Feigenbaum's Constant and the Sommerfeld Fine-Structure Constant

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ABSTRACT

A numerical correlation exists between two unitless, universal physical constants: the Sommerfeld fine-structure constant and Feigenbaum's constant. The relationship has a simple geometrical form.

FINE-STRUCTURE CONSTANT

The Sommerfeld fine-structure constant (α) is a dimensionless quantity comprised of the four basic physical constants, electronic charge (e), speed of light in a vacuum (c), the Planck constant (h), and the permitittivity of free space (ε_0) [1]. This constant was discovered when it first appeared in connection with the fine structure of atomic spectra. The fine-structure constant is sometimes also referred to as the electromagnetic coupling constant. It characterizes the force of coupling between the elementary electric charge and the electromagnetic field. The equation for the fine structure constant is:

$$\alpha = e^2/2hc\varepsilon_0 \qquad (1)$$

where:

$$e = \text{the charge of an electron: } 1.6021917 \times 10^{-19}\ C \qquad (2)$$

$$h = \text{Planck's constant: } 6.626196 \times 10^{-34}\ J{*}S \qquad (3)$$

$$c = \text{the velocity of light: } 2.9979250 \times 10^{8}\ M{*}S^{-1} \qquad (4)$$

$$\varepsilon_0 = \text{the permittivity of free space: } 8.854\ 187\ 817 \times 10^{-12}\ F{*}M^{-1} \qquad (5)$$

therefore:

$$\alpha = 7.297\ 350\ 47 \text{ X } 10^{-3} \approx 1/137 \qquad (6)$$

In some cases, the fine-structure constant is expressed as α^{-1} or 137. The best experimental value for α presently is based on the quantum Hall effect[2]:

$$\alpha^{-1} = 137.035\ 997\ 9(33) \qquad (7)$$

The equation $\alpha = e^2/2hc\varepsilon_0$ contains the fundamental constants of quantum physics (h), of relativity (c) and of electromagnetic theory (e and ε_0). The constant α is also unitless, as shown here:

$$\alpha = e^2/2hc\varepsilon_0$$

$$= C^2/(V*C*S)(M*S^{-1})((C/V)M^{-1})$$

$$= C^2/(V*C*S)(s^{-1})(C/V)$$

$$= C^2/(V*C)(C/V)$$

$$= C^2/(C*C)$$

$$= \text{unitless} \qquad (8)$$

α appears in several equations which describe the hydrogen atom[3]:

First Bohr orbit:	$a_0 = \lambda'_c/\alpha$	(9)
Ground state energy:	$-\varpi_0 = -1/2\alpha^2 m_0 c^2$	(10)
Rydberg constant:	$R\gamma_\infty = \alpha^2/4\pi\lambda'_c$	(11)
Orbital velocity:	$v = \alpha c$	(12)

Where:

λ'_c = Compton wavelength divided by 2π

m_0 = the electron rest mass

FEIGENBAUM'S CONSTANT

As a non-linear system goes into a state of turbulence (chaos), an infinite sequence of bifurcations or period-doublings occur. Figure 1 is a bifurcation tree illustrating the period doubling sequence[4]. Bifurcations occur simultaneously; each branch is an approximate "scale model" of the previous branch and the scaling factor converges on a number, the Feigenbaum constant, $\delta_f = 4.6692016....$

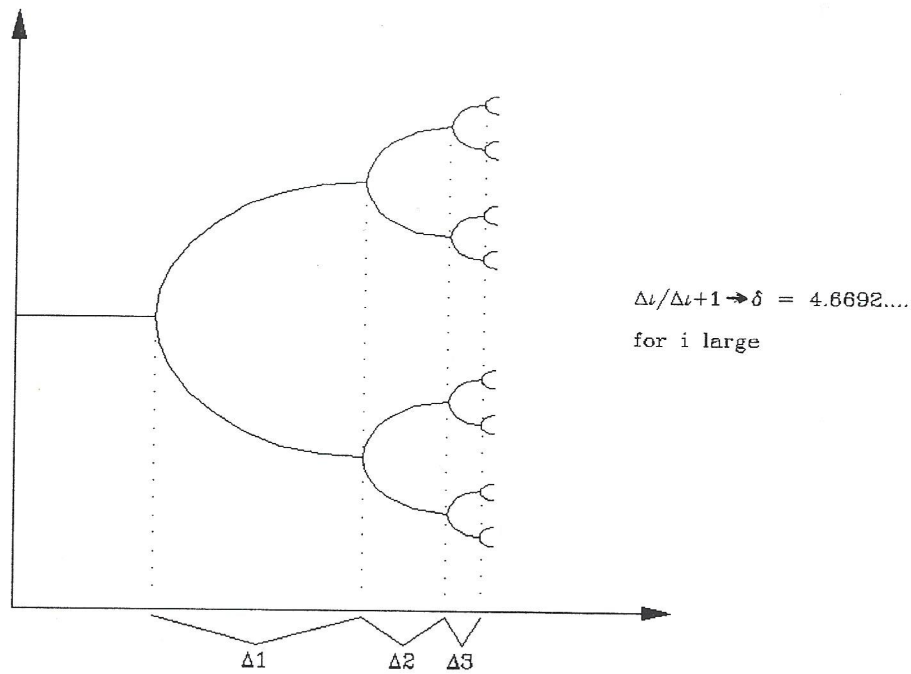

Figure 1: A Typical "Bifurcation Tree"

To quote Feigenbaum:

> "What is quite remarkable (beyond the fact that there is always a geometric convergence) is that, for all systems under going this period doubling, the value of δ is predetermined at the universal value δ = 4.6692016...[5]"

A RELATIONSHIP

A mathematical association exists between the fine-structure constant, α and Feigenbaum's scaling factor, δ_f:

$$\delta' = (2\pi\alpha)^{-1/2} = 4.670113801 \approx \delta_f = 4.6692016090 \qquad (13)$$

$$\delta' - \delta_f = 0.000912192 \qquad (14)$$

In addition, another relationship exists between δ' and α:

$$\delta' = (2\pi\alpha)^{-1/2} \text{ expressed as}$$

$$\alpha^{-1} = 2\pi\delta'^2 \tag{15}$$

also resembles a familiar equation,

$$SA_{sphere} = 4\pi r^2 , \tag{16}$$

where SA_{sphere} is the surface area of a sphere. If the radius of a sphere, $r = \delta'$, then

$$SA_{sphere} = 4\pi\delta'^2 = 2\alpha^{-1} \text{ (Figure 2)} \tag{17}$$

Put another way, if the radius of a sphere is Feigenbaum's constant, then the surface area of that sphere is twice the inverse of the fine-structure constant or $\cong 2 \times 137 = 274$.

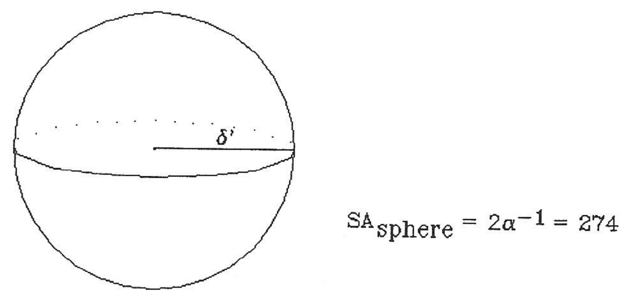

Figure 2: A Sphere, with radius $r = \delta'$

CONCLUSIONS

There is a numerical correlation between two unitless constants, Feigenbaum's number and the Sommerfeld fine-structure constant. This relationship has a simplicity expressed in the equation $\delta' = (2\pi\alpha)^{-1/2}$. All terms of this equation are unitless, therefore the relationship is universal. A geometrical relationship exists between δ' and α; for a sphere with a radius of δ', the surface area of that sphere is $2\alpha^{-1}$.

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[2] Encyclopedia of Physics, 2nd Edition, 1991, pg. 972

[3] Modern Physics and Quantum Mechanics, Elmer E. Anderson, Phd., W.B. Saunders & Co.

[4] Universality in Chaos, Predrag Cvitanovic, Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø

[5] Universal Behavior in Nonlinear Systems, Mitchell J Feigenbaum, Los Alamos Science 1 4-27 (1980)