism; equivalence principle

I. Introduction

Mass is a fundamental concept in Physics, Chemistry and Philosophy. However, the progresses of modern Physics urge us to propose these questions: What is mass? Is mass a real physical quantity? Can the concept of mass be completely superseded by energy?

The word "mass" comes from Greek μᾶζα, "barley cake or lump (of dough)". People thought that mass is the amount of matter an object has and depends on its volume and density. Newton[1] found that mass causes an object to remain in its state of rest or motion at a constant velocity. This is so-called inertia. Newton did not distinguish these two concepts, mass and inertia. He thought the bigger the amount of matter an object has, the more difficult to change its state of rest or motion at a constant velocity. Later Newton also found that mass can produce a gravitational force and can be attracted by a gravitational force. The bigger the amount of matter an object has, the stronger gravitational force can be produced and acted on. This is so-called gravitational mass (or active gravitational mass and passive gravitational mass). Obviously, in Newton's mind, there is only one "mass", the amount of matter an object has. Mass is the measure of the inertia of an object and the ability that an object produces a gravitational force and is acted on by a gravitational force. His opinion could be affected by Democritus' atomism that there are the indivisible "atoms" which all matter are composed of. How to measure the amount of matter? Newton used his second law or law of gravity to measure mass.

In modern physics, the concept of mass has developed into a very complicated concept. There are many opinions and debates. [1-34] There are three main different opinions:

1) Following Newton's opinion of mass, mass is the amount of matter an object has. Duo to the conservation law of matter, the law of mass conservation is true. The formulas of Special Relativity, such as relativistic mass and equivalence of mass and energy are open to doubt. Their reasons are that matter does not increase with velocity and also does not increase with kinetic energy. Although this opinion is far less common today, one still sometimes hears.

2) There is only one mass that is Newtonian mass and is invariant. The equivalence of mass and energy is restricted to rest energy and invariant mass ($E_0 = mc^2$). The law of mass conservation is not correct anymore, only the law of conservation of energy is true. Matter has both mass and energy, while field only has energy. Mass of photon is zero. The concept of relativistic mass is not a physical reality and should be abandoned.[15, 18, 24]

3) The concept of relativistic mass is necessary. Photons have mass $h\nu / c^2$. So the law of mass conservation is correct.[16, 17] Mass and energy are completely equivalence ($E = mc^2$). Matter and field both have mass and energy.

Basing on above different opinions, regarding the relationship between mass and energy, there are various interpretations. [16, 29, 33, 34] The interpretations are mainly based around three questions: [34]

1) Are mass and energy the "same" property of physical systems and is that what is meant by asserting that they are "equivalent"?

2) Is mass "converted" into energy in some physical interactions, and if so, what is the relevant sense of "conversion"?

3) Does $E = mc^2$ or $E_0 = mc^2$ have any ontological consequences, and if so, what are they?

Flores[33, 34] has roundly introduced the interpretations and pointed out their merits and demerits. Just what Lev Okun said, "There is no doubt that the problem of mass is one of the key problems of modern physics, though there is no common opinion even among the experts what is the essence of this problem (p.1-2)."[25]

To solve this problem is necessary not only in pedagogy, but also in understanding the essence of nature. In following sections, the concepts of mass and energy and their relationship will be reviewed through five logically correlative questions. We will be aware that the concept of mass is not a real physical quantity, is superfluous artificial concept, and should be superseded by the concept of energy. All of the issues in relation to mass and energy are caused by incorrectly introducing and understanding the concept of mass. If the concept of mass is superseded by the concept of energy (or more precisely, rest energy), the puzzles relating to mass and energy can be completely solved, the physical equations are more beautiful, and their meanings are clearer.

II. Should the concept of relativistic mass be introduced into Physics?

Regarding this question, many authors [2-34] have discussed. At early period, Einstein accepted the idea that mass is speed-dependent but changed his mind after being confronted by a far more compelling insight.[2, 4] Later, Einstein came to realize that the inertial mass m is not speed-dependent. In 1948, Einstein wrote a letter to Lincoln Barnett. He said: "It is not good to introduce the concept of the mass $M = m / \left(1 - v^2/c^2\right)^{1/2}$ of a moving body for which no clear definition can be given (p.32)."[15]

Lev Okun[15, 22, 25, 32] has been putting big effort to discuss this problem. He has emphasized that in Relativity there is only one mass which is the invariant mass and appealed to abandon the concept of relativistic mass. Gary Oas also pointed out that the concept of velocity dependent mass, relativistic mass, is examined and is found to be inconsistent with the geometrical formulation of special relativity (p.1).[24] He said: "By introducing a concept that is inferred and not primitive, that destroys the Lorentz covariance of the theory, relativistic mass is in direct conflict with the kinematical structure of special relativity. As the concept can in no way be considered a primitive concept of the theory, the statement that it is merely a matter of choice whether to use it or not is flagrantly incorrect (p.9)." [24] "For those who insist on continuing to use the concept of relativistic mass, serious reflection and examination are required. Is it possible to have a completely consistent velocity-dependent mass integrated into the full theory of relativity? Here we have seen it cannot (p.9)."[24] Frank Wilczek also mentioned: "The early literature of relativity employed some compromise definitions of mass – specifically, velocity-dependent mass, in both longitudinal and transverse varieties. Those notions have proved to be more confusing than useful. They do not appear in modern texts or research work, but they persist in some popularizations, and of course in old books." [27]

In modern Physics, the correct formulas relating to mass and energy ought to be:

$$E_0 = mc^2 \tag{1}$$

$$E^2 - \boldsymbol{p}^2 c^2 = E_0{}^2 \tag{2}$$

$$\boldsymbol{p} = \boldsymbol{v}\frac{E}{c^2} \tag{3}$$

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} \tag{4}$$

Here E is the total energy; E_0 is the rest energy; m is the mass, the same as in Newtonian Mechanics; $\boldsymbol{p}$ is the momentum; $\boldsymbol{v}$ is the velocity; c is the speed of light; $\boldsymbol{F}$ is the force; t is the time. The relationship between mass and energy only is formula (1). The concept of relativistic mass should not be introduced in Physics.

III. Is mass the amount of matter an object has?

According to the discussion in section 2, we are aware that the relativistic mass should not be introduced to Physics. Then, what is the mass of photon? Now most physicists believe that the mass of photon is zero. This implies that photon completely has no mass! Here two questions naturally arise: Is a photon matter? Is mass the amount of matter an object has?

In order to answer these questions, firstly, we need to know what matter is. The definition of matter seams no unified expression in Physics and Philosophy. "The generally accepted definition of matter does not exist even today. Some authors of physics text-books do not consider photons – particles of light – as particles of matter, because they are massless. For the same reason they do not consider as matter the electromagnetic field. It is not quite clear whether they consider as matter almost massless neutrinos, which usually move with velocity close to that of light (p.1)."[25]

When Einstein-Infeld interpreted the famous formula of mass and energy, they used a rudimentary distinction between "matter" and "fields." Matter is the thing that is ponderable material stuff; Fields are physical fields such as the electromagnetic field. Matter has both mass and energy, whereas fields only have energy. "Einstein and Infeld, on the other hand, in places seem to argue that we can infer that the fundamental stuff of physics is fields. In other places, however, Einstein and Infeld seem a bit more cautious and suggest only that one can construct a physics with only fields in its ontology."[34]

Matter should be everything, objects, particles, field and some types of matter which we do not know yet, such as dark matter, dark energy, even vacuum. Matter has different forms and can be converted from one type of form to others. The massless particles (or fields) are one type of matter. Photons are one type of matter! In universe, at different periods of evolution and on different levels of matter, there are different forms of matter.

How to measure "the amount of matter"? Newton thought that "the quantity of matter is the measure of the same, arising from its density and bulk conjunctly (p.1)."[1] He believed that the inertia, the gravitational mass, and the amount of matter were identical and the same physical quantity. In another words, he considered the inertial mass and the gravitational mass as "the amount of matter". He did not distinguish their differences and used the inertial mass or the gravitational mass to represent "the amount of matter".

For same kind of substance, for example, iron, we can measure the amount of matter an object has according to its density and volume. However, for different substances, water and alcohol, we cannot compare their "amount of matter". We do not know which one contains more amount of matter for one liter of water and one liter of alcohol because they are not same thing. If a fundamental "atom" which all matter is composed of cannot be found, we cannot compare their amount of matter for different substances. Normally, in macroscopic world, using the gravitational mass to represent the amount of matter seems no serious problem. However, in microscopic world, we even don't know what the "amount of matter" means.

Strictly speaking, even in macroscopic world, two completely same things which contain same "amount of matter", for instance, two exactly same (density and volume) gold bar which contain same amount of gold atoms, if they are respectively taken from a very cold fridge and a very hot oven, should not be same mass because their internal energy are not equal (ignore the change of size, $m = E_0 / c^2$). Two objects containing same amount of matter do not have same mass.

Regarding the reaction of annihilation, "the mass of a system of two particles depends not only on masses and energies of these particles, but also on the angle between their velocities. Thus for two photons *m* is maximal when this angle is π and vanishes when it is zero (p.12)."[25] "The mass of system has lost its Newtonian meaning of an amount of substance, its main characteristic being now rest energy (in units, where c = 1) (p.12)."[25] This example obviously indicates that the mass of a system do not only depend on how many "atoms" are contained, but also lie on the structure and moving state of the "atoms". That is to say, even if a fundamental "atom" which all matter is composed of has been found, we still cannot say that mass is the amount of matter an object has.

Mass is not the amount of matter an object has. "Matter never disappears" is not equal to "the conservation of mass". "Matter never disappears" means one type of matter can always be converted to other types of matter. During their conversion process, they obey the law of energy conservation. The law of mass conservation is not correct anymore.

IV. Is mass the measure of inertia?

Regarding the measure of inertia, Lev Okun has given a clear elaboration. "In the theory of relativity (unlike the mechanics of Newton) the measure of inertia is not mass *m* but the total energy *E* of the body. The momentum $\boldsymbol{p}$ of a body is connected with its velocity *v* not by the Newton's relation $\boldsymbol{p} = m\boldsymbol{v}$ but by the relation $p = \left(E/c^2\right)v$. As a result it is the more difficult to change the momentum of a body, the higher its total energy E. And $E/c^2 = m$ only at zero momentum, when the total energy equals the rest energy E_0 (p.4)." [32] "Thus, the measure of inertia of a particle is its total energy (p.4)." [32]

"People infected by this virus (they often call themselves relativists) believe that Relativistic Mass is the main portal to Relativity Theory because mass is the measure of inertia. They ignore the fact that mass is the measure of inertia only for very slowly moving bodies and particles for which the rest energy E_0 is much larger than the kinetic energy E_k. When velocities are not very low, mass is only an approximate measure of inertia. For fast particles for which $E_k + E_0$ (photons, neutrinos, protons in LHC) the measure of inertia is the total energy E (p.6)."[32] Note: the energy of measuring the inertia of an object is their internal energy (or rest energy) E_0, plus kinetic energy E_k, namely $E_I = E_0 + E_k$, we call this energy E_I as "inertial energy" in order to distinguish with the total energy E in the situation of the potential involved.

From formulas (3) and (4), we can be clearly aware that energy is the measure of the inertia of matter. Actually, the essence of inertia is the conservation of energy. The Einstein's formula of energy and mass indicates that the so-called mass is just its rest energy divided by a constant c^2, is an invariant, and surely is not the measure of inertia. If, at the beginning, Newton realized this fact, he would not propose the concept of mass and just employ the concept of energy directly. In that case, the term mc^2 would not appear in Relativity, but only the rest energy. In formula (2), the second term should be ${E_0}^2$, not m^2c^4.

V. What is the essence of gravitational mass?

Perhaps because of Galileo's experiment at Pisa, Newton did not distinguish the inertial mass and the gravitational mass. In Newton mechanics, there is only one mass which is the amount of matter an object has. In modern Physics, this empirical fact is called the Weak Equivalence Principle (WEP), namely the gravitational mass and the inertial mass of an object are equal. Considering the equivalence of mass and energy (rest energy) and the WEP easily obtain a conclusion that all types of internal energy (rest energy) of an object or a system contribute to the gravitational field and the essence of gravitational mass is rest energy.

G. Liu has obtained this conclusion: "Therefore, the essence of the gravitational mass is the rest energy of matter. The gravitational mass and the inertial mass are the different manifestations of the rest energy of matter. They are actually one physical quantity. The energy of matter is the component of the energy-momentum tensor of matter. The more general expression ought to be the energy-momentum tensor. The source of gravitational field is the energy-momentum tensor of matter (p.5-6)".[35]"We can deduce that the energy-momentum tensor of matter is the source of its gravitational field according to the WEP. The other way round, if supposing the source of gravitational field is the energy-momentum tensor of matter, we can reason and arrive at that the gravitational mass is essentially the inertial mass, and both are the different manifestations of the rest energy of matter in different circumstances (p.6)."[35]

The classical theory offers no compelling reason why the gravitational mass equal to the inertial mass. That it does is merely an empirical fact. So "the equality of inertial and active gravitational mass [...] remains as puzzling as ever."[32] According to above discussion, we know that the essence of inertia is energy. The WEP is equal to the statement that the source (or charge) of gravitational field is energy, more precisely, the energy-momentum tensor, because energy is a component of energy-momentum tensor. If, at the beginning, Newton had realized that the physical quantity being used to measure inertia of an object is not the amount of matter the object has, whereas is the energy (or rest energy when an object is at rest or at low speed), he would have proposed that the source of gravitational field is the energy according to Galileo's experiment at Pisa. So the equality of inertial mass and gravitational mass is a matter-of-course because they are same thing, the rest energy divided by *c* square. Certainly, there is still a question: why is the source of gravitational field the energy-momentum tensor of matter? This is an open question we must answer!

VI. If there is no concept of mass in Physics

If the concept of mass is abandoned and only the concept of energy including the concept of rest energy is employed, all puzzles relating to mass, energy and matter can be solved. Especially, we easily apply this point of view to understand Higgs mechanism.[37] So-called "particles obtain mass after coupling with Higgs field" actually is that the particles having zero rest energy and moving at speed of light are leaded to slow down after coupling with Higgs field, thereby obtain rest energy. So-called the mass term being given in Higgs mechanism is just an expression of the rest energy of the new particle after coupling with Higgs field. In the past, the concept of mass, although is a fundamental concept in Physics, but has not been clearly recognized. When we face to Higgs mechanism, we are puzzle how the particle gains mass and what physical meaning it is. Now it is easy and clear to be understood. All other mechanisms of mass generation can also be understood by applying this point of view.

In classic Physics, Relativity and Quantum theory, if the mass m in physical equations is replaced by E_0/c^2 (or E_0, if $c=1$), the physical equations are completely perfect and can reveal clearer physical meanings. In the dimension of system, mass (kg) should be superseded by energy (eV, GeV, etc). Some famous concepts, equations and laws can be written as following:

1) The formula (3) is the definition of momentum and the formula (4) is Newton second law. The energy E is the time-component of 4-dimentional momentum p_μ . From these concepts and law, we can easily obtain the formula (2).
2) The law of gravity,

$$\boldsymbol{F} = \alpha \frac{E_{I1}E_{I2}}{\boldsymbol{r}^2} \ . \tag{5}$$

   Here, $\alpha = G/c^2$ is constant; E_{I1} and E_{I2} are the inertial energies of the objects (rest energy plus kinetic energy); $\boldsymbol{r}$ is the distance of two objects. By the way, we point out here that this new law of gravity obviously reveals that gravity is relating to the moving state of the object because the energy of objects are relating to its moving state, whereas the Newtonian law of gravity does not indicate this meaning because the masses in Newtonian law of gravity are constant. This expression also reveals that the more accurate law of gravity should involve the energy-momentum tensor of objects or systems because the energy is a component of the energy-momentum tensor.
3) For Schrödinger, Klein-Gordon and other equations, etc, we just need to replace the mass m to E_0/c^2 , then we can obtain the new equations.

VII. Summary

Basing on above discussion, we have been aware that Mass is neither the amount of matter an object has nor the measure of inertia and the real source of gravitational field. At best, when speed is very low, mass could be regarded as an approximate measure of inertia and source of gravitational field. Energy is a measure of inertia and the source of gravitational field. The essence of inertia is conservation of energy!

Then, what is mass? What is mass used for? Mass is not a real physical quantity! It is equal to the rest energy divided by c^2 and does not have any other meaning! Einstein formula of mass and energy is nothing but the relationship of an artificial quantity and the rest energy. If we employ energy to measure the inertia of an object, Einstein formula of mass and energy will not appear in Physics. Further, it would not take more than one hundred years for physicists to argue and interpret the relationship of mass and energy! Since we are aware of mass is rest energy divided by a constant and energy is a more general concept, why we need the concept of mass? The concept of mass should be superseded by energy. Even though one adopts the concept of relativistic mass and believes mass and energy are same properties of matter, why we need two same physical quantities. That implies that one of them must be superseded in Physics. This is just like the obsolete scientific concept of Caloric that was thought of as a weightless gas that could pass in and out of pores in solids and liquids and flow from hotter bodies to colder bodies.

Relativistic mass is not a physical reality. Energy is either the measure of inertia or the source of gravitational field, so the WEP is a matter-of-course. That matter never disappears does not imply that mass is conservative. The law of mass conservation is not true. Matter is everything including objects, particles, fields, even vacuum, and something which we do not know yet, like dark matter. Matter can be converted into different types through interacting each other, but cannot be created or destroyed. During their conversion process, they obey the law of energy conservation. In Physics, if the concept of mass is abandoned and superseded by rest energy, all equations are completely perfect and their meanings are clearer. The mechanism of mass generation is that the particles moving at speed of light and having zero rest energy are leaded to slow down after coupling with Higgs field, thereby obtain rest energy.

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