The Relativistic Mass Ratio in Ultrarelativistic Photon Rockets

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Abstract

In this paper we take a closer look at the initial mass relative to the relativistic mass of the payload for an ideal photon rocket travelling at its maximum velocity. Haug has recently suggested that for all known subatomic particles, a minimum of two Planck masses of fuel are needed to accelerate the fundamental particle to its suggested maximum velocity (see [1]).

Here we will show how this view is consistent with insight given by Tipler at a NASA Breakthrough Propulsion Physics Workshop Proceedings in 1999 (see [2]). Tipler suggested that the mass ratio of the initial rest mass of an ultra-relativistic rocket relative to the relativistic mass of the payload is likely "just" two. An ultrarelativistic rocket is one travelling at a velocity very close to the speed of light. We will here show that the Tipler factor is consistent with results derived from Haug's suggested maximum velocity for any known observed subatomic particle.

However, we will show that the Tipler factor of two is unlikely to hold for ultra-heavy subatomic particle payloads. With ultra-heavy particles, we think of subatomic particles with mass close to that of the Planck mass. Our analysis indicates that the initial mass relative to the relativistic mass of the payload for any type of subatomic particle rocket must be between one and two. Remarkably, the mass ratio is only one for a Planck mass particle. This at first sounds absurd until we understand that the Planck mass particle is probably the very collision point between two photons. Even if a photon's speed "always is" considered to be the speed of light, we can think of it as standing still at the instant it collides with another photon (backscattering). The mass ratio to accelerate a particle that only exists at velocity zero is naturally one. This is true since no fuel is needed to go from zero to zero velocity. Remarkably this indicates that the Planck mass particle and the Planck length likely are invariant. This can only happen if the Planck mass particle only lasts for an instant before it bursts into energy, which is what we could expect for the collision between two photons.

Key words: Relativistic rocket equation, mass ratio, photon propulsion, maximum velocity.

1 Introduction

Haug [1, 3, 4, 5] has suggested that the maximum velocity for anything with rest mass is given by

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}, \tag{1}$$

where $\bar{\lambda}$ is the reduced Compton wavelength of the particle we are trying to accelerate, and l_p is the Planck length ([6]). This maximum velocity puts an upper boundary condition on the kinetic energy, momentum, and relativistic mass, as well as on the relativistic Doppler shift in relation to subatomic particles. The formula above can be derived from assuming that the shortest possible reduced Compton wavelength is equal to the Planck length, or that the maximum length contraction leads to the Planck length. The same formula can also be derived from mathematical atomism, where one assumes that the ultimate building blocks of both matter and energy consist of indivisible particle and void (empty space) (see [3, 7]). The maximum velocity for known subatomic particles is below the speed of light but above what is currently achieved at the Large Hadron Collider (LHC) (see [1, 3, 4]).

The Ackeret [8] relativistic rocket equation[1] is given by

$$m_0 = m_1 \left(\frac{1 + \frac{\Delta v}{c}}{1 - \frac{\Delta v}{c}} \right)^{\frac{c}{2I_{sp}}}, \tag{2}$$

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[1]See also [9], [10] [11], [13], [14] and [15] .

where I_{SP} is the specific impulse, which is a measure of the "rocket's" efficiency, m_1 is the rocket's final rest mass (payload), and m_0 is the rocket's initial rest mass (payload plus fuel). We will assume that rocket drive's internal efficiency is 100 per cent, that is $\frac{I_{SP}}{c} = 1$. Combining the Ackeret rocket equation with Haug's maximum velocity, we have the following relationship (see [1] for more details):

$$m_0 = m_1 \left(\frac{1+\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{1-\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}} \right)^{\frac{1}{2}}. \tag{3}$$

Here we will use this to take a closer look at the ratio of the photon rocket's initial rest mass divided by the relativistic mass of the payload. We will call this the relativistic mass ratio. Due to variation in the interpretation of what exactly a relativistic mass represents, some would likely prefer to examine the energy ratio instead of the mass ratio. This would not alter any of the main conclusions in this paper.

2 Relation to the Tipler Factor of Two, a modified Tipler Factor

Haug's result of two Planck masses as the initial mass needed to accelerate a payload for any[2] fundamental particle to its maximum velocity, $v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}$, seems to be consistent with an interesting result presented by Tipler in 1999 (see [2]). Tipler discussed what he called "ultrarelativistic rockets", which are rockets basically moving very close to the speed of light. Tipler has demonstrated that the total initial mass (energy) needed to accelerate a photon rocket's payload to a velocity very close to that of light only is twice the payload mass (energy) as measured from the Earth (the payload's relativistic mass when it has reached its final velocity, very close to the speed of light). The initial mass is still enormous compared to the payload's rest mass but is only two in relation to the final relativistic payload mass.

Haug's maximum velocity formula yields the same result as Tipler's formula. The relativistic mass of any fundamental particle travelling at its maximum velocity is

$$\frac{m_1}{\sqrt{1-\frac{v_{max}^2}{c^2}}} = \frac{m_1}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} = \frac{m_1}{\sqrt{1-\frac{c^2\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{c^2}}} = m_1\frac{l_p}{\bar{\lambda}} = m_p. \tag{4}$$

That is, every subatomic particle in our theory has a limit on their maximum relativistic mass equal to the Planck mass. The relativistic mass ratio must be given by

$$\frac{m_0}{\frac{m_1}{\sqrt{1-\frac{v_{max}^2}{c^2}}}} = \frac{m_1\left(\frac{1+\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{1-\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}\right)^{\frac{1}{2}}}{m_1\frac{\bar{\lambda}}{l_p}} = \left(\frac{1+\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{1-\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}\right)^{\frac{1}{2}}\frac{l_p}{\bar{\lambda}}. \tag{5}$$

When $\bar{\lambda} >> l_p$ we can approximate with a series expansion: $\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \approx 1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}$, yielding

$$\frac{m_0}{\frac{m_1}{\sqrt{1-\frac{v_{max}^2}{c^2}}}} \approx \left(\frac{1+\left(1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}\right)}{1-\left(1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}\right)}\right)^{\frac{1}{2}}\frac{l_p}{\bar{\lambda}} \approx \left(4\frac{\bar{\lambda}^2}{l_p^2}\right)^{\frac{1}{2}}\frac{l_p}{\bar{\lambda}} = 2. \tag{6}$$

The result above is a very good approximation when $\bar{\lambda} >> l_p$. Our result can be seen as the subatomic derivation/connection leading to the same result as given by Tipler in 1999. Tipler calls this an energy ratio. However, our result derived for subatomic particles seems to provide a deeper as well as important additional insight. For payloads of fundamental subatomic particles with rest mass close to the Planck mass, this factor is actually slightly below two. Here we cannot use the approximation above but must instead use equation 5. In the special case of a Planck mass particle, we must have

$$\left(\frac{1+\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{1-\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}\right)^{\frac{1}{2}}\frac{l_p}{\bar{\lambda}} = \left(\frac{1+\sqrt{1-\frac{l_p^2}{l_p^2}}}{1-\sqrt{1-\frac{l_p^2}{l_p^2}}}\right)^{\frac{1}{2}}\frac{l_p}{l_p} = 1. \tag{7}$$

[2] Except the Planck mass particle itself.

This, as suggested by [12], can be interpreted as the Planck mass particle being the very collision point between two photons. The Planck mass particle only lasts for an instant, actually one Planck second as measured with Einstein-Poincarè synchornized clocks.

Figure 1 plots the relativistic mass ratio given by equation 5 as a function of the number of Planck lengths (equivalent) in the reduced Compton wavelength of the payload's rest mass. We can see that the relativistic mass ratio starts at one for a Planck mass particle but that it very quickly converges towards two for subatomic payloads lighter than the Planck mass particle.

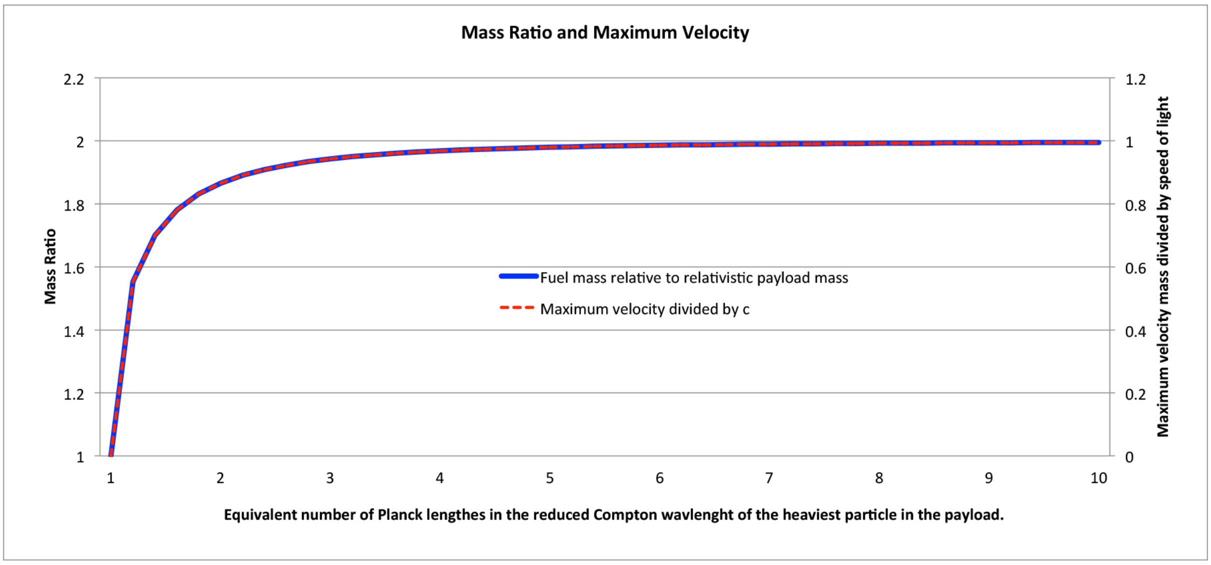

Figure 1: The figure shows the relativistic mass ratio as well as the maximum velocity divided by the speed of light. As the mass of the payload particle approaches that of the Planck mass, the relativistic mass ratio and the maximum velocity falls rapidly.

In the same figure, we have also plotted the maximum velocity divided by the speed of light; for this we use a second x-axis to the right. The mass ratio and the maximum velocity have differing values but exactly the same shape and are actually laying on top of each other. This is not a surprise, since the maximum velocity of a subatomic particle is actually indirectly linked to the mass ratio of an ideal photon rocket:

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} = c\sqrt{1 - \frac{m_0^2}{m_p^2}}, \tag{8}$$

where m_0 is the particle's initial rest mass and m_p is the Planck mass.

If our theory is correct, then the Tipler factor of two is only a very good approximation for subatomic particle payloads with reduced Compton wavelength significantly larger than the Planck length – in other words, for fundamental subatomic particles with mass much smaller than the Planck mass particle. For subatomic particles with mass approximating the Planck mass, the mass ratio goes below two.

Further in the special case of a Planck mass particle (that is, a particle with reduced Compton wavelength equal to the Planck length), we surprisingly have a maximum velocity of zero, and no additional fuel is needed to accelerate the object at rest. This at first sounds absurd, but we have reason to believe that a rest mass particle with mass equal to the Planck mass is the very collision point of two photons. The Planck mass particle only lasts for an instant. The two photons change course of direction; they are backscattering. Stated differently, a Planck mass particle uses its own rest mass energy to accelerate to the speed of light in an instant, by converting itself into energy. The Planck mass particle has no payload, and it likely converts itself to pure energy within a Planck second as measured with Einstein-Poincarè synchronized clocks.

We can "conclude" that the mass ratio in an ideal photon rocket travelling at maximum velocity must be between one and two:

$$1 \leq \frac{m_0}{\frac{m_1}{\sqrt{1-\frac{v_{max}^2}{c^2}}}} \leq 2. \tag{9}$$

A deeper level is represented by

$$1 \leq \left(\frac{1+\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{1-\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}} \right)^{\frac{1}{2}} \frac{l_p}{\bar{\lambda}} \leq 2. \tag{10}$$

We can also generalize this for full-size rockets. As explained by Haug, the maximum velocity for a macroscopic rocket has a upper limit linked to the reduced Compton wavelength of the heaviest subatomic particles making up the rocket.

Unfortunately only extremely high energy experiments, far above what can be done at LHC, are likely to descern if this theory should or should not hold true. Still, we will argue that Haug's maximum velocity formula seems to remove a series of infinity challenges in modern physics and is a simple and logical way to merge key concepts from Max Planck into our current theory without changing existing formulas, but rather giving them exact boundary conditions that no experiment thus far contradicts.

3 Summary and Conclusion

The initial mass relative to the relativistic mass of the payload in an ideal photon rocket must be between one and two. For a Planck mass particle, the ratio is remarkably one, because the Planck mass particle is likely the very collision point between two photons. The Planck mass particle can probably only last for an instant before it burst into energy. As soon as we deal with "fundamental" particles with mass less than the Planck mass particle, then the relativistic mass ratio very quickly converges towards two. The two factors found here seem to be consistent with the Tipler factor of two, as first suggested in 1999 at the NASA Breakthrough Propulsion Physics Workshop Proceedings. However, we have reason to think that the Tipler factor is only a good approximation as long as we are working with rockets built from subatomic particles considerably lighter than the Planck mass particle.

Our findings here also support the author's view that the Planck mass and the Planck length are extremely unique entities. The Planck length is the reduced Compton wavelength of the Planck mass particle, and the Planck mass and the Planck length are likely the same as observed from any reference frame. This can only hold true if the Planck mass (and thereby the Planck length in the form of a Compton wavelength) can only last for an instant. This is exactly what we could expect to happen at the very collision point of two photons.

To test this out in practice, unfortunately, we would need a technological breakthrough to make particle accelerators far more powerful than LHC. However, with the enormous progress we have seen in science during the last hundred years, this will probably be possible in the decades to come.

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