Negative Gravitational Mass in a Superfluid Bose-Einstein Condensate

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Newton's 2nd law of motion tells us that objects accelerate in the same direction as the applied force. However, recently it was shown experimentally that a Superfluid Bose-Einstein Condensate (BEC) accelerates in the *opposite direction* of the applied force, due to the inertial mass of the BEC becoming *negative* at the specifics conditions of the mentioned experiment. Here we show that is not the inertial mass but the *gravitational mass* of the BEC that becomes *negative*, due to the electromagnetic energy absorbed from the trap and the Raman beams used in the experimental set-up. This finding can be highly relevant to the gravitation theory.

Key words: Negative Gravitational Mass, Bose-Einstein Condensates, Superfluids.

1. Introduction

A recent paper described an experiment that shows a Superfluid Bose-Einstein Condensate (BEC) with *negative* mass, and accelerating in the *opposite direction* of an applied force [1]. The experiment starts with a BEC of approximately 10^5 ^{87}Rb atoms confined in a cigar-shaped trap oriented along the x-axis of a far-detuned crossed dipole trap. Using an adiabatic loading procedure, the BEC is initially prepared such that it occupies the lowest minimum of the lower spin-orbit coupled (SOC) BEC. By suddenly switching off one of the two dipole *trap beams*, the condensate is allowed to spread out along the x-axis. Then, the BEC is imaged in-situ for expansion times of 0, 10 and 14 ms. In the negative x-direction, the BEC encounters an essentially parabolic dispersion, while in the positive x-direction, it enters a *negative mass* region. This leads to a marked asymmetry in the expansion.

Obviously, *negative mass* does not mean *anti-matter*. Anti-matter is simply matter which has the opposite electric charge from normal matter, whereas negative mass means more exactly *negative gravitational mass*. If one particle had ordinary positive gravitational mass, and one had *negative* gravitational mass, then the gravitational force between the masses would be *repulsive* differently of in the case of two positive gravitational masses where the force would be of attraction.

In this article, we show that is not the inertial mass but the *gravitational mass* of the BEC that becomes *negative*, due to the electromagnetic energy absorbed from the trap and the Raman beams used in the experimental set-up. The consequences of this finding can be highly relevant to the gravitation theory.

2. Theory

Some years ago I wrote a paper [2] where a correlation between gravitational mass and inertial mass was obtained. In the paper I pointed out that the relationship between gravitational mass, m_g, and *rest* inertial mass, m_{i0}, is given by

$$\chi = \frac{m_g}{m_{i0}} = \left\{ 1 - 2\left[\sqrt{1 + \left(\frac{U n_r}{m_{i0} c^2} \right)^2} - 1 \right] \right\} \quad (1)$$

where U is *the electromagnetic energy absorbed or emitted by the particle;* $n_r = c/v$ is the index of refraction of the particle; c is the speed of light.

Equation (1) can be rewritten as follows

$$\chi = \frac{m_g}{m_{i0}} = \left\{ 1 - 2\left[\sqrt{1 + \left(\frac{W}{\rho c \, v}\right)^2} - 1 \right] \right\} \qquad (2)$$

where ρ is the matter density, v is the velocity of radiation through the particle, and W is the density of absorbed electromagnetic energy. Substitution of the well-known relation $W = 4D/v$ into Eq. (2) yields

$$\chi = \frac{m_g}{m_{i0}} = \left\{ 1 - 2\left[\sqrt{1 + \left(\frac{4D}{\rho c v^2}\right)^2} - 1 \right] \right\} \qquad (3)$$

where D is the power density of the radiation *absorbed* by the particle.

In order to apply the Eq. (3) to the BEC previously mentioned, we start calculating the *rest inertial mass* of the BEC, which is given by

$$m_{i0(BEC)} \cong 1 \times 10^5 \left(86.909187 \times 1.66 \times 10^{-27} kg \right) =$$

$$= 1.4 \times 10^{-20} kg$$

Assuming that the average radius of the BEC is approximately 40 μm (See reference [1]), then we can calculate the density of the BEC, i.e.,

$$\rho_{BEC} = \frac{m_{i0(BEC)}}{V_{BEC}} = \frac{1.4 \times 10^{-20} kg}{\frac{4}{3}\pi \left(40 \times 10^{-6} m\right)^3} \cong 5.2 \times 10^{-8} kg.m^{-3}$$

Substitution of the values of ρ_{BEC} into Eq. (3) gives

$$\chi_{BEC} = \frac{m_{g(BEC)}}{m_{i0(BEC)}} = \left\{ 1 - 2\left[\sqrt{1 + 0.065\frac{D^2}{v_{BEC}^4}} - 1 \right] \right\} \qquad (4)$$

The variable D, in Eq. (4), refers now to the total power density of the radiation *absorbed* by the BEC (from t*he trap and the Raman beams*, used in the experimental set-up of reference [1]). According to the authors of the experiment the power of the Raman beams are of approximately 3mW (2.9 mW in one of the two beams,3.3 mW in the other), focused to a beam waist of 120 microns ($60\mu m$ radius); the absorption coefficient is $1E_R/2.5E_R = 0.4$. Thus, we can write that

$$D = \frac{P_{abs}}{S_{BEC}} = \frac{0.4 P_{beams}}{4\pi\left(60 \times 10^{-6} m\right)^2} =$$

$$= \frac{0.4 \times \left(2.9 mW + 3.3 mW\right)}{4\pi\left(60 \times 10^{-6} m\right)^2} \cong 5.4 \times 10^4 W.m^{-2} \quad (5)$$

Substitution of this value into Eq. (4) gives

$$m_{g(BEC)} = \left\{ 1 - 2\left[\sqrt{1 + \frac{1.8 \times 10^8}{v_{BEC}^4}} - 1 \right] \right\} m_{i0(BEC)} \qquad (6)$$

Note that, for $v_{BEC} < 109.5 m.s^{-1}$ the *gravitational mass* of the BEC ($m_{g(BEC)}$) becomes *negative*. Lene Hau et al., [3] showed that light speed through a BEC reduces to values much smaller than $100 m.s^{-1}$.

Consequently, we can conclude that it is the *gravitational mass* of the BEC of ^{87}Rb atoms that becomes *negative* and not its inertial mass.

Also it was deduced in the reference [2] a generalized expression for the Newton's 2nd law of motion, which shows that the expression for *inertial forces* is given by

$$\vec{F} = m_g \vec{a} \qquad (7)$$

The presence of m_g in this equation shows that the inertial forces have origin in the *gravitational interaction* between the particle and the others particles of the Universe, just as *Mach's principle* predicts. In this way, the new equation expresses the incorporation of the Mach's principle into Gravitation Theory, and reveals that the inertial effects upon a body can be strongly reduced by means of the decreasing of its gravitational mass. Note that only when m_g reduces to m_{i0} is that we have the well-know expression ($\vec{F} = m_i \vec{a}$) of the Newton's law.

Taking Eq. (6) for an arbitrary value of $v_{BEC} < 109.5 m.s^{-1}$, we obtain $m_{g(BEC)} = -K m_{i0(BEC)}$, where K is a positive number. Substitution of this equation into Eq. (7) yields

$$\vec{F}_{BEC} = -K m_{i0(BEC)} \vec{a} \qquad (8)$$

The sign $(-)$ in this expression reveals clearly why the BEC accelerates in the *opposite direction* of the *applied force*, i.e.,

$$\vec{F}_{BEC} = -\vec{F}_{BEC} = K m_{i0(BEC)} \vec{a} \qquad (9)$$

Recently it was created a BEC with 2.506×10^{17} ^{23}**Na** atoms [4]. The inertial mass of this BEC is

$$m_{i0(BEC)} = 2.506 \times 10^{17}(23)(1.66 \times 10^{-27}) = 3.8 \times 10^{-9}\,kg$$

The number of atoms/cm^3, as showed in reference [4], is $n_0 = 2.742 \times 10^6 cm^3 = 2.742 \times 10^{22} m^{-3}$. Thus, the density of the BEC is given by

$$\rho = (23)(1.66 \times 10^{-27})2.742 \times 10^{22} = 1.04 \times 10^{-3}\,kg.m^{-3}$$

Substitution of these values into Eq. (3) gives

$$m_{g(BEC)} = \left\{1 - 2\left[\sqrt{1 + 1.6 \times 10^{-10}\frac{D^2}{v_{BEC}^4}} - 1\right]\right\}m_{i0(BEC)} \quad (10)$$
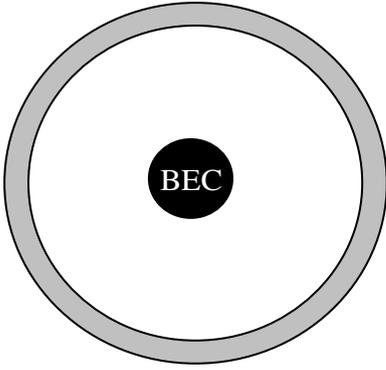

Fig. 1 – A BEC inside a hollow sphere.

Now consider the system showed in Fig.1. Inside the hollow sphere, whose *gravitational* mass is $m_{g(sphere)} \cong m_{i0(sphere)}$, there is a BEC whose *gravitational* mass is given by Eq. (10). Thus, the *total gravitational* mass of the system, $m_{g(SYS)}$, is given by

$$m_{g(SYS)} = m_{g(sphere)} + m_{g(BEC)} =$$

$$\cong m_{i0(sphere)} + \left\{1 - 2\left[\sqrt{1 + 1.6 \times 10^{-10}\frac{D^2}{v_{BEC}^4}} - 1\right]\right\}m_{i0(BEC)} \quad (11)$$

In 2001 Lene Hau et al., have shown that the light speed through a BEC could have values very close to *zero*. Therefore, v_{BEC} with these magnitudes are not unusual. This shows that the value of $m_{g(SYS)}$, given Eq. (11), can be strongly reduced even the value of D be small, for example, of the order of $1 W/m^2$.

Then, consider a system in which the inertial mass of the hollow sphere is $m_{i0(sphere)} = 10,000\,kg$; $m_{i0(BEC)} = 3.8 \times 10^{-9}\,kg$; $D \cong 1 W/m^2$ and $v_{BEC} = 10^{-9}\,m.s^{-1}$ [*], then the gravitational mass of the system will have the following value:

$$m_{g(SYS)} = m_{i0(sphere)} - 2.5 \times 10^{13} m_{i0(BEC)} =$$

$$= -8.5 \times 10^4\,kg \quad (12)$$

Thus, if this system is subjected to a gravity acceleration $\vec{g} = 9.8\,m.s^{-1}$, as shown in Fig.2, then its weight force will be given by $\vec{P} = \left|m_{g(SYS)}\right|\vec{g} = 8.5 \times 10^4\,\vec{g}$, $\left(P = 8.3 \times 10^5\,N\right)$. This value is greater than the thrust of the fighter aircraft F-22 Raptor (fifth-generation), which reaches 160,000N.
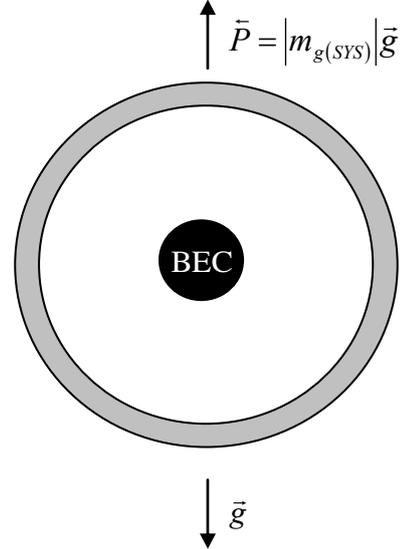

Fig. 2 – The system sphere-BEC, with *negative* gravitational mass ($m_{g(SYS)} < 0$), in a gravitational field.

It is important to note that, in spite the light speed reach values very close to zero through the BEC in the Lene Hau experiments, the number of atoms in the BEC, in this case is very small $\left(10^5 - 10^6\,atoms\right)$ in comparison with the 2.506×10^{17} atoms of the BEC of ^{23}**Na,** here mentioned. Consequently, the weight force, acting on the BEC in the case of the Lene Hau experiments is approximately 10^{-12} times smaller than in the case of the BEC of Na atoms.

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[*] With this velocity the light beam would travel about 0.1mm in a day.

References

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