Mathematics, the Continuous or the Discrete Which is Better to Reality of Things

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Abstract: There are 2 contradictory views on our world, i.e., continuous or discrete, which results in that only partially reality of a thing T can be understood by one of continuous or discrete mathematics because of the universality of contradiction and the connection of things in the nature, just as the philosophical meaning in the story of the blind men with an elephant. Holding on the reality of natural things motivates the combination of continuous mathematics with that of discrete, i.e., an envelope theory called *mathematical combinatorics* which extends classical mathematics over topological graphs because a thing is nothing else but a multiverse over a spacial structure of graphs with conservation laws hold on its vertices. Such a mathematical object is said to be an *action flow*. The main purpose of this report is to introduce the powerful role of action flows, or mathematics over graphs with applications to physics, biology and other sciences, such as those of G-solution of non-solvable algebraic or differential equations, Banach or Hilbert $\overrightarrow{G}$-flow spaces with multiverse, multiverse on equations, $\cdots$ and with applications to, for examples, the understanding of particles, spacetime and biology. All of these make it clear that holding on the reality of things by classical mathematics is only on the coherent behaviors of things for its homogenous without contradictions, but the mathematics over graphs G is applicable for contradictory systems because contradiction is universal only in eyes of human beings but not the nature of a thing itself.

Key Words: Graph, Banach space, Smarandache multispace, $\overrightarrow{G}$-flow, observation, natural reality, non-solvable equation, mathematical combinatorics.

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§1. Introduction

Generally, the reality of a thing T is its state of existed, exists, or will exist in the world, whether or not they are observable or comprehensible by human beings. However, the recognized reality maybe very different from that of the truth because it depends on the way of the observer and his world view is continuous or discrete, i.e., view the behavior of thing T a continuous function f, or an infinite or finite sequence $x_1, x_2, \cdots, x_n$ with $n \geq 1$ on time t.

Is our world continuous or discrete? Certainly not because there exist both continuous or discrete things in the eyes of human beings. For example, all apples on a tree is discrete but the moving of a car on the road is continuous, such as those figures (a) and (b) shown in Fig.1.

(a) (b)

Fig.1

And historically, holding on the behavior of things mutually develops the continuous and discrete mathematics, i.e., research a discrete (continuous) question by that of continuous (discrete) mathematical methods. For example, let x, y be the populations in a self-system of cats and rats, such as Tom and Jerry shown in Fig.2,

Fig.2

then they were continuously characterized by Lotka-Volterra with differential equations ([4])

$$\begin{cases} \dot{x} &= \ \ x(\ \lambda - by), \\ \dot{y} &= y(-\mu - cx). \end{cases} \tag{1.1}$$

Similarly, all numerical calculations by computer for continuous questions are carried out by discrete methods because algorithms language recognized by computer is essentially discrete. Such a typical example is the movies by discrete images for a continuous motion shown in Fig.3. Thus, the reality of things needs the combination of the continuous mathematics with that of the discrete.

Fig.3

Physically, the behavior of things T is usually characterized by differential equation

$$\mathscr{F}\left(t, x_1, x_2, x_3, \psi_t, \psi_{x_1}, \psi_{x_2}, \cdots, \psi_{x_1x_2}, \cdots\right) = 0 \tag{1.2}$$

established on observed characters of $\mu_1, \mu_2, \cdots, \mu_n$ for its state function $\psi(t, x)$ in reference system $\mathbb{R}^3$ by Newtonian and $\mathbb{R}^4$ by Einstein ([2]).

Fig.4

Usually, these physical phenomenons of a thing is complex, and hybrid with other things. *Is the reality of particle P all solutions of that equation* (1.2) *in general?*

Certainly not because the equation (1.2) only characterizes the behavior of P on some characters of $\mu_1, \mu_2, \cdots, \mu_n$ at time t abstractly, not the whole in philosophy. For example, the behavior of a particle is characterized by the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + U\psi \tag{1.3}$$

in quantum mechanics ([24]) but observation shows it in two or more possible states of being, i.e., superposition such as the skying question of Schrödinger for the alive or dead of the cat in the box with poison switch shown in Fig.4. We can not even say which solution of the Schrödinger equation (1.3) is the particle because each solution is only for one determined state.

Furthermore, *can we conclude the equation* (1.2) *is absolutely right for a particle* P*?* Certainly not also because the dynamic equation (1.2) is always established with an additional assumption, i.e., the geometry on a particle P is a point in classical mechanics or a field in quantum mechanics and dependent on the observer is out or in the particle. For example, a water molecule H_2O consists of 2 hydrogen atoms and 1 oxygen atom such as those shown in Fig.5. If an observer receives information on the behaviors of hydrogen or oxygen atom but stands out of the water molecule H_2O by viewing it a geometrical point, then he can only receives coherent information on atoms H and O with the water molecule H_2O.

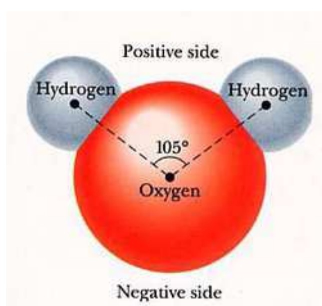


Fig.5

But if he enters the interior of the molecule, he will view a different sceneries for atoms H and O, which are respectively called *out-observation* and *in-observation*, and establishes equation (1.3) on H_2O or 3 dynamic equations

$$\begin{cases} -i\hbar\dfrac{\partial\psi_O}{\partial t} = \dfrac{\hbar^2}{2m_O}\nabla^2\psi_O - V(x)\psi_O \\ -i\hbar\dfrac{\partial\psi_{H_1}}{\partial t} = \dfrac{\hbar^2}{2m_{H_1}}\nabla^2\psi_{H_1} - V(x)\psi_{H_1} \\ -i\hbar\dfrac{\partial\psi_{H_2}}{\partial t} = \dfrac{\hbar^2}{2m_{H_2}}\nabla^2\psi_{H_2} - V(x)\psi_{H_2} \end{cases} \tag{1.4}$$

on atoms H and O. *Which is the right model on H_2O, the* (1.3) *or* (1.4) *dynamic equations*? The answer is not easy because the equation model (1.3) can only characterizes those of coherent behavior of atoms H and O in H_2O, but equations (1.4) have no solutions, i.e., non-solvable in mathematics ([17]).

The main purpose of this report is to clarify that the reality of a thing T should be a contradictory system in one's eyes, or multiverse with non-solvable systems of equations in geometry, conclude that they essentially describe its nature, which results in mathematical combinatorics, i.e., mathematics over graphs in space, and show its powerful role to mathematics with applications to elementary particles, gravitational field and other sciences, such as those of extended Banach or Hilbert $\overrightarrow{G}$-flow spaces, geometry on non-solvable systems of solvable differential equations, $\cdots$ with applications to the understanding of particles, population biology and other sciences.

For terminologies and notations not mentioned here, we follow references [1] for mechanics, [4] for biological mathematics, [8] for combinatorial geometry, [23]-[24] for elementary particles, and [25] for Smarandache systems and multispaces, and all phenomenons discussed in this paper are assumed to be true in the nature.

§2. Contradiction, a By-product of Non-complete Recognizing

Notice that classical mathematical systems are homogenous without contradictions but contradictions exist everywhere in our world. Thus, let $\mathscr{R}$, $\mathscr{M}\mathscr{R}$ be respectively the sets of reality and the reality known by classical mathematics on things. Then, it is concluded that

$$\mathscr{M}\mathscr{R} \subset \mathscr{R} \quad \text{and} \quad \mathscr{M}\mathscr{R} \neq \mathscr{R} \tag{2.1}$$

in philosophy and we need an envelope theory on mathematics for reality of things, i.e., a mathematical theory including contradictions.

2.1 Thinking Models

Let us discuss 3 thinking models following.

T1. The Blind Men with an Elephant. This is a famous story in Buddhism which implies the entire consisting of its parts but we always hold on parts. In this story, there are six blind men were asked to determine what an elephant looked like by feeling different parts of an elephant's body. The man touched the elephant's leg, tail, trunk, ear, belly or tusk respectively claims it's like a pillar, a rope, a tree branch, a hand fan, a wall or a solid pipe, such as those shown in Fig.6.

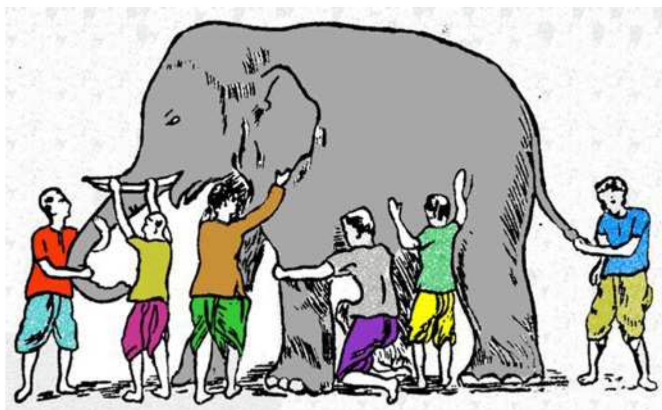


Fig.6

Each of these blind men insisted on his own's right, not accepted others, and then entered into an endless argument. *All of you are right*! A wise man explains to them: *why are you telling it differently is because each one of you touched the different part of the elephant. So, actually the elephant has all those features what you all said*. Hence, the wise man told these blind man that an elephant seemingly looked

$$\begin{aligned}\text{An elephant} &= \{4 \text{ pillars}\}\bigcup\{1 \text{ rope}\}\bigcup\{1 \text{ tree branch}\} \\ &\bigcup \{2 \text{ hand fans}\}\bigcup\{1 \text{ wall}\}\bigcup\{1 \text{ solid pipe}\} \qquad (2.2)\end{aligned}$$

What is the implication of this story for human beings? It lies in the situation that human beings understand things in the world is analogous to these blind men. Usually, a thing T is understand by its known characters at one by one time and known gradually. For example, let $\mu_1, \mu_2, \cdots, \mu_n$ be known and $\nu_i, i \geq 1$ unknown characters on a thing T at time t. Then, T is understood by

$$T=\left(\bigcup_{i=1}^{n}\{\mu_i\}\right)\bigcup\left(\bigcup_{k\geq 1}\{\nu_k\}\right) \tag{2.3}$$

in logic and with an approximation $T^{\circ}=\bigcup_{i=1}^{n}\{\mu_i\}$ at time t. The equation (2.3) is called the *Smarandache multispace* ([8], [25]), a combination of discrete characters for understanding a thing T.

T2. Everett's Multiverse on Superposition. The multiverse interpretation by H.Everett [3] on wave function of equation (1.2) in 1957 answered the superposition of particles in machinery. By an assumption that the wave function of an observer would be interacted with a superposed object, he concluded different worlds in different quantum system obeying equation (1.2) and the superposition of a particle be liked those separate arms of a 2-branching universe ([16], [17]) such as those shown in Fig.7

Fig.7

which revolutionary changed an ambiguous interpretation in quantum mechanics before him, i.e., an observer will cause the wave function to collapse randomly into one of the alternatives with all others disappearing. Everett's multiverse interpretation on the superposition of particle is in fact alluded in thinking model $T1$, i.e., the story of blind men with an elephant because if one views each of these pillar, rope, tree branch, hand fan, wall and solid pipe by these blind men feeling on different parts of the elephant to be different spaces, then the looks of an elephant of the wise man told these blind men (2.2) is nothing else but an Everett's multiverse.

T3. Quarks Model. The divisibility of matter initiates human beings to search elementary constituting cells of matter, i.e., elementary particles such as those of quarks, leptons with interaction quanta including photons and other particles of mediated interactions, also with those of their antiparticles at present ([23], [24]), and unmatters between a matter and its antimatter which is partially consisted of matter but others antimatter ([26], [27]). For example, a baryon is predominantly formed by three quarks, and a meson is mainly composed of a quark and an antiquark in the models of Sakata, or Gell-Mann and Ne'eman, such as those shown in Fig.2, where there is also a particle composed of 5 quarks.

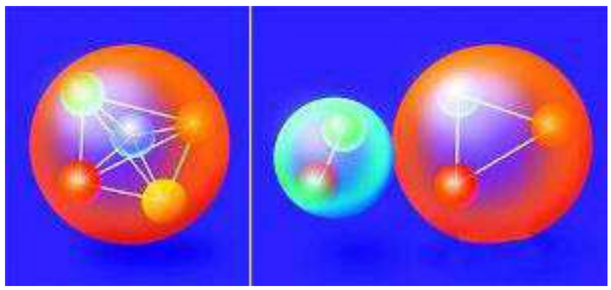


Fig.8

However, a free quark was never found in experiments. We can not even conclude the Schrödinger equations (1.3) is the right equation on quarks. But *why is it believed without a shadow of doubt that the dynamical equation of elementary particles such as those of quarks, leptons with interaction quanta is* (1.3) *in physics*? The reason is because that all observations come from a macro viewpoint, the human beings, not the quarks, and which can only lead to coherent behaviors, not the individuals. In mathematics, it is just an equation on those of particles viewed abstractly to be a geometrical point or an independent field from a macroscopic point, which results in physicists assuming the internal structures mechanically for understanding behaviors of particles, such as those shown in Fig.8. However, such an assumption is a little ambiguous in logic, i.e., we can not even distinguish who is the geometrical point or the field, the particle or its quark.

2.2 Contradiction Originated in Non-complete Recognizing

If we completely understand a thing T, i.e., $T = T^o$ in formula (2.3) at time t, there are no contradiction on T. However, this is nearly impossible for human beings, concluded in the first chapter of *TAO TEH KING* written by *Lao Zi*, a famous ideologist in China, i.e., "Name named is not the eternal; the without is the nature and naming the origin of things", which also implies the universality of contradiction and an generalization of equation (2.1).

Certainly, the looks (2.2) of the wise man on the elephant is a complete recognizing but these of the blind men is not. However, *which is the right way of recognizing*? The answer depends on the standing view of observer. The observation of these blind men on the elephant are a microscopic or in-observing but the wise man is macroscopic or out-observing. If one needs only for the macroscopic of an elephant, the wise man is right, but for the microscopic, these blind men are right on the different parts of the elephant. For understanding the reality of a thing T, we need the complete by individual recognizing, i.e., the whole by its parts. Such an observing is called a *parallel observing* ([17]) for avoiding the defect that each observer can only observe one behavior of a thing, such as those shown in Fig.9 on the water molecule H_2O with 3 observers.

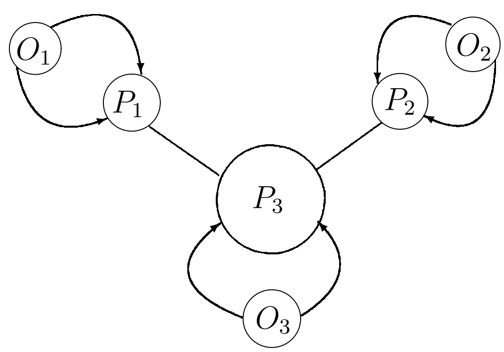


Fig.9

Thus, the looks of the wise man on an elephant is a collection of parallel observing by these 6 blind men and finally results in the recognizing (2.2), and also the Everett's multiverse interpretation on the superposition, the models of Sakata, or Gell-Mann and Ne'eman on particles. This also concludes that multiverse exists everywhere if we observing a thing T by in-observation, not only those levels of $I - IV$ classified by Max Tegmark in [28].

However, these equations (1.2) established on parallel observing datum of multiverse, for instance the equations (1.4) on 2 hydrogen atoms and 1 oxygen atom ([17]), and generally, differential equations (1.2) on population biology with more than 3 species are generally non-solvable. Then, *how to understand the reality of a* *thing T by mathematics holding with an equality $\mathscr{M}\mathscr{R} = \mathscr{R}$?* The best answer on this question is the combination of continuous mathematics with that of the discrete, i.e., turn these non-mathematics in the classical to mathematics by a combinatorial manner ([13]), i.e., *mathematical combinatorics*, which is the appropriated way for understanding the reality because all things are in contradiction.

§3. Mathematical Combinatorics

3.1 Labeled Graphs

A *graph* G is an ordered 2-tuple (V, E) with $V \neq \emptyset$ and $E \subset V \times V$, where V and E are finite sets and respectively called the vertex set, the edge set of G, denoted by $V(G)$ or $E(G)$, and a graph G is said to be embeddable into a topological space $\mathcal{T}$ if there is a $1-1$ continuous mapping $\phi : G \to \mathcal{T}$ with $\phi(p) \neq \phi(q)$ if $p, q \notin V(G)$. Particularly, if $\mathcal{T} = \mathbb{R}^3$ such a topological graph is called *spacial graph* such as those shown in Fig.10 for cube $C_4 \times C_4$,

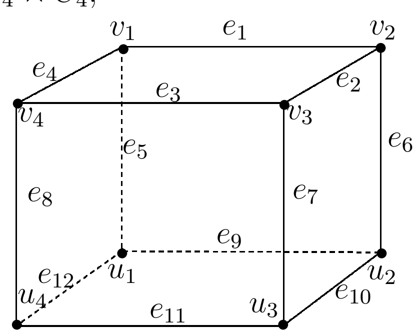


Fig.10

and a labeling on a graph G is a mapping $L : V(G) \bigcup E(G) \to \mathscr{L}$ with a labeling set $\mathscr{L}$ such as $\mathscr{L} = \{v_i, u_i, e_j, 1 \leq i \leq 4, 1 \leq j \leq 12\}$ in Fig.10.

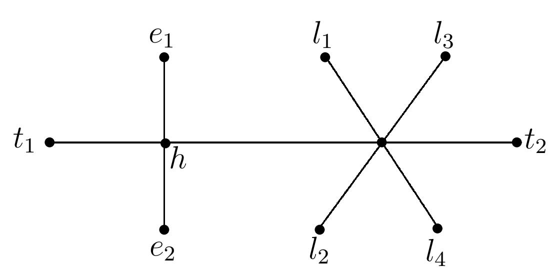


Fig.11

Notice that the underlying structure of an elephant by these blind men is a labeled tree shown in Fig.11. Then, *how can we rebuilt the looks of elephant from the labeled tree in Fig.*11? First, one blows up all edges, i.e., $e \to$ a cylinder for $\forall e \in E(G^L)$ and then, homeomorphically transforms these cylinders as parts of an elephant. After these transformations, a 3-dimensional elephant is built again in $\mathbb{R}^3$ such as those shown in Fig.12.

Fig.12

All of these discussions implies that labeled graph should be a mathematical element for understanding things ([20]), not only a labeling game because of

$$\text{Labeled Graphs in } \mathbb{R}^n \quad \Leftrightarrow \quad \text{Inherent Structure of Things.}$$

But *what are labels on labeled graphs, is it just different symbols*? And *are such labeled graphs a mechanism for the reality of things, or only a labeling game*? In fact, labeled graphs are researched mainly on symbols, not mathematical elements. If one puts off this assumption, i.e., labeling a graph by elements in mathematical systems, *what will happens*? *Are these resultants important for understanding things in the world*? The answer is certainly yes ([6], [7]) because this step will enable one to pullback more characters of things, particularly the metrics in physics, characterize things precisely and then holds on the reality of things.

3.2 G-Solutions on Equations

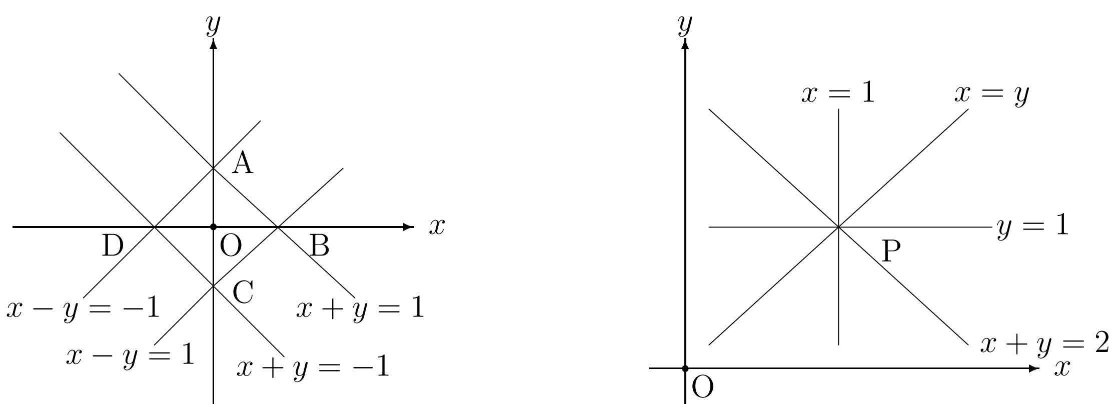
Let $\mathscr{F}: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ be a $\mathbb{C}^k$, $1 \leq k \leq \infty$ mapping with $\mathscr{F}(\overline{x}_0, \overline{y}_0) = \overline{0}$ for $\overline{x}_0 \in \mathbb{R}^n$, $\overline{y}_0 \in \mathbb{R}^m$ and a non-singular $m \times m$ matrix $(\partial \mathscr{F}^j / \partial y^i(\overline{x}_0, \overline{y}_0))$. Then the implicit mapping theorem concludes that there exist opened neighborhoods $V \subset \mathbb{R}^n$ of $\overline{x}_0$, $W \subset \mathbb{R}^m$ of $\overline{y}_0$ and a $\mathbb{C}^k$ mapping $\phi: V \to W$ such that $T(\overline{x}, \phi(\overline{x})) = \overline{0}$, i.e.,equation (1.2) is always solvable.

Let $\mathscr{F}_1, \mathscr{F}_2, \cdots, \mathscr{F}_m$ be m mappings holding with conditions of the implicit mapping theorem and let $S_{\mathscr{F}_i} \subset \mathbb{R}^n$ be a manifold such that $T_i: \ S_{\mathscr{F}_i} \to 0$ for integers $1 \leq i \leq m$. Consider the equations

$$\left\{\begin{array}{l} \mathscr{F}_1(x_1, x_2, \cdots, x_n) = 0 \\ \mathscr{F}_2(x_1, x_2, \cdots, x_n) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \mathscr{F}_m(x_1, x_2, \cdots, x_n) = 0 \end{array}\right. \tag{3.1}$$

in Euclidean space $\mathbb{R}^n, n \geq 1$. Geometrically, the system (3.1) is non-solvable or not dependent on $\bigcap\limits_{i=1}^{m} S_{\mathscr{F}_i} = \emptyset$ or $\neq \emptyset$.

Now, *is the non-solvable case meaningless for understanding the reality of things*? Certainly not because the non-solvable case of (3.1) only concludes the intersection $\bigcap\limits_{i=1}^{m} S_{\mathscr{F}_i} = \emptyset$, the behavior of the solvable and non-solvable cases should be both characterized by the union $\bigcup\limits_{i=1}^{m} S_{\mathscr{F}_i}$ such as those shown in (2.2) for the elephant.



(LES_4^N) (LES_4^S)

Fig.13

For example, if things T_1, T_2, T_3, T_4 and T_1', T_2', T_3', T_4' are respectively characterized by systems of equations following

$$
(LES_4^N)\quad \begin{cases} x+y &= \ \ 1 \\ x+y &= -1 \\ x-y &= -1 \\ x-y &= \ \ 1 \end{cases} \qquad\qquad (LES_4^S)\quad \begin{cases} x=y \\ x+y=2 \\ x=1 \\ y=1 \end{cases}
$$

it is clear that (LES_4^N) is non-solvable because $x+y=-1$ is contradictious to $x+y=1$, and so that for equations $x-y=-1$ and $x-y=1$, i.e., there are no solutions x_0, y_0 hold with this system. But (LES_4^S) is solvable with $x=1$ and $y=1$. *Can we conclude that things T_1', T_2', T_3', T_4' are $x=1, y=1$ and T_1, T_2, T_3, T_4 are nothing*? Certainly not because $(x,y)=(1,1)$ is the intersection of straight line behavior of things T_1', T_2', T_3', T_4' and there are no intersection of T_1, T_2, T_3, T_4 in plane $\mathbb{R}^2$. However, they are indeed exist in $\mathbb{R}^2$ such as those shown in Fig.13.

Let $L_{a,b,c} = \{(x,y)|ax+by=c, ab \neq 0\}$ be points in $\mathbb{R}^2$. We are easily know the straight line behaviors of T_1, T_2, T_3, T_4 and T_1', T_2', T_3', T_4' are nothings else but the unions $L_{1,-1,0} \bigcup L_{1,1,2} \bigcup L_{1,0,1} \bigcup L_{0,1,1}$ and $L_{1,1,1} \bigcup L_{1,1,-1} \bigcup L_{1,-1,-1} \bigcup L_{1,-1,1}$, respectively.

Definition 3.1 *A G-solution of system* (3.1) *is a labeling graph G^L defined by*

$$V(G) = \{S_{F_i},\ 1 \leq i \leq n\};$$

$$E(G) = \left\{(S_{F_i}, S_{F_j})\ \textit{if}\ S_{F_i} \bigcap S_{F_j} \neq \emptyset \text{ for integers } 1 \leq i,j \leq n\right\}\ \textit{with a labeling}$$

$$L:\ S_{F_i} \to S_{F_i}, \qquad (S_{F_i}, S_{F_j}) \to S_{F_i} \bigcap S_{F_j}.$$

For Example, the G-solutions of (LES_4^N) and (LES_4^S) are respectively labeling graphs C_4^L and K_4^L shown in Fig.14 following.

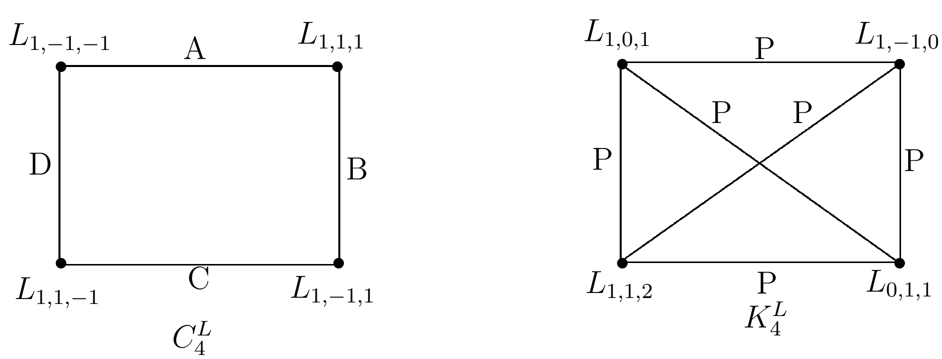


Fig.14

Theorem 3.2 *A system* (3.1) *of equations is G-solvable if $\mathscr{F}_i \in \mathbb{C}^1$ and $\mathscr{F}_i|_{(x_1^0, x_2^0, \cdots, x_n^0)}$ $= 0$ but $\left.\frac{\partial \mathscr{F}_i}{\partial x_i}\right|_{(x_1^0, x_2^0, \cdots, x_n^0)} \neq 0$ for any integer i, $1 \leq i \leq n$.*

More results on combinatorics of non-solvable algebraic, ordinary or partial differential equations can be found in references [9]-[14]. For example, let $(LDES_m^1)$ be a system of linear homogeneous differential equations

$$\begin{cases} \ddot{x} - 3\dot{x} + 2x = 0 & (1) \\ \ddot{x} - 5\dot{x} + 6x = 0 & (2) \\ \ddot{x} - 7\dot{x} + 12x = 0 & (3) \\ \ddot{x} - 9\dot{x} + 20x = 0 & (4) \\ \ddot{x} - 11\dot{x} + 30x = 0 & (5) \\ \ddot{x} - 7\dot{x} + 6x = 0 & (6) \end{cases}$$

where $\ddot{x} = \frac{d^2x}{dt^2}$ and $\dot{x} = \frac{dx}{dt}$. Clearly, this system is non-solvable with solution bases $\{e^t, e^{2t}\}$, $\{e^{2t}, e^{3t}\}$, $\{e^{3t}, e^{4t}\}$, $\{e^{4t}, e^{5t}\}$, $\{e^{5t}, e^{6t}\}$, $\{e^{6t}, e^t\}$ respectively on equations $(1) - (6)$ and its G-solution is shown in Fig.15,

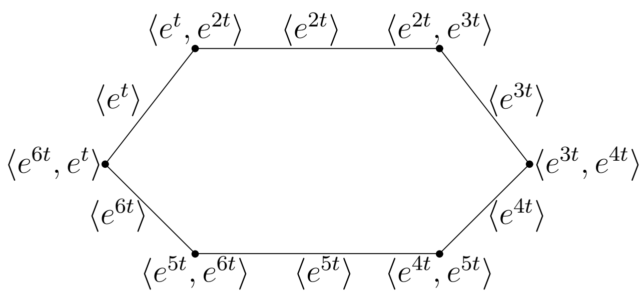


Fig.15

where $\langle\Delta\rangle$ denotes the linear space generalized by elements in Δ.

3.3 Mathematics Over Graph

Let $(\mathscr{A}; \circ_1, \circ_2, \cdots, \circ_k)$ be an algebraic system, i.e., $a \circ_i b \in \mathscr{A}$ for $\forall a, b \in \mathscr{A}$, $1 \leq i \leq k$ and let $\overrightarrow{G}$ be an oriented graph embedded in space $\mathcal{T}$. Denoted by $\overrightarrow{G}^L_{\mathscr{A}}$ all of those labeled graphs $\overrightarrow{G}^L$ with labeling $L:\ E\left(\overrightarrow{G}\right) \rightarrow \mathscr{A}$ constraint with ruler:

R1 : *For $\forall \overrightarrow{G}^{L_1}, \overrightarrow{G}^{L_2} \in \overrightarrow{G}^L_{\mathscr{A}}$, define $\overrightarrow{G}^{L_1} \circ_i \overrightarrow{G}^{L_2} = \overrightarrow{G}^{L_1 \circ_i L_2}$, where $L_1 \circ_i L_2 :$ $e \rightarrow L_1(e) \circ_i L_2(e)$ for $\forall e \in E\left(\overrightarrow{G}\right)$ and integers $1 \leq i \leq k$.*

For example, such a ruler on graph $\overrightarrow{C}_4$ is shown in Fig.16, where $\mathbf{a}_3=\mathbf{a}_1\circ_i\mathbf{a}_2$, $\mathbf{b}_3=\mathbf{b}_1\circ_i\mathbf{b}_2$, $\mathbf{c}_3=\mathbf{c}_1\circ_i\mathbf{c}_2$, $\mathbf{d}_3=\mathbf{d}_1\circ_i\mathbf{d}_2$.

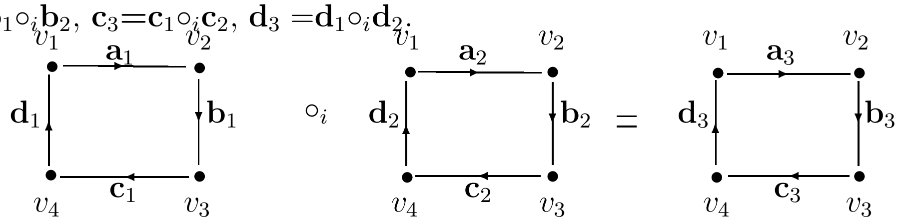


Fig.16

Then, $\overrightarrow{G}^{L_1}\circ_i\overrightarrow{G}^{L_2}=\overrightarrow{G}^{L_1\circ_i L_2}\in\overrightarrow{G}^L_{\mathscr{A}}$ by the ruler **R1**, and generally,

$$\overrightarrow{G}^{L_1}\circ_{i_1}\overrightarrow{G}^{L_2}\circ_{i_2}\cdots\circ_{i_s}\overrightarrow{G}^{L_{s+1}}\in\overrightarrow{G}^L_{\mathscr{A}}$$

for integers $1\leq i_1,i_2,\cdots,i_s\leq k$, i.e., $\overrightarrow{G}^L_{\mathscr{A}}$ is also an algebraic system, and it is commutative on an operation $\circ_i$ if $(\mathscr{A};\circ_1,\circ_2,\cdots,\circ_k)$ is commutative on an operation $\circ_i$ for an integer i, $1\leq i\leq k$. Particularly, if $k=1$, $\overrightarrow{G}^L_{\mathscr{A}}$ is a group if $(\mathscr{A};\circ_1)$ is a group. Thus, we extend $(\mathscr{A};\circ_1,\circ_2,\cdots,\circ_k)$ and obtain an algebraic system over graph $\overrightarrow{G}$ underlying a geometrical structure in space $\mathcal{T}$.

Notice that such an extension $\overrightarrow{G}^L_{\mathscr{A}}$ is only a pure extension of algebra over $\overrightarrow{G}$ without combining the nature of things, i.e., the conservation of matter which states that the amount of the conserved quantity at a point or within a volume can only change by the amount of the quantity which flows in or out of that volume. Thus, understanding the reality of things motives the extension of mathematical systems $(\mathscr{A};\circ_1,\circ_2,\cdots,\circ_k)$ over graph $\overrightarrow{G}$ constrained also on the laws of conservation

R2 : $\sum\limits_l \mathbf{F}(v)_l^- = \sum\limits_s \mathbf{F}(v)_s^+$, *where* $\mathbf{F}(v)_l^-$, $l\geq 1$ *and* $\mathbf{F}(v)_s^+$, $s\geq 1$ *denote respectively the output and input amounts at vertex* $v\in E(\overrightarrow{G})$.

This notion brings about a new mathematical element finally, i.e., action flows, which combines well the continuous mathematics with that of the discrete.

Definition 3.3([19]) *An action flow* $\left(\overrightarrow{G};L,A\right)$ *is an oriented embedded graph* $\overrightarrow{G}$ *in a topological space* $\mathscr{S}$ *associated with a mapping* $L:(v,u)\to L(v,u)$, *2 end-operators* $A^+_{vu}:L(v,u)\to L^{A^+_{vu}}(v,u)$ *and* $A^+_{uv}:L(u,v)\to L^{A^+_{uv}}(u,v)$ *on a Banach space* $\mathscr{B}$

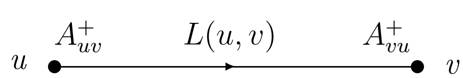


Fig.17

with $L(v,u) = -L(u,v)$ *and* $A^+_{vu}(-L(v,u)) = -L^{A^+_{vu}}(v,u)$ *for* $\forall(v,u) \in E\left(\overrightarrow{G}\right)$ *holding with conservation laws*

$$\sum_{u\in N_G(v)} L^{A^+_{vu}}(v,u) = \mathbf{c}_v \quad for \quad \forall v \in V\left(\overrightarrow{G}\right)$$

such as those shown for vertex v *in Fig.*18 *following*

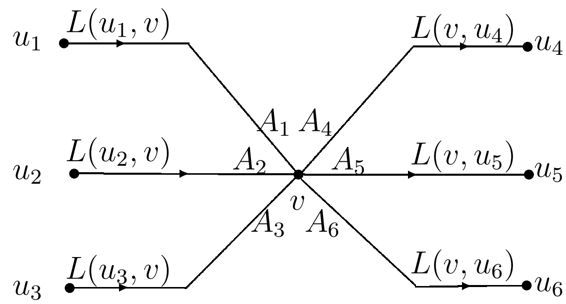


Fig.18

with a conservation law

$$-L^{A_1}(v,u_1) - L^{A_2}(v,u_2) - L^{A_4}(v,u_3) + L^{A_4}(v,u_4) + L^{A_5}(v,u_5) + L^{A_6}(v,u_6) = \mathbf{c}_v,$$

where $\mathbf{c}_v$ *is the surplus flow on vertex* v*, and usually, let* $\mathbf{c}_v = \mathbf{0}$.

Indeed, action flow is an element both with the character of continuous and discrete mathematics. For example, the conservation laws on an action flow over dipole shown in Fig.19

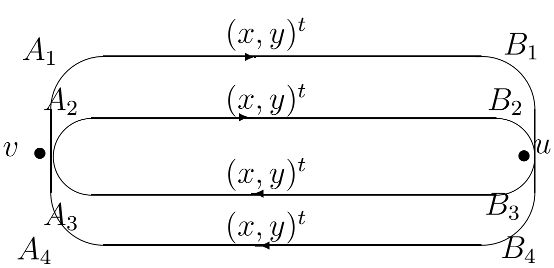


Fig.19

are partial differential equations

$$\left\{ \begin{array}{l} a_1\dfrac{\partial^2 x}{\partial t^2} + b_1\dfrac{\partial^2 y}{\partial t^2} - a_3\dfrac{\partial x}{\partial t} + (a_2 - a_4)x + (b_2 - b_3 - b_4)y = 0 \\ c_2\dfrac{\partial^2 x}{\partial t^2} + d_2\dfrac{\partial^2 y}{\partial t^2} - d_4\dfrac{\partial y}{\partial t} + (c_1 - c_3 - c_4)x + (d_1 - d_3)y = 0 \end{array} \right. ,$$

where, $A_1 = (a_1\partial^2/\partial t^2, b_1\partial^2/\partial t^2)$, $A_2 = (a_2, b_2)$, $A_3 = (a_3\partial/\partial t, b_3)$, $A_4 = (a_4, b_4)$, $B_1 = (c_1, d_1)$, $B_2 = (c_2\partial^2/\partial t^2, d_2\partial^2/\partial t^2)$, $B_3 = (c_3, d_3)$, $B_4 = (c_4, d_4\partial/\partial t)$.

Certainly, not all mathematical systems can be extended over a graph $\overrightarrow{G}$ constraint with the laws of conservation at $v \in V(\overrightarrow{G})$ unless $\overrightarrow{G}$ with special structure but such an extension of linear space $\mathscr{A}$ can be always done.

Theorem 3.4([20]) *Let $(\mathscr{A}; +, \cdot)$ be a linear space, $\overrightarrow{G}$ an embedded graph in space $\mathcal{T}$ and $A^+_{vu} = A^+_{uv} = \mathbf{1}_{\mathscr{A}}$ for $\forall (v,u) \in E(\overrightarrow{G})$. Then, $\left(\overrightarrow{G}^L_{\mathscr{A}}; +, \cdot\right)$ is also a linear space under rulers* **R1** *and* **R2** *with dimension* $\dim \mathscr{A}^{\beta(\overrightarrow{G})}$ *if* $\dim \mathscr{V} < \infty$*, or infinite.*

An action flow $\left(\overrightarrow{G}; L, \mathbf{1}_{\mathscr{A}}\right)$, i.e., $A^+_{vu} = A^+_{uv} = \mathbf{1}_{\mathscr{A}}$ for $\forall (v,u) \in E(\overrightarrow{G})$ is usually called $\overrightarrow{G}$*-flows*, denoted by $\overrightarrow{G}^L$ and the linear space $\left(\overrightarrow{G}^L_{\mathscr{A}}; +, \cdot\right)$ extended over $\overrightarrow{G}$ by $\overrightarrow{G}^{\mathscr{A}}$ for simplicity.

§4. Banach $\overrightarrow{G}$-Flow Spaces with Multiverses

4.1 Banach $\overrightarrow{G}$-Flow Space

A Banach or Hilbert space is respectively a linear space $\mathscr{A}$ over a field $\mathbb{R}$ or $\mathbb{C}$ equipped with a complete norm $\|\cdot\|$ or inner product $\langle \cdot, \cdot \rangle$, i.e., for every Cauchy sequence $\{x_n\}$ in $\mathscr{A}$, there exists an element x in $\mathscr{A}$ such that

$$\lim_{n\to\infty} \|x_n - x\|_{\mathscr{A}} = 0 \quad \text{or} \quad \lim_{n\to\infty} \langle x_n - x, x_n - x \rangle_{\mathscr{A}} = 0,$$

which can be extended over graph $\overrightarrow{G}$ by introducing the norm of a $\overrightarrow{G}$-flow $\overrightarrow{G}^L$ following

$$\left\|\overrightarrow{G}^L\right\| = \sum_{(v,u)\in E(\overrightarrow{G})} \|L(v,u)\|,$$

where $\|L(v,u)\|$ is the norm of $L(v,u)$ in $\mathscr{A}$.

Theorem 4.1([15]) *For any graph $\overrightarrow{G}$, $\overrightarrow{G}^{\mathscr{A}}$ is a Banach space, and furthermore, if $\mathscr{A}$ is a Hilbert space, $\overrightarrow{G}^{\mathscr{A}}$ is a Hilbert space too.*

We can also consider operators action on the Banach or Hilbert $\overrightarrow{G}$-flow space $\overrightarrow{G}^{\mathscr{A}}$. Particularly, an operator $\mathbf{T} : \overrightarrow{G}^{\mathscr{A}} \to \overrightarrow{G}^{\mathscr{A}}$ is *linear* if

$$\mathbf{T}\left(\lambda\overrightarrow{G}^{L_1} + \mu\overrightarrow{G}^{L_2}\right) = \lambda\mathbf{T}\left(\overrightarrow{G}^{L_1}\right) + \mu\mathbf{T}\left(\overrightarrow{G}^{L_2}\right)$$

for $\forall \overrightarrow{G}^{L_1}, \overrightarrow{G}^{L_2} \in \overrightarrow{G}^{\mathscr{A}}$, $\lambda, \mu \in \mathscr{F}$, which enables one to generalize the representation theorem of Fréchet and Riesz on linear continuous functionals of Hilbert space to Hilbert $\overrightarrow{G}$-flow space $\overrightarrow{G}^{\mathscr{A}}$ following.

Theorem 4.2([15]) *Let* $\mathbf{T} : \overrightarrow{G}^{\mathscr{A}} \to \mathbb{C}$ *be a linear continuous functional. Then there is a unique* $\overrightarrow{G}^{\widehat{L}} \in \overrightarrow{G}^{\mathscr{A}}$ *such that* $\mathbf{T}\left(\overrightarrow{G}^{L}\right) = \left\langle \overrightarrow{G}^{L}, \overrightarrow{G}^{\widehat{L}} \right\rangle$ *for* $\forall \overrightarrow{G}^{L} \in \overrightarrow{G}^{\mathscr{A}}$.

Notice that linear continuous functionals exist everywhere in mathematics, particularly, the differential and integral operators. For example, let $\mathscr{A}$ be a Hilbert space consisting of measurable functions $f(x_1, x_2, \cdots, x_n)$ on a set

$$\Delta = \{(x_1, x_2, \cdots, x_n) \in \mathbb{R}^n | a_i \le x_i \le b_i, 1 \le i \le n\},$$

which is a functional space $L^2[\Delta]$ with inner product

$$\langle f(\mathbf{x}), g(\mathbf{x}) \rangle = \int_{\Delta} \overline{f(\mathbf{x})} g(\mathbf{x}) d\mathbf{x} \ \text{ for } \ f(\mathbf{x}), g(\mathbf{x}) \in L^2[\Delta],$$

where $\mathbf{x} = (x_1, x_2, \cdots, x_n)$. By Theorem 4.1, $\mathscr{A}$ can be extend to Hilbert $\overrightarrow{G}$-flow space $\overrightarrow{G}^{\mathscr{A}}$, and the *differential* or *integral operators*

$$D = \sum_{i=1}^{n} a_i \frac{\partial}{\partial x_i} \quad \text{and} \quad \int_{\Delta}$$

on $\mathscr{A}$ are extended to $\overrightarrow{G}^{\mathscr{A}}$ respectively by $D\overrightarrow{G}^{L} = \overrightarrow{G}^{DL(v,u)}$ and

$$\int_{\Delta} \overrightarrow{G}^{L} \ = \ \int_{\Delta} K(\mathbf{x}, \mathbf{y}) \overrightarrow{G}^{L[\mathbf{y}]} d\mathbf{y} = \overrightarrow{G}^{\int_{\Delta} K(\mathbf{x},\mathbf{y}) L(v,u)[\mathbf{y}] d\mathbf{y}}$$

for $\forall (v, u) \in E(\overrightarrow{G})$, where a_i, $\dfrac{\partial a_i}{\partial x_j} \in \mathbb{C}^0(\Delta)$ for integers $1 \le i, j \le n$ and $K(\mathbf{x}, \mathbf{y}) : \Delta \times \Delta \to \mathbb{C} \in L^2(\Delta \times \Delta, \mathbb{C})$ with

$$\int_{\Delta \times \Delta} K(\mathbf{x}, \mathbf{y}) d\mathbf{x} d\mathbf{y} < \infty.$$

Theorem 4.3([15]) *The differential or integral operator* $D : \overrightarrow{G}^{\mathscr{A}} \to \overrightarrow{G}^{\mathscr{A}}$, $\int_{\Delta} : \overrightarrow{G}^{\mathscr{A}} \to \overrightarrow{G}^{\mathscr{A}}$ *both are linear operators on* $\overrightarrow{G}^{\mathscr{A}}$.

For example, let let $f(t) = t$, $g(t) = e^t$, $K(t, \tau) = t^2 + \tau^2$ for $\Delta = [0, 1]$ and let $\overrightarrow{G}^{L}$ be the $\overrightarrow{G}$-flow shown on the left in Fig.20,

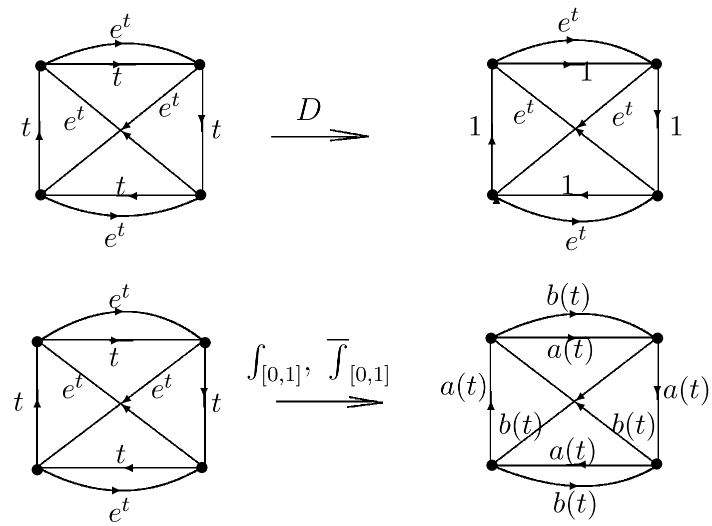


Fig.20

where $a(t) = \dfrac{t^2}{2} + \dfrac{1}{4}$ and $b(t) = (e-1)t^2 + e - 2$.

4.2 Multiverses on Equations

Notice that solving Schrödinger equation (1.3) with initial data only get one state of a particle P but the particle is in superposition, which brought the H.Everett multiverse on superposition and the quark model of Sakata, or Gell-Mann and Ne'eman on particles machinery. However, Theorems $4.1 - 4.3$ enables one to get multiverses constraint with linear equations (3.1) in $\overrightarrow{G}^{\mathscr{A}}$.

For example, we can consider the Cauchy problem

$$\frac{\partial X}{\partial t} = c^2 \sum_{i=1}^{n} \frac{\partial^2 X}{\partial x_i^2}$$

with initial values $X|_{t=t_0}$ in $\overrightarrow{G}^{\mathbb{R}^n\times\mathbb{R}}$, i.e., Hilbert space $\mathbb{R}^n \times \mathbb{R}$ over graph $\overrightarrow{G}$, and get multiverse solutions of heat equation following.

Theorem 4.4([15]) *For $\forall \overrightarrow{G}^{L'} \in \overrightarrow{G}^{\mathbb{R}^n\times\mathbb{R}}$ and a non-zero constant c in $\mathbb{R}$, the Cauchy problems on differential equations*

$$\frac{\partial X}{\partial t} = c^2 \sum_{i=1}^{n} \frac{\partial^2 X}{\partial x_i^2}$$

with initial value $X|_{t=t_0} = \overrightarrow{G}^{L'} \in \overrightarrow{G}^{\mathbb{R}^n\times\mathbb{R}}$ is solvable in $\overrightarrow{G}^{\mathbb{R}^n\times\mathbb{R}}$ if $L'(v,u)$ is continuous and bounded in $\mathbb{R}^n$ for $\forall (v,u) \in E\left(\overrightarrow{G}\right)$.

And then, the H.Everett's multiverse on the Schrödinger equation (1.3) is nothing else but a 2-branch tree

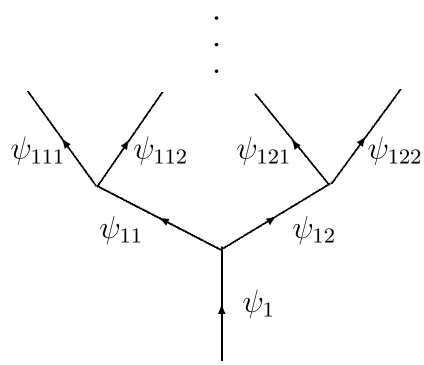


Fig.21

with equalities $\psi_1 = \psi_{11} + \psi_{12}$, $\psi_{11} = \psi_{111} + \psi_{112}$, $\psi_{12} = \psi_{121} + \psi_{122}$, $\cdots$ ([16], [17]).

If the equations (3.1) is not linear, we can not immediately apply Theorems 4.1 − 4.3 to get multiverse over any graphs $\overrightarrow{G}$. However, if the graph $\overrightarrow{G}$ is prescribed with special structures, for instance the circuit decomposable, we can also solve the Cauchy problem on an equation in Hilbert $\overrightarrow{G}$-flow space $\overrightarrow{G}^{\mathscr{A}}$ if it is solvable in $\mathscr{A}$ and obtain a general conclusion following, which enable us to interpret also the superposition of particles ([17]), biological diversity and establish multiverse model of spacetime in Einstein's gravitation.

Theorem 4.5([15]) *If the graph $\overrightarrow{G}$ is strong-connected with circuit decomposition*

$$\overrightarrow{G} = \bigcup_{i=1}^{l} \overrightarrow{C}_i$$

such that $L(v,u) = L_i(\boldsymbol{x})$ for $\forall (v,u) \in E\left(\overrightarrow{C}_i\right)$, $1 \leq i \leq l$ and the Cauchy problem

$$\left\{\begin{array}{l} \mathscr{F}_i\left(\boldsymbol{x}, u, u_{x_1}, \cdots, u_{x_n}, u_{x_1x_2}, \cdots\right) = 0 \\ u|_{\boldsymbol{x}_0} = L_i(\boldsymbol{x}) \end{array}\right.$$

is solvable in a Hilbert space $\mathscr{A}$ on domain $\Delta \subset \mathbb{R}^n$ for integers $1 \leq i \leq l$, then the Cauchy problem

$$\left\{\begin{array}{l} \mathscr{F}_i\left(\boldsymbol{x}, X, X_{x_1}, \cdots, X_{x_n}, X_{x_1x_2}, \cdots\right) = 0 \\ X|_{\boldsymbol{x}_0} = \overrightarrow{G}^{L} \end{array}\right.$$

such that $L\left(v,u\right) = L_i(\boldsymbol{x})$ for $\forall (v,u) \in X\left(\overrightarrow{C}_i\right)$ is solvable for $X \in \overrightarrow{G}^{\mathscr{A}}$.

Theorem 4.5 enables one to explore the multiverse, particularly, the solutions of Einstein's gravitational equations

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} + \lambda g^{\mu\nu} = -8\pi G T^{\mu\nu},$$

where $R^{\mu\nu} = R_{\alpha}^{\mu\alpha\nu} = g_{\alpha\beta}R^{\alpha\mu\beta\nu}$, $R = g_{\mu\nu}R^{\mu\nu}$ are the respective Ricci tensor, Ricci scalar curvature, $G = 6.673 \times 10^{-8} cm^3/gs^2$, $\kappa = 8\pi G/c^4 = 2.08 \times 10^{-48} cm^{-1} \cdot g^{-1} \cdot s^2$ ([24]). In fact, Einstein's general relativity is established on $\mathbb{R}^4$. However, *if the dimension of the universe* > 4, *how can we characterize the structure of spacetime for the universe*? If the dimension of the universe > 4, all observations are nothing else but a projection of the true faces on our six organs because the dimension of human beings is 3. We can characterize the spacetime by a complete graph K_m^L labeled by $\mathbb{R}^4$ (See [7]-[8] for details). For example, if $m = 4$ there are 4 Einstein's gravitational equations respectively on $v \in V\left(K_4^L\right)$. We solve it one by one by the spherically symmetric solution in $\mathbb{R}^4$ and construct a K_4^L-solution labeled by $S_{\mathscr{F}_1}, S_{\mathscr{F}_2}, S_{\mathscr{F}_3}, S_{\mathscr{F}_4}$ in Fig.22,

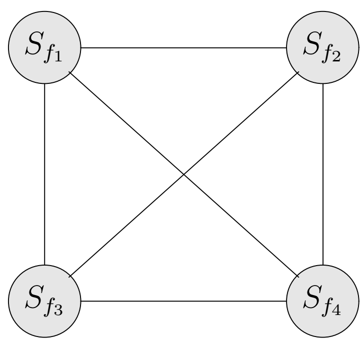


Fig.22

where, each $S_{\mathscr{F}_i}$ the geometrical space of the Schwarzschild spacetime

$$ds^2 = f(t)\left(1 - \frac{r_s}{r}\right)dt^2 - \frac{1}{1 - \frac{r_s}{r}}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

for integers $1 \leq i \leq 4$.

Notice that $m = 4$ is only an assumption. We do not know the exact value of m at present. Similarly, by Theorem 4.5 we also get a conclusion on multiverse of the Einstein's gravitational equations and we do not even know which is the real spacetime of our universe.

Theorem 4.6([15], [19]) *There are infinite many $\overrightarrow{G}$-flow solutions on Einstein's gravitational equations*

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = -8\pi GT^{\mu\nu}$$

in $\overrightarrow{G}^{\mathbb{C}}$, particularly on those graphs with circuit-decomposition

$$\overrightarrow{G} = \bigcup_{i=1}^{m} \overrightarrow{C}_i$$

labeled with Schwarzschild spacetime on their edges.

For example, let $\overrightarrow{G} = \overrightarrow{C}_4$. We are easily find $\overrightarrow{C}_4$-flow solution of Einstein's gravitational equations such as those shown in Fig.23.

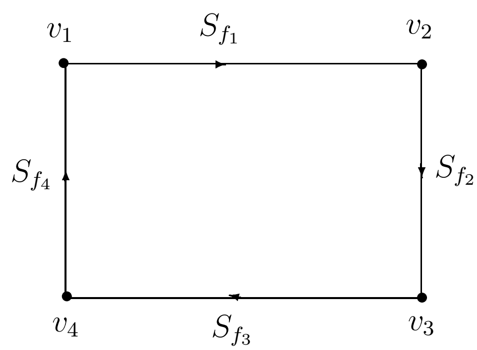


Fig.23

Then, the spacetime of the universe is nothing else but a curved ring such as those shown in Fig.24.

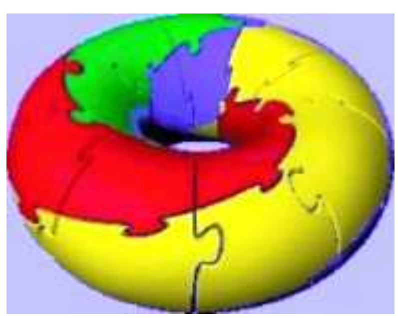


Fig.24

Generally, if $\overrightarrow{G}$ is the union of m orientated circuits $\overrightarrow{C}_i$, $1 \leq i \leq m$, Theorem 4.6 implies the spacetime of Einstein's gravitational equations is a multiverse consisting of m curved rings over graph $\overrightarrow{G}$.

Notice that a graph $\overrightarrow{G}$ is circuit decomposable if and only if it is an Eulerian graph. Thus, Theorems $4.1-4.5$ can be also applied to biology with global stability of food webs of n species following.

Theorem 4.7([21]) *A food web $\overrightarrow{G}^L$ with initial value $\overrightarrow{G}^{L_0}$ is globally stable or asymptotically stable if and only if there is an Eulerian multi-decomposition*

$$\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)^{\widehat{L}}=\bigoplus_{i=1}^{s}\overrightarrow{H}_i^L$$

*with solvable stable or asymptotically stable conservative equations on Eulerian subgraphs $\overrightarrow{H}_i^L$ for integers $1\leq i\leq s$, where $\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)^{\widehat{L}}$ is the bi-digraph of $\overrightarrow{G}$ defined by $\overrightarrow{G}\bigcup\overleftarrow{G}$ with a labeling $\widehat{L}:V(\overrightarrow{G}\bigcup\overleftarrow{G})\rightarrow L\left(V(\overrightarrow{G})\right)$, $\widehat{L}:E\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)\rightarrow L\left(E\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)\right)$ by $\widehat{L}:\ (u,v)\rightarrow\{0,(x,y),yf'\}$, $(v,u)\rightarrow\{xf,(x,y),0\}$ if $L:(u,v)\rightarrow\{xf,(x,y),yf'\}$ for $\forall(u,v)\in E(\overrightarrow{G})$, such as those shown in Fig.*25*,*

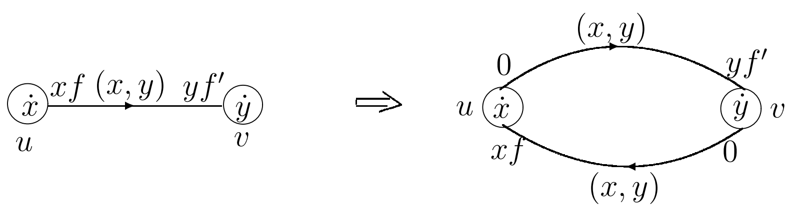


Fig.25

and a multi-decomposition $\bigoplus\limits_{i=1}^{s}\overrightarrow{H}_i^L$ of $\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)^{\widehat{L}}$ is defined by

$$\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)^{\widehat{L}}=\bigcup_{i=1}^{s}\overrightarrow{H}_i$$

with $\overrightarrow{H}_i\neq\overrightarrow{H}_j$, $\overrightarrow{H}_i\bigcap\overrightarrow{H}_j=\emptyset$ or $\neq\emptyset$ for integers $1\leq\neq j\leq s$.

Theorem 4.8([21]) *A food web $\overrightarrow{G}^L$ with initial value $\overrightarrow{G}^{L_0}$ is globally asymptotically stable if there is an Eulerian multi-decomposition*

$$\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)^{\widehat{L}}=\bigoplus_{k=1}^{s}\overrightarrow{H}_k^L$$

with solvable conservative equations such that $Re\lambda_i<0$ for characteristic roots λ_i of A_v in the linearization $A_vX_v=0_{h_v\times h_v}$ of conservative equations at an equilibrium point $\overrightarrow{H}_k^{L_0}$ in $\overrightarrow{H}_k^L$ for integers $1\leq i\leq h_v$ and $v\in V(\overrightarrow{H}_k^L)$, where $V(\overrightarrow{H}_k^L)=$

$\{v_1, v_2, \cdots, v_{h_v}\}$,

$$A_v = \begin{pmatrix} a_{11}^v & a_{12}^v & \cdots & a_{1h_v}^v \\ a_{21}^v & a_{22}^v & \cdots & a_{2h_v}^v \\ \cdots & \cdots & \cdots & \cdots \\ a_{h1}^v & a_{h2}^v & \cdots & a_{hh_v}^v \end{pmatrix}$$

a constant matrix and $X_k = (x_{v_1}, x_{v_2}, \cdots, x_{v_{h_v}})^T$ *for integers* $1 \leq k \leq l$.

§5. Conclusion

Answer the question which is better to the reality of things, the continuous or discrete mathematics is not easy because our world appears both with the continuous and discrete characters. However, contradictions exist everywhere, which are all artificial, not the nature of things. Thus, holding on the reality of things motivates us to turn contradictory systems to compatible systems, i.e., giving up the notion that contradiction is meaningless and establish an envelope theory on mathematics, which needs the combination of the continuous mathematics with that of discrete, i.e., mathematical combinatorics because a non-mathematics in classical is in fact a mathematics over a graph $\overrightarrow{G}$ ([13]), and action flow $\overrightarrow{G}^L$ is a candidate for this objective.

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