Formula to generate a set of Poulet numbers from a Poulet number P and its factor d lesser than sqr P

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Abstract. In this paper I make the following observation: Let d be a factor (not necessarily prime) of the Poulet number P such that d < sqr P and m the least number such that m*d*(d - 1) > (P - 1)/2. Let n be equal to P - m*d*(d - 1). Then often exist a set of Poulet numbers Q such that Q mod(m*d*(d - 1)) = n. For example, for P = 2047 = 23*89 and d = 23, where d < sqr 2047, the least m such that m*23*22 > (P - 1)/2 is equal to 3 (1518 > 1023, while, for 2, 1012 < 1023); so, n = 2047 - 3*23*22 = 2047 - 1518 = 529 and indeed there exist a set of Poulet numbers Q such that Q mod 1518 = 529; the formula 1518*x + 529 gives the Poulet numbers 2047, 6601, 15709, 30889 (...) for x = 1, 4, 10, 20 (...).

Observation:

Let d be a factor (not necessarily prime) of the Poulet number P such that d < sqr P and m the least number such that m*d*(d - 1) > (P - 1)/2. Let n be equal to P - m*d*(d - 1). Then often exist a set of Poulet numbers Q such that Q mod(m*d*(d - 1)) = n.

Example: for P = 2047 = 23*89 and d = 23, where d < sqr 2047, the least m such that m*23*22 > (P - 1)/2 is equal to 3 (1518 > 1023, while, for 2, 1012 < 1023); so, n = 2047 - 3*23*22 = 2047 - 1518 = 529 and indeed there exist a set of Poulet numbers Q such that Q mod 1518 = 529; the formula 1518*x + 529 gives the Poulet numbers 2047, 6601, 15709, 30889 (...) for x = 1, 4, 10, 20 (...).

Few sets of Poulet numbers obtained:

: for P = 341 = 11*31, d = 11, d < sqr 341; the least m such that m*110 > 170 is m = 2 so n = 341 - 220 = 121; then Q mod 220 = 221 and formula 220*x + 121 gives the Poulet numbers 341, 561, 8481 (...) for x = 1, 2, 38 (...);

: for P = 561 = 3*11*17, d = 11, d < sqr 561; the least m such that m*110 > 280 is m = 3 so n = 561 – 330 = 231; then Q mod 330 = 231 and formula 330*x + 231 gives the Poulet numbers 561, 8481 (...) for x = 1, 25 (...);

: for P = 645 = 3*5*43, d = 15, d < sqr 645; the least m such that m*210 > 322 is m = 2 so n = 645 – 420 = 225; then Q mod 420 = 225 and formula 420*x + 225 gives the Poulet numbers 645, 1905 (...) for x = 1, 4 (...);

: for P = 1105 = 5*13*17, d = 13, d < sqr 1105; the least m such that m*156 > 552 is m = 4 so n = 1105 – 624 = 481; then Q mod 624 = 481 and formula 624*x + 481 gives the Poulet numbers 1105, 1729, 16705 (...) for x = 1, 2, 26 (...);

: for P = 1729 = 7*13*19, d = 13, d < sqr 1729; the least m such that m*156 > 864 is m = 6 so n = 1729 – 936 = 793; then Q mod 936 = 793 and formula 936*x + 793 gives the Poulet numbers 1729, 16705 (...) for x = 1, 17 (...);

: for P = 1729 = 7*13*19, d = 19, d < sqr 1729; the least m such that m*342 > 864 is m = 3 so n = 1729 – 1026 = 703; then Q mod 1026 = 703 and formula 1026*x + 703 gives the Poulet numbers 1729, 8911 (...) for x = 1, 8 (...);

: for P = 2701 = 37*73, d = 37, d < sqr 2701; the least m such that m*1332 > 1350 is m = 2 so n = 2701 – 2664 = 37; then Q mod 2664 = 37 and formula 2664*x + 37 gives the Poulet numbers 2701, 29341 (...) for x = 1, 11 (...);

: for P = 2821 = 7*13*31, d = 31, d < sqr 2821; the least m such that m*930 > 1410 is m = 2 so n = 2821 – 1860 = 961; then Q mod 1860 = 961 and formula 1860*x + 961 gives the Poulet numbers 2821, 4681, 10261, 13981, 15841, 75361, 93961, 172081, 285541 (...) for x = 1, 2, 5, 7, 8, 40, 50, 92, 153 (...).