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Why Do Planets Rotate Around Themselves ?

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It is commonly believed that the self-rotation angular momentum of planets is due to an original angular momentum of dense interstellar clouds at the formation stage of the stars. However, the study shows something completely different: a test planet in free-fall, in fact, follows two geodesics; the first is the usual Schwarzschild path, and the second is a Schwarzschild-like path, defined (spatially) locally: an elliptical orbit in the plane ($U(1)$-variable, azimuthal angle). The analysis leads to the fact that: the motion along these geodesics (physically) is exactly the self-rotation of a charged test planet in Reissner-Nordstrom spacetime. The results reveal a more general understanding of Einstein equivalence principle: locally, gravitational field can be (in the Reissner-Nordstrom space) replaced with an accelerated and rotated local frame.

1. Introduction

In general theory of relativity,[1,2] an exact solution of Einstein equations is the mathematical relation relating the spacetime coordinates and the stress-energy tenor in. In literature, several types of solutions have been studied, including: (a) Vacuum solutions,[3] (b) Electro-vacuum solutions whose the mass-energy considered is the electromagnetic field,[4] (c) Null dust solutions whose the only mass-energy tensor considered is that of mass-less radiation,[5] and (d) Fluid solutions whose the stress-energy tensor is produced totally by the momentum, mass, and a density of fluid.[6] Having said that, the physical understanding of the solutions does not parallel the theoretical progress, so the case of the ReissnerNordstrom solution.[8] In the present paper we show that the ReissnerNordstrom metric leads naturally to the observed self-rotation angular momentum of planets.

2. Reissner-Nordstrom spacetime

The ReissnerNordstrom metric corresponds to the gravitational field of a non-rotating, charged, spherically symmetric gravity-source. The metric takes the form

$$ds^2 = -\left(1 - \frac{r_s}{r} + \frac{a}{r^2}\right) dt^2 + \frac{1}{1 - \frac{r_s}{r} + \frac{a}{r^2}} dr^2 + r^2 d\varphi^2 \tag{1}$$

where $r_s = \frac{2GM}{c^2}$, c is the speed if light, M the mass of the gravity source, G is the constant of Newton, $a = \frac{Q^2 G}{4\pi\epsilon_0 c^4}$, Q the electric charge of the gravity-source, $\frac{1}{4\pi\epsilon_0}$

is the coulomb force constant. Of course, taking the limit of the value a to zero recovers the Schwarzschild solution, so the case of r_s: taking the value of r_s to zero (or fixing it at a given spatial point on the usual Schwarzschild geodesic) gives the solution corresponding to the geometric-electromagnetic field, which has the form

$$ds^2 = -\left(1 + \frac{a}{r^2}\right) dt^2 + \frac{1}{1 + \frac{a}{r^2}} dr^2 + r^2 d\varphi^2 \tag{2}$$

3. The physical understanding of the solution

It is clear that gravity is a geometric theory defined with the group $SL(2, C)$, and cannot be observed locally according to Einstein equivalence principle. Also, we know that classical electromagnetism is a $U(1)$–group theory, i.e. it corresponds to rotations on two-dimensional space. Thus, the last equation defines the curvature of the spacetime resulting from the stress-energy tensor of the electromagnetic sector (the charge in the gravity-source), i.e. the last equation corresponds to a free-fall on geodesics, but it takes place (spatially) locally.

On the other hand, the rotation (i.e. $U(1)$) can be defined with an angle variable φ, thus we deal with the variable $\acute{r} = r_0\varphi$, where r_0 is constant (with respect to the study). Also, we denote the proper time as $\acute{s}$, the time as $\acute{t}$ and the angle phi as $\acute{\varphi}$. The angle $\acute{\varphi}$ should not be confused with φ. The second is related to the electromagnetic rotation, and the first to the usual phi-coordinate. The metric, therefor, takes the form:

$$d\acute{s}^2 = -\left(1 + \frac{a}{\acute{r}^2}\right) d\acute{t}^2 + \frac{1}{1 + \frac{a}{\acute{r}^2}} d\acute{r}^2 + \acute{r}^2 d\acute{\varphi}^2 \tag{3}$$

where we have used the prime on the coordinates in order not the study to be confused with the usual coordinates in the Schwarzschild spacetime, because the two metrics affect the test particle at the same time: locally the metric of the primed coordinates, and globally the usual Schwarzschild metric. Consequently, the ReissnerNordstrom spacetime has two Schwarzschild metrics. Simply, a test particle in free-fall in the ReissnerNordstrom spacetime follows to paths: one in the plane $(r_0, \acute{\varphi})$, and the other in the plane $(\acute{r}, \acute{\varphi})$. The manifestation of the first motion is the observed planetary motion around sun, and that of the second is the observed self-rotation of planets. The fact that the term $1/\acute{r}$ corresponds to $1/\acute{r}^2$ in the new Schwarzschild metric is that: we are (globally) dealing with the variable $r_0\varphi$ (not φ alone), thus the factor r_0 results in the term of the curvature as a curvature-factor with the value $f = 1/\acute{r}$, thus $f \cdot 1/\acute{r} = 1/\acute{r}^2$ see Ref. 7 for more details.

4. Results

The physical understanding of the ReissnerNordstrom solution is crucial: *a test particle, in fee-fall along a ReissnerNordstrom geodesic, in fact follows two elliptical orbits: one in the plane $(\acute{r}, \acute{\varphi})$ and the second in the plane $(r_0, \acute{\varphi})$.*

4.1. *Geodesics equations*

Using the geodesic equations together with the metric of the interval in Eq. (3), we get the differential-system

$$
\begin{aligned}
ds^2 &= -(1+\frac{a}{\acute{r}^2})d\acute{t}^2 + \frac{1}{1+\frac{a}{\acute{r}^2}}d\acute{r}^2 + \acute{r}^2 d\acute{\varphi}^2, &(4)\\
\frac{d^2\acute{t}}{d\acute{s}^2} &= \frac{2a}{\acute{r}(\acute{r}^2+a)}\frac{d\acute{t}}{d\acute{s}}\frac{d\acute{r}}{d\acute{s}}, &(5)\\
\frac{d^2\acute{\varphi}}{d\acute{s}^2} &= \frac{-2}{r}\frac{d\acute{\varphi}}{d\acute{s}}\frac{d\acute{r}}{d\acute{s}}, &(6)\\
\frac{d^2\acute{r}}{d\acute{s}^2} &= \frac{-a}{\acute{r}(\acute{r}^2+a)}\left(\frac{d\acute{r}}{d\acute{s}}\right)^2 + \frac{\acute{r}^2+a}{\acute{r}}\left(\frac{d\acute{\varphi}}{d\acute{s}}\right)^2 + \frac{a(\acute{r}^2+a)}{\acute{r}^5}\left(\frac{d\acute{t}}{d\acute{s}}\right)^2, &(7)
\end{aligned}
$$

which give

$$
\begin{aligned}
\frac{d\acute{t}}{d\acute{s}} &= \beta\frac{\acute{r}^2}{\acute{r}^2+a}, &(8)\\
\frac{d\acute{\varphi}}{d\acute{s}} &= \frac{\gamma}{\acute{r}^2}, &(9)
\end{aligned}
$$

and

$$\frac{d^2\acute{r}}{d\acute{s}^2} = \frac{-a}{\acute{r}(\acute{r}^2+a)}\left[1+\beta^2+\frac{a}{\acute{r}^2}-\frac{a+\acute{r}^2}{\acute{r}^4}\gamma^2\right] + \left(\acute{r}+\frac{a}{\acute{r}}\right)\frac{\gamma^2}{\acute{r}^4} + \frac{a(\acute{r}^2+a)}{\acute{r}^5}\frac{\acute{r}^4}{(\acute{r}^2+a)^2}\beta^2 \qquad (10)$$

where γ, β are constants: particularly, γ represents the angular momentum of the test particle; since the test particle rotates around itself at a any spatial point of the manifold, the constant of angular momentum can be written as $\gamma = I\omega$, where I is the moment of inertia of the test particle (or planet) and ω is the corresponding angular velocity (i.e. $\frac{d\varphi}{d\acute{s}}$).

4.2. *The Newtonian approximation*

Using the Newtonian approximation (that is dealing with the elliptical orbits in the context of the newton approximation: $\frac{a}{\acute{r}} \approx 0, \frac{\gamma}{\acute{r}} \approx 0$), we get the following equation of motion

$$\frac{d^2\acute{r}}{d\acute{s}^2} = -\frac{a}{\acute{r}^3} + \frac{\gamma^2}{\acute{r}^3} \qquad (11)$$

or

$$r_0\frac{d^2\varphi}{d\acute{s}^2} = -\frac{a}{\acute{r}^3} + \frac{\gamma^2}{\acute{r}^3} \qquad (12)$$

Using Einstein equivalence principle: for a free-fall motion, the angular acceleration is equivalent to the gravitational field, we get our main result:

$$a = \gamma^2 \qquad (13)$$

this is the equation of the conservation of angular momentum (a is constant; γ represents the angular momentum of the test planet), i.e. the angular momentum of the planet ($\gamma = I\omega$) is conserved along the full orbit.

4.3. *The physical interpretation*

physically, the above analysis can be understood using the tangential velocity (to the curve), which is $\frac{r_0 d\acute{\varphi}}{d\acute{s}} = v$, thus Eq.(12) becomes:

$$-\frac{a}{r_0^3} + \frac{\sigma^2 v^2}{r_0} \qquad (14)$$

where $\gamma = \sigma J$, and J is the orbital angular momentum (note that in this dynamic, $\frac{d\acute{\varphi}}{d\acute{s}} = \frac{\gamma}{\acute{r}^2}$, also in the Schwarzschild dynamic the change in $\acute{\varphi}$ is defined with the same mathematical equation $\frac{d\acute{\varphi}}{ds} = \frac{J}{r_0^2}$ – with respect to r_0 –; from this consistency the constant σ can be understood).

The right hand side of Eq. (14) is the centripetal force. The left hand side of Eq. (14) is the electromagnetic force.

Note that: the fact that the electromagnetic force is proportional to $\frac{1}{r_0^3}$ is expected; the electromagnetic stress-energy tensor in the case of the ReissnerNordstrom spacetime is proportional to $\frac{1}{r_0^2}$, thus the gradient of the potential results the proportionality in question.

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