ABC Conjecture–An Ambiguous Formulation

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Abstract

Due to exist forevermore uncorrelated limits of values of real number $\varepsilon\geq0$, enable ABC conjecture to be able to be both proved and negated. In this article, we find a representative equality $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[Rad(1, 2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, then both prove the ABC conjecture and negate the ABC conjecture according to two limits of values of ε.

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1. Introduction

The ABC conjecture was proposed by Joseph Oesterle and David Masser in 1985. The conjecture states that if A, B and C are three co-prime positive integers satisfying A+B=C, then for any real number $\varepsilon>0$, there is merely at most a finite number of solutions to the inequality $C>(Rad(A, B, C))^{1+\varepsilon}$, where Rad(A, B, C) denotes the product of all distinct prime divisors of A, B and C. Yet it is still both unproved and un-negated a conjecture hitherto, although somebody claiming proved it on the internet.

2. The proof and the negation coexist

As everyone knows, whether anybody wants to prove the ABC conjecture by one side or unilaterally negate it, all in all, that is a very difficult thing.

Such being the case, we have to find an equality such that difference of C minus Rad(A, B, C) is small as far as possible. Self-evidently, not only the way of doing is simple and convenient, but also it implies that once solved the equality, actually solved other equalities that are represented by it.

So let A or B to equal 1, and another equals O^2-1, then C is equal to O^2 according to A+B=C, where O expresses an odd number ≥ 3.

Then, the equality A+B=C satisfying $C > (Rad(A, B, C))^{1+\varepsilon}$ is changed into the equality $1+(O^2-1)=O^2$ satisfying $O^2 > (Rad(1, O^2-1, O^2))^{1+\varepsilon}$ in the case that regards ε as an infinitesimal real number >0.

If O is a positive prime P, then the equality $1+(O^2-1)=O^2$ satisfying $O^2 > (Rad(1, O^2-1, O^2))^{1+\varepsilon}$ is turned into the equality $1+(P^2-1)=P^2$ satisfying $P^2 > (Rad(1, P^2-1, P^2))^{1+\varepsilon}$. In the case that regards ε as an infinitesimal real number >0, $P^2 > (Rad(1, P^2-1, P^2))^{1+\varepsilon}$ approximates to $P > (Rad(P^2-1))^{1+\varepsilon}$, both deviation is only a very tiny P^{ε}. When $P \geq 7$, see also APPENDIX at the back of this article for reference.

Thus it can be seen, that the equality $1+(P^2-1)=P^2$ satisfying $P > (Rad(P^2-1))^{1+\varepsilon}$ by and large, seemingly should last forever in the case that regards ε as an infinitesimal real number>0, although the densities of satisfactory primes are getting sparser and sparser along with which the values of P are getting greater and greater, but there are infinitely many primes after all.

To say nothing of the conjecture including all positive integers, presumably satisfactory positive integers must be even more.

Well then, let the equality $1+(O^2-1)=O^2$ be endowed with certain peculiar values, enable it to turn into a representative equality, and that apply the representative equality to prove and negative the conjecture in ambiguity.

From $O^2-1=(O+1)(O-1)$, we know that O+1 and O-1 are two even numbers, further let O+1 to equal 2^N, then not only 2 is a common prime factor of O+1 and O-1, but also 2 is the unique prime factor of O+1, where $N \geq 2$.

From $O+1=2^N$, get $O=2^N-1$, $O-1=2^N-2$, $O^2=(2^N-1)^2$ and $O^2-1=2^N(2^N-2)$, so the equality $1+(O^2-1)=O^2$ satisfying $O^2>(Rad(1, O^2-1, O^2))^{1+\varepsilon}$ is transformed into equality $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[Rad(1, 2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ i.e. $(2^N-1)^2>[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon}$ in the case that regards ε as an infinitesimal real number >0.

Since N≥2, thus there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$.

Also the large-small symbol between $(2^N-1)^2$ and $[Rad(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ is alterable, and illustrate with example as follows.

Let N=2, then it has $(2^2-1)^2=9$, and $[Rad(2^2(2^2-2), (2^2-1)^2)]^{1+\varepsilon}=(2\times3)^{1+\varepsilon}$, evidently $(2^2-1)^2 >[Rad(2^2(2^2-2), (2^2-1)^2)]^{1+\varepsilon}$ where $\varepsilon< \log_6 9-1$.

In the inequality, if $\varepsilon>\log_6 9-1$, then it has $(2^2-1)^2<[Rad(2^2(2^2-2), (2^2-1)^2)]^{1+\varepsilon}$; if $\varepsilon=\log_6 9-1$, then it has $(2^2-1)^2 =[Rad(2^2(2^2-2), (2^2-1)^2)]^{1+\varepsilon}$.

By this token, after N=a positive integer, different valuations of ε decide large or small of $[Rad(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ as compared with $(2^N-1)^2$.

As thus, suppose that $(2^N-1)^2=[\mathrm{Rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, then it has $1+\varepsilon= \log_{\mathrm{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2$, and there is $\varepsilon=[\log_{\mathrm{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$.

So if $\varepsilon=[\log_{\mathrm{rad}(2^N(2^N-2),(2^N-1)^2)}(2^N-1)^2]-1$, then $(2^N-1)^2=[\mathrm{Rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$;

If $0<\varepsilon<[\log_{\mathrm{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, then $(2^N-1)^2>[\mathrm{Rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, and that there are infinitely many real numbers of ε between 0 and $[\log_{\mathrm{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$;

If $\varepsilon>[\log_{\mathrm{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, then $(2^N-1)^2<[\mathrm{Rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, of course, there are infinitely many real numbers of ε in the case too.

Hereinafter we will divide the range of values of ε into four parts as compared with requirements of the conjecture, and from this decide the take or the abandonment for each part.

Firstly, when $\varepsilon=0$, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ with $N\geq 2$ satisfying $(2^N-1)^2>[\mathrm{Rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$. Evidently this part has nothing to do with the conjecture because $\varepsilon=0$ is inconformity to the requirement of the conjecture.

Secondly, when $0<\varepsilon<[\log_{\mathrm{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ with $N\geq 2$ satisfying $(2^N-1)^2>[\mathrm{Rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$. Namely there are infinitely many pairs of N plus ε to satisfy infinitely many equalities plus inequalities in the case monogamously.

Thirdly, when $\varepsilon=[\log_{\mathrm{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, there is only an equality $1+2^N(2^N-2)=(2^N-1)^2$ with $N\geq 2$ satisfying $(2^N-1)^2=[\mathrm{Rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$. This part has nothing to do with the conjecture either because $(2^N-1)^2=[\mathrm{Rad}$

$(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ is inconformity to the requirement of the conjecture.

Fourthly, when $\varepsilon>[\log_{rad(2^N(2^N-2),\ (2^N-1)^2)}(2^N-1)^2]-1$, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ with $N\geq 2$ satisfying $(2^N-1)^2 < [Rad\ (2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$. This part has nothing to do with the conjecture likewise because $(2^N-1)^2 < [Rad(2^N(2^N-2),\ (2^N-1)^2)]^{1+\varepsilon}$ is inconformity to the requirement of the conjecture .

By this token, whether anybody wants to prove the ABC conjecture or negate the ABC conjecture, he/she can only comes from aforesaid second part i.e. when $0<\varepsilon<[\log_{rad(2^N(2^N-2),\ (2^N-1)^2)}(2^N-1)^2]-1$ to consider it.

Below list $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[Rad(2^N(2^N-2),\ (2^N-1)^2)]^{1+\varepsilon}$ according to headmost values of N, where $0<\varepsilon<[\log_{\ rad(2^N(2^N-2),\ (2^N-1)^2)}(2^N-1)^2]-1$, but values of ε which satisfy each inequality are incomplete alike as compared with others.

N	$2^N(2^N-2)$	$(2^N-1)^2 >[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon}$	$1+2^N(2^N-2)=(2^N-1)^2$
2,	8,	$9 >6^{1+\varepsilon}$,	1+8=9
3,	48,	$49 >(2*3*7)^{1+\varepsilon}=42^{1+\varepsilon}$,	1+48=49
4,	224,	$225 >(2*3*5*7)^{1+\varepsilon}=210^{1+\varepsilon}$,	1+224=225
5,	960,	$961 >(2*3*5*31)^{1+\varepsilon}=930^{1+\varepsilon}$,	1+960=961
6,	3968,	$3969 >(2*3*7*31)^{1+\varepsilon}=1302^{1+\varepsilon}$,	1+3968=3969
7,	16128,	$16129 >(2*3*7*127)^{1+\varepsilon}=5334^{1+\varepsilon}$,	1+16128=16129
8,	65024,	$65025 >(2*3*5*17*127)^{1+\varepsilon}=64770^{1+\varepsilon}$,	1+65024=65025
9,	261120,	$261121>(2*3*5*7*17*73)^{1+\varepsilon}=260610^{1+\varepsilon}$,	1+261120=2611 21
10,	1046528,	$1046529 >(2*3*7*11*31*73)^{1+\varepsilon}=1045506^{1+\varepsilon}$,	1+1046528=1046529
11,	4190208,	$4190209 >(2*3*11*23*31*89)^{1+\varepsilon}=4188162^{1+\varepsilon}$,	1+4190208=4190209
12,	16769024,	$16769025 >(2*3*5*23*89*91)^{1+\varepsilon}=5588310^{1+\varepsilon}$,	1+16769024=16769025
13,	67092480,	$67092481>(2*3*5*7*13*8191)^{1+\varepsilon}=22361430^{1+\varepsilon}$,	1+67092480=67092481
14,	268402688,	$268402689 >(2*3*43*127*8191)^{1+\varepsilon}=268386306^{1+\varepsilon}$,	1+268402688=268402689
15,	1073676288,	$1073676289>(6*7*31*43*127*151)^{1+\varepsilon}=1073643522^{1+\varepsilon}$,	1+1073676288=1073676289
…	…	…	…

From listed above inequalities and predicting inequalities infinitely extend, we are not difficult to make out that values of ε are getting smaller and smaller up to infinitesimal along with which values of N are getting greater and greater up to infinite.

When $0<\varepsilon<[\log_{rad(2^N(2^N-2),\,(2^N-1)^2)}(2^N-1)^2]-1$, if successive valuations of ε begin with some point near to $[\log_{rad(2^N(2^N-2),\,(2^N-1)^2)}(2^N-1)^2]-1$, then the conjecture can be proved; if successive valuations of ε begin with some point near to 0, then the conjecture will be negated. Nobis, be necessary to expound them on aforementioned two aspects respectively, ut infra.

3. Proving the ABC conjecture

Prove the ABC conjecture, obviously this implies that we are unable to find a fixed value of ε, such that there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ with N≥2 satisfying $(2^N-1)^2>[Rad(2^N(2^N-2),\,(2^N-1)^2)]^{1+\varepsilon}$.

Namely for any real number ε >0, there are merely finitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon}$ in the case that regards ε as a fixed value.

Since N≥2, on the one hand, values of N are getting more and more up to infinite many along with which values of N are getting greater and greater up to infinite, so they form infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$.

On the other hand, begin with a greater suited value of ε in correspondence with a value of N, then ε is getting smaller and smaller successively up to infinitesimal along with which N is getting greater and greater successively up to infinite. As thus, pairs of ε plus N are getting more and more up to infinitely many, so form infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon}$ monogamously.

Since N and ε appear in pairs within equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon}$, thus start from any given value of ε, when lessen successively values of ε to reach any very tiny fixed value ε_x in finite field, N in correspondence with ε_x is too a finite natural number in finite field, accordingly there are unquestionably finitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon_x}$.

Further speak with emphasis, begin with any given pair of N and ε, although natural numbers of N are getting greater and greater successively and corresponding real numbers of ε are getting smaller and smaller successively to form more and more equalities like $1+2^N(2^N-2) =(2^N-1)^2$ satisfying $(2^N-1)^2 >[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon}$, but since forever cannot reach greatest natural number and forever cannot reach smallest positive real number, therefore, for any tiny fixed ε_x in finite field, there are only finitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon_x}$.

On balance, 1, $2^N(2^N-2)$ and $(2^N-1)^2$ are three co-prime positive integers satisfying $1+2^N(2^N-2)=(2^N-1)^2$, for any real number ε >0, there is merely at most a finite number of solutions to $(2^N-1)^2>[Rad(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$.

Now that satisfactory smallest constant 2 within the equality cause only finite many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 >$[Rad(1,

$2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, not excepting each and every integer >2 surely too.

Consequently, the ABC conjecture is proven by us to be tenable.

4. Negating the ABC conjecture

Negate the ABC conjecture, undoubtedly this implies that we must find at least a value of ε between 0 and $[\log_{rad(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$ such that there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ with N≥2 satisfying $(2^N-1)^2 >[Rad(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$.

For the half that there are infinitely many equalities like $1+2^N(2^N-2)= (2^N-1)^2$, this is out of question. The problem is to confirm a satisfactory real number.

Now that there are infinitely many positive real numbers of ε between 0 and $[\log_{rad(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, then the positive real number which and 0 border on each other is certainly the smallest positive real number.

Suppose that we named the smallest positive real number "ε_0", then, there is not a real number between 0 and ε_0. Then again, there are still infinitely many positive real numbers between ε_0 and $[\log_{rad(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$.

Consequently, if N is endowed with infinite many values, then there are infinitely many values of ε between ε_0 and $[\log_{rad(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$ too, enable them one-to-one pairing such that $1+2^N(2^N-2)=(2^N-1)^2$ with N≥2 satisfying $(2^N-1)^2>[Rad(1, 2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_0}$.

In other words, when $\varepsilon=\varepsilon_0$ and N≥2, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 >[Rad(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_0}$.

Moreover, start from ε_0, we name orderly-increasing and orderly-adjacent real numbers "ε_0, ε_1, $\varepsilon_2 \ldots \varepsilon_y$", where y is a concrete natural number which consists of Arabic numerals.

Without doubt, for real number ε_y, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ with $N\geq 2$ satisfying $(2^N-1)^2>[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon_y}$ because infinitely many values of ε between $[\log_{rad(2^N(2^N-2),\,(2^N-1)^2)}(2^N-1)^2]-1$ and ε_y and infinitely many values of $N\geq 2$ form monogamously pairs to satisfy $(2^N-1)^2>[Rad(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_y}$. Thus, begin with any given fixed value of ε, the given fixed value forever cannot be decreased to ε_y.

That is to say, when $\varepsilon=\varepsilon_y$, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ with $N\geq 2$ satisfying $(2^N-1)^2>[Rad(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_y}$.

So far, let the representative equality as compared with the definition of the conjecture as follows.

First, three terms 1, $2^N(2^N-2)$ and $(2^N-1)^2$ in the representative equality are co-prime positive integers.

Secondly, $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[Rad(2^N(2^N-2),(2^N-1)^2)]^{1+\varepsilon_0}$ are completely in conformity with the requirements of the conjecture.

By this token, if regard ε_0 as a fixed real number, then the ABC conjecture has to be negated by infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 >(rad(1,\ 2^N(2^N-2),\ (2^N-1)^2))^{1+\varepsilon}$ where $N\geq 2$, $0<\varepsilon=\varepsilon_0, \varepsilon_1, \varepsilon_2 \ldots \varepsilon_y$, and y is a concrete natural number.

That is to say, the ABC conjecture is untenable. As thus, the ABC conjecture can only be regarded as a fallacy or a defective expression.

After y is endowed with a natural number, can ε_0 or ε_y be a fixed real number? At present, we only know that ε_0 or ε_y has the designation and the fixed location, therein ε_0 neighbors 0. In addition to this, it can compare out large or small between any real number and ε_0 or ε_y.

5. The eventual statement

What causes both proving and negating the ABC conjecture? In my opinion, the key to the settlement of the question lies in mathematical circles, whether they can admit ε_0 as a fixed real number.

If ε_0 is admitted as a fixed real number, then the ABC conjecture thereupon is negated either according to the disproof of preceding fourth section.

If ε_0 can not be admitted as a fixed real number, then the ABC conjecture is tenable too according to the proof of preceding third section.

In this article, the author has analyzed merely two aspects which the ABC conjecture is both proved and negated, this is for reader's reference only.

Is on earth right or wrong the ABC conjecture? I am convinced of either judgment of reader adequately.

PS. In the course that forms this article stepwise, the author once got successively the guidance of Professors Nigel Byott at the University of Exeter, Zeev Rudnick at Tel-Aviv University, Alan Haynes at the University of York, Akshay Venkatesh at Stanford University and Kathrin Bringmann at University of Cologne, hereon, I as the author sincerely thank them.

APPENDIX: Prime number P and equality $1+(P^2-1)=P^2$ satisfying $P^2 >(\text{Rad}(1, P^2-1, P^2))^{1+\varepsilon}$ i.e. satisfying $P>(\text{Rad}(P^2-1))^{1+\varepsilon}$ by and large after the evaluations of headmost P are listed as follows, but limits of real number ε which satisfy each inequality are incomplete alike as compared with others.

P,	P^2-1,	Rad (P^2-1)
7,	48,	2*3=6
17,	288,	2*3=6
31,	960,	2*3*5=30
97,	9408,	2*3*7=42
127,	16128,	2*3*7=42
251,	63000,	2*3*5*7=210
449,	201600,	2*3*5*7=210
487,	237168,	2*3*61=366
577,	332928,	2*3*17=102
1151,	1324800,	2*3*5*23=690
1249,	1560000,	2*3*5*13=390
1567,	2455488,	2*3*7*29=1218
1999,	3996000,	2*3*5*37=1110
2663,	7091568,	2*3*11*37=2442
4801,	23049600,	2*3*5*7=210
4999,	24990000,	2*3*5*7*17=3570
7937,	62995968,	2*3*7*31=1302
8191,	67092480,	2*3*5*7*13=2730
12799,	163814400,	2*3*5*79=2370
13121,	172160640,	2*3*5*41=1230
13183,	173791488,	2*3*13*103=8034
15551,	241833600,	2*3*5*311=9330
31249,	976500000,	2*3*5*7*31=6510
31751,	1008126000,	2*3*5*7*127=26670
32257,	1040514048,	2*3*7*127=5334
33857,	1146296448,	2*3*11*19*23=28842
35153,	1235733408,	2*3*7*13*31=16926
39367,	1549760688,	2*3*7*19*37=29526
65537,	4295098368,	2*3*11*331=21846
79201,	6272798400,	2*3*5*11*199=65670
81919,	6710722560,	2*3*5*37*41=45510
85751,	7353234000,	2*3*5*7*397=83370
115249,	13282332000,	2*3*5*7*461=96810
117127,	13718734128,	2*3*11*241=15906

124001,	15376248000,	2*3*5*31*83=77190
126001,	15876252000,	2*3*5*7*251=52710
131071,	17179607040,	2*3*5*17*257=131070
153089,	23436241920,	2*3*5*7*13*23=62790
160001,	25600320000,	2*3*5*2963=88890
161839,	26191861920,	2*3*5*7*17*37=132090
165887,	27518496768,	2*3*7*17*41=29274
196831,	38742442560,	2*3*5*6151=184530
215297,	46352798208,	2*3*29*443=77082
281249,	79101000000,	2*3*5*11*17*47=263670
442367,	195688562688,	2*3*29*263=45762
474337,	224995589568,	2*3*61*487=178242
511757,	261895227048,	2*3*7*13*373=203658
524287,	274876858368,	2*3*7*19*73=58254
538001,	289445076000,	2*3*5*41*269=330870
665857,	443365544448,	2*3*17*577=58854
715823,	512402567328,	2*3*71*1657=705882
902501,	814508055000,	2*3*5*19*619=352830
911249,	830374740000,	2*3*5*13*337=131430
988417,	976968165888,	2*3*11*13*19*37=603174
1039681,	1080936581760,	2*3*5*7*19*103=410970
1062881,	1129716020160,	2*3*5*7*13*73=199290
1102249,	1214952858000,	2*3*5*7*4409=925890
1179649,	1391571763200,	2*3*5*23593=707790
1229311,	1511205534720,	2*3*5*7*29*157=956130
1246589,	1553984134920,	2*3*5*7*19*211=841890
1272833,	1620103845888,	2*3*11*97*113=723426
...	...	...

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