# The Dark side of Gravity vs MOND/DM

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We review the foundations of the Dark Gravity Theory and find that, inter alia, the strength of the dark side gravity could be enhanced relative to our side gravity in some space-time domains and that a MOND radius arises naturally so that hopefully the best of both MOND and Dark Matter phenomenology may finally be within reach for Dark Gravity.

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## 1. Introduction

In the seventies, theories with a flat non dynamical background metric and/or implying many kinds of preferred frame effects became momentarily fashionable and Clifford Will has reviewed some of them (Rosen theory, Rastall theory, BSLL theory ...) in his book [34]. Because those attempts were generically roughly conflicting with accurate tests of various versions of the equivalence principle, the flat non dynamical background metric was progressively given up. The Dark Gravity (DG) theory we support here is a remarkable exception as it can easily reproduce most predictions of GR up to Post Newtonian order (as we shall remind in the two following sections) and for this reason deserves much attention since it might call into question the assumption behind most modern theoretical avenues: background independence.

DG follows from a crucial observation: in the presence of a flat non dynamical background $\eta_{\mu\nu}$, it turns out that the usual gravitational field $g_{\mu\nu}$ has a twin, the "inverse" metric $\tilde{g}_{\mu\nu}$. The two being linked by :

$$\tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}\left[g^{-1}\right]^{\rho\sigma} = \left[\eta^{\mu\rho}\eta^{\nu\sigma}g_{\rho\sigma}\right]^{-1} \tag{1}$$

are just the two faces of a single field (no new degrees of freedom) that we called a Janus field [2][3][6][13][14]. See also [4][7][8][5] [29][30][31][32][33][27] for alternative approaches to anti-gravity with two metric fields.

The action treating our two faces of the Janus field on the same footing is achieved by simply adding to the usual GR and SM (standard model) action, the similar action with $\tilde{g}_{\mu\nu}$ in place of $g_{\mu\nu}$ everywhere.

$$\int d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \int d^4x(\sqrt{g}L + \sqrt{\tilde{g}}\tilde{L}) \tag{2}$$

where R and $\tilde{R}$ are the familiar Ricci scalars respectively built from $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ as usual and $L$ and $\tilde{L}$ the Lagrangians for respectively SM F type fields propagating along $g_{\mu\nu}$ geodesics and $\tilde{F}$ fields propagating along $\tilde{g}_{\mu\nu}$ geodesics. This is invariant under the permutation of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. This theory symmetrizing the roles of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ is Dark Gravity (DG) and the field equation satisfied by the Janus field derived from the minimization of the action is:

$$\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}G^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{G}^{\rho\mu} = -8\pi G(\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}T^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{T}^{\rho\mu}) \qquad (3)$$

with $T^{\mu\nu}$ and $\tilde{T}^{\mu\nu}$ the energy momentum tensors for F and $\tilde{F}$ fields respectively and $G^{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ the Einstein tensors (e.g. $G^{\mu\nu} = R^{\mu\nu} - 1/2g^{\mu\nu}R$). Of course from the Action extremization with respect to $g_{\mu\nu}$ (see the detailed computation in the Annex), we first obtained an equation for the dynamical field $g_{\mu\nu}$ in presence of the non dynamical $\eta_{\mu\nu}$. Then $\tilde{g}_{\mu\nu}$ has been reintroduced using (1) and the equation was reformatted in such a way as to maintain as explicit as possible the symmetrical roles played by the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ of the Janus field. The contracted form of the DG equation simply is :

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \qquad (4)$$

It is well known that GR is the unique theory of a massless spin 2 field. However DG is not the theory of one field but of two fields: $g_{\mu\nu}$ and $\eta_{\mu\nu}$. Then it is also well known that there is no viable (ghost free) theory of two interacting massless spin 2 fields. However, even though $\eta_{\mu\nu}$ is a genuine order two tensor field transforming as it should under general coordinate transformations[a], $\eta_{\mu\nu}$ actually propagates no degrees of freedom : it is really non dynamical, not in the sense that there is no kinetic (Einstein-Hilbert) term for it in the action, but in the sense that all it's degrees of freedom were frozen a priori before entering the action and need not extremize the action : we have the pre-action requirement that Riemm($\eta_{\mu\nu}$)=0 like in the BSLL, Rastall and Rosen theories [34]. So DG is also not the theory of two interacting spin 2 fields.

We will later carry out the complete analysis of the stability of the theory however we already found that, at least about a Minkowskian background common to the two faces of the Janus field, the worst kind of classical instabilities might be avoided (reduced to a well acceptable level) because:

- Fields minimally coupled to the two different sides of the Janus field never meet each other from the point of view of the other interactions (EM, weak, strong) so stability issues could only arise in the purely gravitational sector.

[a] in contrast to a background Minkowski metric $\hat{\eta}_{\mu\nu}$ such as when we write $g_{\mu\nu} = \hat{\eta}_{\mu\nu} + h_{\mu\nu}$, which by definition is invariant since only the transformation of $h_{\mu\nu}$ is supposed to reflect the effect of a general coordinate transformation applied to $g_{\mu\nu}$

- The run away issue [9] [10] is avoided between two masses propagating on $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ respectively, because those just repel each other, anti-gravitationally as in all other versions of DG theories [8][5] rather than one chasing the other ad infinitum.
- The energy of DG gravitational waves almost vanishes about a common Minkowski background (we remind in a forthcoming section that DG has an almost vanishing energy momentum pseudo tensor $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ in this case) avoiding or extremely reducing for instance the instability of positive energy matter fields through the emission of negative energy gravitational waves.

In particular the first two points are very attractive so we were not surprised discovering that recently the ideas of ghost free dRGT bimetric massive gravity [35] have led to a PN phenomenology identical to our though through an extremely heavy, unnatural and Ad Hoc collection of mass terms fine tuned just to avoid the so called BD ghost[b]. Anyway, all such kind of bimetric constructions seriously question the usual interpretation of the gravitational field as being the metric describing the geometry of space-time itself. There is indeed no reason why any of the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, which describe a different geometry should be preferred to represent the metric of space-time. At the contrary our non dynamical flat $\eta_{\mu\nu}$ is now the perfect candidate for this role.

We think the theoretical motivations for studying as far as possible a theory such as DG are very strong and three-fold : challenge the idea of background independence, bridge the gap between the discrete and the continuous and challenge the standard understanding of time reversal.

- Challenge the idea of background independence because DG is the straightforward generalization of GR in presence of a background non dynamical metric so either there is no such background and GR is most likely the fundamental theory of gravity or there is one and DG is the most obvious candidate for it.
- Bridge the gap between the discrete and the continuous because we here have both the usual continuous symmetries of GR but also a permutation symmetry which is a discrete symmetry between the two faces of the Janus field.
- Challenge the standard understanding of time reversal because as we shall see the two faces of the Janus field are related by a global time reversal symmetry.

[b]Indeed the first order differential equation in [31] is exactly the same as our: see e.g eq (3.12) supplemented by (4.10) and for comparison our section devoted to the linearized DG equations. This is because the particular coupling through the mass term between the two dynamical metrics in dRGT eventually constrains them to satisfy a relation Eq (2.4) which for $\alpha = \beta$ [31] becomes very similar to our Eq (1) to first order in the perturbations which then turn out to be opposite (to first order) as Eq (4.10) makes it clear.

The two last points require more clarification and the reader is invited to find it in the detailed analysis of our previous publications which we may summarize as follows:

Basically modern physics incorporates two kinds of laws: continuous and local laws based on continuous symmetries, most of them inherited from classical physics, and discrete and non local rules of the quanta which remain largely as enigmatic today as these were for their first discoverers one century ago. Though there are many ongoing attempts to "unify" the fundamental interactions or to "unify" gravity and quantum mechanics, the unification of the local-continuous with the non-local-discrete laws would be far more fundamental as it would surely come out with a genuine understanding of QM roots. However such unification would certainly require the identification of fundamental discrete symmetry principles underlying the discontinuous physics of the quanta just as continuous and local laws are related to continuous symmetries. The intuition at the origin of DG is that the Lorentz group which both naturally involves discrete P (parity) and T (time reversal) symmetries as well as continuous space-time symmetries might be a natural starting point because the structure of this group itself is already a kind of unification between discrete and continuous symmetries. However neither P nor the Anti-Unitary T in the context of QFT seem to imply a new set of dynamical discrete laws. Moreover our investigation in [6] (see also [13] section 3) revealed that following the alternative non- standard option of the Unitary T operator to understand time reversal led to a dead-end at least in flat spacetime. However we concluded that it might eventually be possible to understand and rehabilitate negative energies and relate them to normal positive energies through time reversal but only in the context of a gravitational theory in which the metric itself would transform non trivially under time reversal. This time reversal not anymore understood as a local symmetry (exchanging initial and final states as does the anti-unitary operator) but as a global symmetry implying a privileged time and a privileged origin of time would jump from one metric to it's conjugate. Only such time reversal would retain it's discrete nature inherited from the Lorentz group even in a generally covariant framework because at the contrary to a mere diffeomorphism but rather like an internal symmetry it would really discretely transform one set of inertial coordinates into another non equivalent one (see [3] section 5), i.e. it would transform a metric into a distinct one describing a different geometry. The DG solutions that we shall remind in the first sections in the homogeneous-isotropic case impressively confirm that our sought privileged time is a cosmological conformal time and that the two faces of the Janus field are just this time reversal conjugate metrics we have been looking for: the conjugate conformal scale factors are indeed found to satisfy $\tilde{a}(t) = 1/a(t) = a(-t)$ (also see [13] section 6.2). The solutions in the isotropic case then also confirm the reversal of the gravific energy as seen from the conjugate metric. In a sense DG had to reinvent the zero and negative values for the time and mass-energies which only became possible thanks to the pivot metric $\eta_{\mu\nu}$. Eventually we are aware that we are not yet ready to derive the Planck-Einstein relations from this new framework but in the following we will have to keep in mind what was our initial motivation: understand the origin of the discrete rules of QM from discrete symmetries to not prohibit oneself the explicit introduction of discrete rules and processes any time the development of the theory seems to require them.

The article is organized as follows: in section 2 we remind and complement the results of previous articles as for the global homogeneous solution and in section 3 the local static isotropic asymptotically Minkowskian solutions of the DG equation. In section 4 we discuss the linearized theory about this common Minkowskian background for $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ and the prediction of the theory as for the emission of gravitational waves. In sections 5 and 6, we give up the hypothesis that the two conjugate metrics are asymptotically the same to derive the isotropic static solution again in this more general case and discuss our pseudo Black Hole and new predictions for gravitational waves. It turns out that the theory of one single Janus field can't account for both the global gravity of section 2 and the local gravity of sections 3, 4, 5 and 6. In sections 7, 8, 9, 10 and 11 we are then led to propose various unification schemes for the global and local Janus field theories, one being based on matter exchanges between the two metrics and another one based on the concept of emerging dynamics, space-time exchange symmetries and an original quantization postulate, resulting in a renewed understanding of global expansion effects and the Pioneer anomaly. We believe both are realized and lead to new phenomena. Section 12 explores the MOND like phenomenology of the unified DG theory. Section 13 emphasizes the need for a theory of gravity such as DG which very principles being based on discrete as well as continuous symmetries, for the first time open a natural bridge to quantum mechanics and hopefully a royal road toward a genuine unification. Section 14 discusses all kind of stability issues to conclude that the theory is safe once understood as a semi-classical theory of gravity. Before the last remarks and outlooks (section 16) and conclusion, section 15 analyses a new plausible Dark Matter candidate within our framework.

## 2. Global gravity

### 2.1. *The scalar-$\eta$ cosmological field*

We found that an homogeneous and isotropic solution is necessarily spatially flat because the two sides of the Janus field about our flat Minkowski background are required to satisfy the same isometries. However, it is also static so that the only way to save cosmology in the DG framework is to introduce a $\eta$-scalar Janus field built from our non dynamical background and a scalar $\Phi$ such that $g_{\mu\nu} = \Phi\eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \frac{1}{\Phi}\eta_{\mu\nu}$. Then our fundamental cosmological single equation obtained by requiring the action to be extremal under any variation of $\Phi(t) = a^2(t)$ is:

$$a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = \frac{4\pi G}{3}(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \quad (5)$$

where $\tilde{a}(t) = \frac{1}{a(t)}$. With this scalar cosmology we avoid all the normal degrees of freedom of a metric and corresponding two Friedmann type equations which for a spatial curvature k=0, could only be satisfied all together by a static solution for any equations of state[c]. That such two cosmological equations would be incompatible unless in the static case is most easily checked in the $a(t) = e^{h(t)}$ , $\tilde{a}(t) = e^{-h(t)}$ domain of small h(t) and will also appear very clearly after rewriting those equations as in Eq 39. The reason for that incompatibility is that though DG equations are of course generally covariant, the gauge invariance of GR is lost[d]: such equations are not invariant under the transformations of $g_{\mu\nu}$ alone but under the combined transformations of $g_{\mu\nu}$ and $\eta_{\mu\nu}$. As a result we have no equivalent of the Bianchi identities to make the DG equations functionally dependent as in GR. If we don't reduce the number of dof of the metric field it is therefore not surprising to get two independent equations for the scale factor (constraining it to remain static) in contrast to GR when the matter and radiation fields equations of motion are satisfied on each metric in the usual way. So we absolutely need the scalar-$\eta$ Janus field and it's single equation to avoid this.

Now this field is also understood to be "genetically homogeneous" i.e. the spatially independent $\Phi(t)$ at any scale and sourced by the mean expectation value of the usual sources averaged over space rather than the sources themselves. So there are no scalar waves associated to this field and there is also no scale related to a loss of homogeneity of the background effects as in GR. We strongly support the idea that the homogeneity of the scalar field is fundamental just because we want to rehabilitate field discontinuities: in a sense the field will sometimes need to vary discontinuously just because it cannot vary continuously in space[e]. The Pioneer effect, as we shall see is a perfect signature of what we should expect from spatial discontinuities of the scale factor. Of course in a given domain it is possible to require this fundamental homogeneity in a fully covariant way : the conjugate metrics

[c]In GR cosmology there is for instance the first order equation $H^2(t) = \frac{8\pi G\rho}{3}$ for k=0. Here for $a >> \tilde{a}$ we can neglect $\tilde{a}$ terms in our equation to get an equation that is also valid within GR. For the scale factor in standard time coordinate, it's just: $\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 = \frac{4\pi G}{3}(\rho - 3p)$. Since we only have this second order equation, in principle the initial conditions i.e. a(t) and $\dot{a}(t)$ could be chosen at will at some particular time yielding $H^2(t)$ very different from $\frac{8\pi G\rho}{3}$ at this time. However for negligible pressure the derivative of $H^2(t) = \frac{8\pi G\rho}{3}$ and matter equations of motion lead to $\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 = \frac{4\pi G}{3}\rho$ so any solution of the first order equation is also solution of the second order equation. The converse is not true and the general solution of our second order equation must involve additional integration constants and terms relative to a solution of the first order equation. However since we know that $a(t) \approx t^{2/3}$ and $H^2(t) \approx \frac{1}{a^3(t)}$ is a solution of the first order equation that correctly fits the data in the cold era we can deduce that $H^2(t) \approx \frac{8\pi G\rho}{3}$ is approximately valid just as in GR in our case for $p \approx 0$ with the same deduction that the baryonic matter is cosmologically not abundant enough to account for the measured Hubble rate: in other words we again have a missing mass issue at the cosmological scale.

[d]So this is rather harmless as compared to theories such as for instance unimodular gravity in their diffeomorphism breaking versions [[42]][[43]]

[e]In the future, we might relax this hypothesis to allow a new complete scalar sector, because its G coupling constant could be different and actually much smaller than the gravitational coupling constant G of the separate spin 2 theory. This weakness of the new scalar coupling constant would of course be necessary to satisfy all known observational constraints.

should share the killing vector of a maximally symmetric sub 3d-space insuring that for each metric there is a coordinate system in which it can be written the way we did and it just remains to assume that in this coordinate system for one of the metrics, we also have $\eta_{\mu\nu} = diag(-1, +1, +1, +1)$ for it to be the common conformal coordinate system for both metrics. The difference with the GR treatment of a cosmological metric is that in the context of GR such symmetry would not prevent the metric to fluctuate in any way it wants i.e. for instance non homogeneously.

Now an independent other Janus field is then of course required to describe all other (other than cosmological) aspects of gravity with all it's usual degrees of freedom, but then a field forced to remain asymptotically static to satisfy all the equations. Thus in DG we would apparently need two different fields to separately describe the homogeneous evolution and fluctuations respectively. For instance the source densities and pressures would be $\bar{\rho}(t)$ and $\bar{p}(t)$ for the homogeneous scalar-$\eta$ field and $\delta\rho = \rho(\mathbf{x}, t) - \bar{\rho}(t)$, $\delta p = p(\mathbf{x}, t) - \bar{p}(t)$, $\rho(\mathbf{x}, t)\mathbf{v}(\mathbf{x}, t)$ ... for the asymptotically Minkowskian spin 2 field, where "bar" denotes spatial averaging. Further discussion of the implications and how to correctly reproduce results of GR theory of small fluctuations in DG are postponed to the relevant section because we first need to understand our isotropic and isotropic-homogeneous solutions.

### 2.2. *Cosmology*

This section reviews and provides a more in depth analysis of results already obtained in [13][14]. Cosmological alternative scenarii are also considered.

#### 2.2.1. *Continuous evolution and discontinuous permutation*

The expansion of our side implies that the dark side of the universe is in contraction. Provided dark side terms can be neglected, our cosmological equation reduces to a cosmological equation known to be also valid within GR. For this reason it is straightforward for DG to reproduce the same scale factor expansion evolution as obtained within the standard LCDM Model at least up to the redshift of the LCDM Lambda dominated era when something new must have started to drive the evolution in case we want to avoid a cosmological constant term. The evolution of our side scale factor before the transition to the accelerated regime is depicted in blue on the top of Figure 1 as a function of the conformal time t and the corresponding evolution laws as a function of standard time t' are also given in the radiative and cold era. Notice however the new behaviour about t=0 meaning that the Big-Bang singularity is avoided.

A discontinuous transition is a natural possibility within a theory involving truly dynamical discrete symmetries as is our permutation symmetry in DG. The basic idea is that some of our beloved differential equations might only be valid piecewise, only valid in the bulk of space-time domains at the frontier of which new discrete rules apply implying genuine field discontinuities. Here this will be the case for the scale factor. Of course a discontinuous process can't be consistent with the continuous process predicted by a differential equation but here the two kind of processes have their own domain of validity (the bulk vs the frontier) which avoids any conflicting predictions. However we would prefer the discontinuous process not to occur arbitrarily but to be governed by the same discrete symmetries readily readable from the equations of motion.

We postulated that a transition occurred billion years ago as a genuine permutation of the conjugate scale factors, understood to be a discrete transition in time modifying all terms explicitly depending on $a(t)$ but not the densities and pressures themselves in our cosmological equation (5): in other words, the equations of free fall for our "average source field" did not apply at the discrete transition in time.

Let's be more specific. The equations of free fall for the perfect fluids on both sides of course apply as usual before and after the transition and for instance on our side in the cold era dominated by non relativistic matter with negligible pressure, we have $\frac{d}{dt}(\rho a^3) = 0$. Such conservation equation is valid just because it follows from our action for the matter fields on our side. But here we not only have the usual invariance of our action under continuous space-time symmetries from which we can derive the corresponding field conservation equations closely related to the continuous field equations of motion valid in the bulk of a space-time domain. We also have the invariance of the action under a permutation which is a discrete symmetry. To continuous symmetries can be associated continuous evolution, interactions and conservation equations of the fields thanks to variational methods. Such methods are of course not available to derive discontinuous processes from discrete symmetries so we postulate and take it for granted that our new permutation symmetry also allows a new kind of process to take place : the actual permutation of the conjugate $a$ and $\tilde{a}$. The process is understood to modify all terms explicitly scale factor dependent in the cosmological equation whereas all density and pressure terms remain unchanged. Because such process is not at all related to the continuous symmetries that generate the continuous field equation there is indeed no reason why the discrete version $(\rho a^3)_{before} = (\rho a^3)_{after}$ of a conservation equation such as $\frac{d}{dt}(\rho a^3) = 0$ should be satisfied by this particular process. The symmetry principles and their domain of validity are the more fundamental so we should not be disturbed by a process which violates the conservation of energy since this process is discontinuous, only valid at the frontier of a space-time domain and related to a new discrete symmetry for which we have no equivalent of the Noether theorem. Here the valid rule when the permutation of the scale factors occurs is rather $\rho_{before} = \rho_{after}$ and the same for the pressure densities.

This permutation (at the purple point depicted on figure 1) could produce the subsequent recent acceleration of the universe. This was demonstrated in previous articles [13] and [14] assuming our side source $a^4(\rho - 3p)$ term has been dominant and therefore has driven the evolution up to the transition to acceleration. If this term is still dominant after the transition we get an accelerated expansion regime $(t' - t'_0)^{-2}$ in standard time coordinate with a Big Rip at future time $t'_0$[14]. However this scenario needs densities on the conjugate side much smaller than on our side and still does not specify a triggering condition so a more interesting possibility is that, following the transition, the dark side source term have started to drive the evolution: $a^4(\rho - 3p) << \tilde{a}^4(\tilde{\rho} - 3\tilde{p})$ resulting from $a(t) << \tilde{a}(t)$ and $\rho - 3p = \tilde{\rho} - 3\tilde{p}$. Then this equality of densities would be the perfect trigger for the transition we were looking for all the more since our cosmological equation (5) is actually invariant under the combined permutations of densities and scale factors rather than permutation of scale factors alone. We can then even specify two kinds of triggering conditions for a permutation to occur according our postulate: either (A) a scale factor permutation can occur at the crossing when we have equal density source terms; or similarly when the scale factors cross each other (which defines the origin of time), it is the permutation of the densities (B) which is allowed corresponding to the two metrics exchanging their matter and radiation content.

2.2.2. *Global time reversal and permutation symmetry*

The evolution of the scale factor is largely determined by initial conditions at t=0. The parameters are the initial densities $\rho_0 - 3p_0$, $\tilde{\rho}_0 - 3\tilde{p}_0$ and initial expanding rate $H_0$ (not to be confused with the present Hubble rate). Considering a scenario with equal initial densities on both sides one needs a non vanishing $H_0$ to get non static solutions which then turn out to satisfy the fundamental relation:

$$\tilde{a}(t) = \frac{1}{a(t)} = a(-t) \tag{6}$$

For this reason, already in our previous publications we could interpret our permutation symmetry as a global time reversal symmetry about privileged origin of conformal time t=0. But from such initial conditions the densities (decreasing on our side while increasing on the dark side) will never have the opportunity to cross again. To get benefit from our scale factors permutation postulated process (A) we thus need to break the initial equilibrium between densities in such a way that the densities can cross each other at a time different from t=0. Then however, we realize that for $a(t) = e^{h(t)}$, h(t) is not anymore an odd function meaning that the condition Eq 6 for interpreting the permutation symmetry as a global time reversal is now broken. Fortunately, the only thing we need to restore Eq 6 is that our other previously postulated density exchange process (B) does really occur at t=0. This is illustrated in fig 2 where h(t) is plotted with (plain line) and without (dotted line) assuming such exchange. Moreover this densities exchange results in the inversion of their evolution laws i.e from decreasing to increasing or vice versa, so that the evolution of both densities and scale factors are cyclic as illustrated in fig 3. This also insures the stability of these homogeneous solutions in the sense that these remain bounded and confirms that we completely avoid any singularity issue.

Once our permutation symmetry is successfully reinterpreted as a time reversal symmetry, for the scale factors to exchange their respective values at the equality of densities, we just need to jump from t to -t as illustrated in fig 1 and 3. If we do not revert the arrow of time i.e. t is still increasing in the negative time domain, the expanding rate is continuous at the discrete transition.

2.2.3. *Testable cosmological scenarios*

Focusing on the transition triggered by equal densities we have two possibilities :

- The conjugate side is currently in a radiative regime, so that our cosmological equation simplifies in a different way[f] :

$$\tilde{a}^2\frac{\ddot{\tilde{a}}}{\tilde{a}} \approx \frac{4\pi G}{3}\tilde{a}^4(\tilde{\rho}-3\tilde{p}) = K\tilde{a}^2 \tag{7}$$

  The solution $\tilde{a}(t) = C.sh(\sqrt{K}(t-t_0)) \approx C\sqrt{K}(t-t_0)$ for $1/C << \sqrt{K}(t-t_0) << 1$ so $a(t) \propto 1/(t-t_0)$ which translates into an exponentially accelerated expansion regime $e^{t'}$ in standard time coordinate. $t_0$ is determined by demanding the continuity of $H(t) = \frac{\dot{a}}{a} = \frac{-1}{t-t_0}$ after the transition which should match 2/t before the transition. This is not in concordance with our understanding of time reversal[13] because it requires a shift and redefinition of the origin of time.
- The dark side is also in a cold era at the transition and satisfies $\tilde{\rho}-3\tilde{p} \approx \tilde{\rho} = \rho - 3p \approx \rho$. Then the continuity of the Hubble rate is automatically satisfied without having to define a new $t_0$ after transition. For this reason this is still our privileged scenario first introduced in [13]. This leads to a constantly accelerated universe $a(t') \approx t'^2$ in standard coordinate following the transition redshift.

In all scenarios, constraining the age of the universe to be the same as within LCDM the transition redshift can be predicted (see [14] equation 6) and confronted to the measured value $z_{tr} = 0.67 \pm 0.1$.

- For the constantly accelerated universe the prediction is $z_{tr} = 0.78$ in very good agreement with the measured transition redshift.
- In the Big Rip scenario, $z_{tr} = 0.27$. Yet the hypothesis that the transition occurred everywhere simultaneously might not be valid. Otherwise the mean transition redshift should be significantly increased by an expected dispersion of transition redshifts due to inhomogeneities (some domains being already in the accelerating regime while others are still in the decelerating one) smoothing the observed transition between decelerated and accelerated expansion after averaging over large regions and making the theory difficult to discriminate from the very progressive LCDM

[f] That a quantity such as $\tilde{\rho}-3\tilde{p}$ is expected to follow a $1/\tilde{a}^2$ evolution in the limit where all species are ultra-relativistic can be deduced from Eq (21)-(25) of [39] and the matter and radiation energy conservation equation rewritten as $\tilde{\rho}-3\tilde{p} = 4\tilde{\rho}+\tilde{a}\frac{d\tilde{\rho}}{d\tilde{a}}$ in a radiation dominated dark side of the universe when $\tilde{\rho}$ and $\tilde{p} \approx 1/\tilde{a}^4(t)$.

transition. The mean measured transition redshift is indeed very sensitive to a smoothing. Our interest in this Big Rip scenario is motivated by the anomaly of the best precision "recent" cosmological measurement of $\bar{H}_0 = 73.03 \pm 1.79 (km/sec)/Mpc$ [17][18] over the two last billion years (300 SNe Ia at $z < 0.15$ having a Cepheid-calibrated distance) appearing to be exceeding by three standard deviations the one predicted by LCDM from Planck data. This is noteworthy because an unexpectedly high recent acceleration could of course be the signature of such Big Rip vs LCDM expectations.

- For the exponentially accelerated expansion scenario (hence just like the one produced by a pure cosmological constant) $z_{tr} \approx 0.4$. This again is assuming a transition occurring everywhere simultaneously which is just equivalent to a fictitious LCDM discrete transition between a purely CDM and a purely Lambda driven expansion regime (the Hubble rate still being continuous at the transition).
  The comparison with $z_{tr} \approx 0.7$ predicted for an actual progressive LCDM transition confirms that a smoothing effect would significantly increase the mean observed $z_{tr}$ again making this scenario even harder to discriminate from a LCDM transition.

Whatever the actual scenario we believe that such alternative to the cosmological constant is more satisfactory as it follows from first principles of the theory and eventually should fit the data without any arbitrary parameter, everything being only determined by the actual matter and luminous content of the two conjugate universes, such content so far not being directly accessible for the dark side. More specifically, the parameter which replaces the cosmological constant in our framework is merely the redshift of densities equality i.e. the transition redshift $z_{tr}$.

## 3. Local gravity: the isotropic case about Minkowski

Another Janus field and it's own separate Einstein Hilbert action are required to describe local gravity with isotropic solution in vacuum of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$.

$$A = e^{\frac{2MG}{r}} \approx 1 + 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} \tag{8}$$

$$B = \frac{1}{A} = -e^{\frac{-2MG}{r}} \approx -1 + 2\frac{MG}{r} - 2\frac{M^2G^2}{r^2} + \frac{4}{3}\frac{M^3G^3}{r^3} \tag{9}$$

perfectly suited to represent the field generated outside an isotropic source mass M. This is different from the GR one, though in good agreement up to Post-Newtonian order. The detailed comparison will be carried out in section 6. It is straightforward to check that this Schwarzschild new solution involves no horizon. The solution also confirms that a positive mass M in the conjugate metric is seen as a negative mass -M from its gravitational effect felt on our side.

## 4. Local gravity : linear equations about Minkowski

The linearized equations about a common Minkowskian background look the same as in GR, the main differences being the additional dark side source term $\tilde{T}_{\mu\nu}$ and an additional factor 2 on the linear lhs:

$$2(R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^{(1)\lambda}_{\lambda}) = -8\pi G(T_{\mu\nu} - \tilde{T}_{\mu\nu} + t_{\mu\nu} - \tilde{t}_{\mu\nu}) \tag{10}$$

however to second order in the perturbation $h_{\mu\nu}$ (plane wave expanded as usual) and given that $\tilde{h}_{\mu\nu} = -h_{\mu\nu} + h_{\mu\rho}h_{\nu\sigma}\eta^{\rho\sigma} + O(3)$ we found that the only non cancelling contributions to $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ on the rhs, vanish upon averaging over a region of space and time much larger than the wavelength and period (this is the way the energy and momentum of any wave are usually evaluated according [1] page 259). This $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ is standing as usual for the energy-momentum of the gravitational field itself because the Linearized Bianchi identities are still obeyed on the left hand side and it therefore follows the local conservation law:

$$\frac{\partial}{\partial x^{\mu}}(T^{\mu\nu} - \tilde{T}^{\mu\nu} + t^{\mu\nu} - \tilde{t}^{\mu\nu}) = 0 \tag{11}$$

We can try to go beyond the second order noticing that the DG equation (3) has the form $X^{\mu\nu} - \tilde{X}^{\nu\mu} = -8\pi G(Y^{\mu\nu} - \tilde{Y}^{\nu\mu})$ and can be split in a $\mu \leftrightarrow \nu$ symmetric, $X^{\mu\nu}_s - \tilde{X}^{\mu\nu}_s = -8\pi G(Y^{\mu\nu}_s - \tilde{Y}^{\mu\nu}_s)$, and a $\mu \leftrightarrow \nu$ anti-symmetric $X^{\mu\nu}_a + \tilde{X}^{\mu\nu}_a = -8\pi G(Y^{\mu\nu}_a + \tilde{Y}^{\mu\nu}_a)$, in which the s (resp a) indices refer to the symmetric (resp anti-symmetric) parts of the tensors. Though the antisymmetric equation could in principle source gravitational waves, its production rate is expected to be extremely reduced vs GR because the dominant source term is at most of order hT rather than T in the Y term.

The value of the $\mu \leftrightarrow \nu$ symmetric equation is the manifest anti-symmetry of its lhs under the permutation of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. Replacing $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ thus $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$, this translates into the odd property of the lhs to all orders in $\bar{h}_{\mu\nu}$. Then we are free to use the plane wave expansion of this new $\bar{h}_{\mu\nu}$ (not to be confused with $h_{\mu\nu}$ nor $\tilde{h}_{\mu\nu}$) instead of $h_{\mu\nu}$ and because each term of the perturbative series has an odd number of such $\bar{h}$ factors, such term will always exhibit a remaining $e^{i\mathbf{kx}}$ factor which average over regions much larger than wavelength and period vanishes (in contrast to [1] page 259 where the computation is carried on for quadratic terms for which we are left with some $x^{\mu}$ independent, hence non vanishing, cross-terms).

Our new interpretation is that any radiated wave of this kind (sourced from the symmetric rather than the anti-symmetric part of the equation) will both carry away a positive energy in $t^{\mu\nu}$ as well as the same amount of energy with negative sign in $-\tilde{t}^{\mu\nu}$ about Minkowski resulting in a total vanishing radiated energy. Thus the DG theory, so far appears to be dramatically conflicting with both the indirect and direct observations of gravitational waves.

Actually, we shall show in the next two sections that, since the asymptotic behaviours of the two sides of the Janus field are not necessarily the same, we could both expect from the theory an isotropic solution approaching the GR Schwarzschild one with it's black hole horizon and the same gravitational wave solutions, including the production rate, as in GR but also, whenever some particular yet to be defined conditions are reached, the above DG solutions, with a vanishingly small production rate of gravitational waves and the B=1/A exponential DG Schwarzschild solution without horizon. Both will be limiting cases of a more general solution.

## 5. Differing asymptotic values

### 5.1. *The C effect*

Due to expansion on our side and contraction on the dark side the common Minkowskian asymptotic value of our previous section is actually not a natural assumption. At the contrary a field assumed to be asymptotically $C^2\eta_{\mu\nu}$ with C constant has its conjugate asymptotically $\eta_{\mu\nu}/C^2$ so their asymptotic values should differ by many orders of magnitude. Given that $g_{\mu\nu}^{C^2\eta} = C^2 g_{\mu\nu}^{\eta}$ and $\tilde{g}_{\mu\nu}^{\eta/C^2} = \frac{1}{C^2}\tilde{g}_{\mu\nu}^{\eta}$, where the $< g^\eta, \tilde{g}^\eta >$ Janus field is asymptotically $\eta$, it is straightforward to rewrite the local DG Janus Field equation now satisfied by this asymptotically Minkowskian Janus field after those replacements. Hereafter, we omit all labels specifying the asymptotic behaviour for better readability and only write the time-time equation satisfied by the asymptotically $\eta_{\mu\nu}$ Janus field.

$$C^2\sqrt{g}\frac{G_{tt}}{g_{tt}} - \frac{1}{C^2}\sqrt{\tilde{g}}\frac{\tilde{G_{tt}}}{\tilde{g}_{tt}} = -8\pi G(C^4\sqrt{g}\delta\rho - \frac{1}{C^4}\sqrt{\tilde{g}}\tilde{\delta\rho}) \qquad (12)$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $\delta\rho$ is the energy density fluctuation for matter and radiation. The tilde terms again refer to the same tensors except that they are built from the corresponding tilde (dark side) fields. Notice that for zero fluctuations, the solutions are Minkowskian as needed, being understood that the background plays it's dynamics in the global Janus field equation rather than in this local Janus field equation.

Then for $C >> 1$ we are back to $G_{tt} = -8\pi G C^2 g_{tt}\delta\rho$, a GR like equation for local gravity from sources on our side because all terms depending on the conjugate field become negligible on the left hand side of the equation while the local gravity from sources on the dark side is attenuated by the huge $1/C^8$ factor (in the weak field approximation, $G_{tt} = 8\pi G\frac{\delta\tilde{\rho}}{C^6}$). From $g_{\mu\nu}^{\eta}$ we then can get back $g_{\mu\nu}^{C^2\eta}$ and of course absorb the C constant by the adoption of a new coordinate system and redefinition of G, so for $C >> 1$ we tend to GR : we expect almost the same gravitational waves emission rate and the almost the same weak field gravitational field. However on the dark side everything will feel the effect of the anti-gravitational field from bodies on our side amplified by the same huge factor relative to the gravity produced by bodies on their own side.

The roles are exchanged in case $C << 1$. Then the GR equation $\tilde{G}_{tt} = -\frac{8\pi}{C^2}G\tilde{g_{tt}}\delta\tilde{\rho}$ is valid on the dark side while the anti-gravity we should feel from the dark side is enhanced by the huge $1/C^8$ factor relative to our own gravity (given in the weak field approximation by solving $\tilde{G}_{tt} = 8\pi G C^6\delta\rho$ for $\tilde{g}_{\mu\nu}$ from which we derive immediately our side $g_{\mu\nu}$ of the Janus field).

Only in case C=1 do we recover our local exponential Dark Gravity, with no significant GW radiations and also a strength of gravity ($G_{tt} = -4\pi G\delta\rho$) reduced by a factor $2C^2$ relative to the above GR gravity ($G_{tt} = -8\pi GC^2\delta\rho$).

It's important to stress that the phenomenology following from different asymptotic behaviours of the two faces of the Janus field here has no peer within GR in which a mere coordinate transformation is always enough to put the gravitational field in an asymptotically Minkowskian form in which a redefinition of the gravitational constant G gives back the usual gravitational potentials. This would still be possible in DG for one face of the Janus field but not for both at the same time. The new physics emerges from their relative asymptotic behaviour which can't be absorbed by any choice of coordinate system.

Eventually, depending on the local C value in a given space-time domain, a departure from GR predictions could be expected or not both for the gravitational waves radiated power and the local static gravitational field e.g. depending on a context able or not to trigger a reset to C=1, we could get either the DG exponential elements or the GR Schwarzschild solution for the static isotropic gravity; and get either no gravitational waves at all or the same radiated power as in General Relativity[g].

### 5.2. *Frontier effects*

We are here interested in specifying the kind of effects related to the occurrence of C and 1/C asymptotic gravity spatial domains and more specifically at the frontier between two such domains. Indeed we earlier emphasized that the scale factor permutation responsible for the transition to acceleration did not occur instantaneously over the whole universe but must have spread progressively implying regions already in the accelerated regime (1/C asymptotic) neighbouring regions still in the decelerating regime (C asymptotic). Let's assume a weak field so that we can for instance approximate the $g_{00}$ metric element by an exponential function. Let's assume we have point masses $M_1$ on our side and $M_2$ on the dark side, both being in the C domain (of our side metric). Then according the previous section results, we have :

$$g_{00} \approx C^2 e^{-G(C^2 M_1/r_1 - C^{-6} M_2/r_2)} \tag{13}$$

anywhere in the C domain at distance $r_1$ from $M_1$ and $r_2$ from $M_2$. This can be extended anywhere in a neighbouring 1/C domain by

$$g_{00} \approx C^{-2} e^{-G(C^2 M_1/r_1 - C^{-6} M_2/r_2)} \tag{14}$$

[g]For $C >> 1$ we also approximately recover the gauge invariance of GR

In other words the metric is simply renormalized by a constant factor at the frontier between two domains. Now let's assume we have two point masses, $M_3$ on our side and $M_4$ on the dark side, both being in the 1/C domain (of our side metric). Then we get:

$$g_{00} \approx C^{-2} e^{-G(C^{-6} M_3/r_3 - C^2 M_4/r_4)} \tag{15}$$

anywhere in this 1/C domain at distance $r_3$ from $M_3$ and $r_4$ from $M_4$. Again this can be extended anywhere in the neighbouring C domain by

$$g_{00} \approx C^2 e^{-G(C^{-6} M_3/r_3 - C^2 M_4/r_4)} \tag{16}$$

At last if we both have the previous two couples of masses we can merely combine the above results in the C domain to get:

$$g_{00} \approx C^2 e^{-G(C^2(M_1/r_1 - M_4/r_4) + C^{-6}(M_3/r_3 - M_2/r_2))} \approx C^2 e^{-G(C^2(M_1/r_1 - M_4/r_4))} \tag{17}$$

and in the 1/C domain to get:

$$g_{00} \approx C^{-2} e^{-G(C^2(M_1/r_1 - M_4/r_4) + C^{-6}(M_3/r_3 - M_2/r_2))} \approx C^{-2} e^{-G(C^2(M_1/r_1 - M_4/r_4))} \tag{18}$$

the last approximations being for $C >> 1$. We realize that in both domains the strengths of gravity and anti-gravity respectively from $M_1$ and $M_4$ are the same! The above combination reflects our intuition that the frontier surface behaves as a secondary source (Huygens principle) when it propagates (renormalizing it in passing) the field from one domain to the neighbouring one so that eventually in a given domain the fields from masses in any domains, non linearly mix just as in GR.

Now that we have clarified how the metric transforms at domain frontiers it just remains to clarify how the matter and radiation fields behave there. Just as the discontinuity in time of the scale factor triggering the acceleration of the universe had no effect on densities, the discontinuity in space from $C^2$ to $C^{-2}$ implied by the different normalization between the two domains (itself implied by the scale factors permutation) is again required not to affect the energy levels of particles crossing the frontier and their associated densities.

## 6. Back to Black-Holes and gravitational waves

Let's consider the collapse of a massive star which according to GR should lead to the formation of a Black Hole. As the radius of the star approaches the Schwarzschild radius the metric becomes singular there so the process lasts an infinite time according to the exterior observer. If the local fields both outside and inside the star have huge asymptotic C values, we already demonstrated that the gravitational equations tend to GR. However this can't be the case when we approach the Schwarzschild radius because C is finite and the metric elements can grow in such a way that we could not anymore neglect the dark side geometrical term. Therefore presumably the horizon singularity is avoided as well for C $\neq$ 1. To check this we need the exact differential equations satisfied in vacuum by C-asymptotic isotropic static metrics of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$. With $A = C^2 e^a$ and $B = C^2 e^b$, we get the differential equations satisfied by a(r) and b(r):

$$a'' + 2a' + \frac{a'^2}{p} = 0 \tag{19}$$

$$b' = -a' \frac{1 + a'r/p}{1 + 2a'r/p} \tag{20}$$

where $p = 4\frac{e^{a+b}C^4+1}{e^{a+b}C^4-1}$. GR is recovered for C infinite thus p=4. Then the integration is straightforward leading as expected to

$$A = (1 + U)^{p=4}; \tag{21}$$

$$B = (\frac{1 - U}{1 + U})^{(p=4)/2} \tag{22}$$

where $U = GM/2r$ and the infinite C can be absorbed by opting to a suitable coordinate system : then there is no dark side. DG C=1 corresponds to b=-a, p infinite and the integration, as expected, gives $A = e^U$, $B = e^{-U}$.

The integration is far less trivial for intermediary Cs because then p is not anymore a constant, however in the weak field approximation, treating p as the constant $4\frac{C^4+1}{C^4-1}$ the PPN development of the above solutions brings to light a possible departure from GR at the PostPostNewtonian level since:

$$A_{GR} \approx 1 + 4U + 6U^2 \tag{23}$$

$$B_{GR} \approx 1 - 4U + 8U^2 - 12U^3 \tag{24}$$

$$A_{p\neq 4} \approx 1 + pU + \frac{p(p-1)}{2}U^2 \tag{25}$$

$$B_{p\neq 4} \approx 1 - pU + \frac{p^2}{2}U^2 - p\frac{2+p^2}{6}U^3 \tag{26}$$

This makes clear that for $p \neq 4$ redefining the coupling constant to match GR at the Newtonian level, which amounts to replace U by 4U/p in the above expressions, a discrepancy would remain at the PPN level relative to GR predictions.

$$A_{p\neq 4} \approx 1 + 4U + 8(\frac{p-1}{p})U^2 \tag{27}$$

$$B_{p\neq 4} \approx 1 - 4U + 8U^2 - \frac{32}{3}(\frac{2+p^2}{p^2})U^3 \tag{28}$$

For $4 \leq p = 4\frac{1+1/C^4}{1-1/C^4} \leq \infty$ the departure from GR is the greatest for p infinite (C=1) :

$$A_{DG} \approx 1 + 4U + 8U^2 \tag{29}$$

$$B_{DG} \approx 1 - 4U + 8U^2 - \frac{32}{3}U^3 \tag{30}$$

but should hopefully soon become testable with the data from neutron stars or black holes mergers if C is not too big.

In the strong field regime we need to rely on numerical approximation methods to understand what's going on near the Schwarzschild radius. The numerical integration in Geogebra (using NRésolEquaDiff) was carried on and the resulting b(r) are shown in Figure 4 for various C values. It is found that as C increases b(r) will closely follow the GR solution near the Schwarzschild radius over an increasing range of b(r) which can be many orders of magnitude and perfectly mimic the GR black hole horizon, however at some point the solution deviates from GR and crosses the Schwarzschild radius without singularity. Therefore, as far as the numerical integration is reliable our theory appears to avoid horizon singularities (true Black Holes) for any finite C and not only C=1. This means that the collapsed star will only behave as a Black Hole for a finite time after which the external observer will be able to learn something about what's going on beyond the pseudo Horizon. Indeed, the resulting object having no true horizon is in principle still able to radiate extremely red-shifted and delayed light or gravitational waves emitted from inside the object.

The classical picture of a collapse toward a central singularity could therefore also be probed which is interesting because we have another mechanism within our framework that could stop the collapse: when the metric reaches some threshold, the inner region (the volume defined by the star itself) global and local fields could respectively be reset to Minkowski and C=1. This discrete transition would produce a huge discontinuity at a spherical surface with radius very close to the Schwarzschild Radius (because this is where the postulated metric threshold is expected to be reached). This surface would behave like the hard shell of a gravastar [44] and likely produce the same kind of phenomenological signatures such as echoes following BH mergers which might already have been detected [22].

Then at the center of such object, the two faces of the Janus field should get very close to each other just because C=1 and because this is where the own star potential vanishes. The crossing of the metrics is the required condition to allow the transfer of matter and radiation between the star and the conjugate side there. The lost of a significant part of its initial mass along with the strength of gravity being reduced by a factor 2C for DG relative to GR should eventually stop the collapse as it would allow new stability conditions to be reached.

To still behave as a very gravific object while it has lost most of it's matter and gravitational strength, the discontinuity itself must be gravific and behave as an equivalent gravific mass as the original one[h]. This is expected as the discontinuity is at a domain boundary and just needs to "store" the original value of the metric

[h]or an even greater gravific mass which then might lead to pseudo BHs much more massive than we believed them to be.

and it's derivative at the surface at the time it became this domain boundary. Then the external Schwarzschild type solution in vacuum is obtained merely thanks to these boundary conditions.

Shocks and matter anti-matter annihilation at the discontinuity (an excess of gamma radiation from our Milky Way giant black hole has indeed been reported [21]) which we remember is also a bridge toward the dark side and it's presumably anti-matter dominated fluid, could also produce further GWs radiation which would be much less natural from a regular GR Black Hole [22].

Eventually in the vicinity of stars as well as in "Black Holes" we can't exclude a transfer of matter and radiation through the discontinuity at crossing metrics that would proceed in the opposite way feeding them and increasing their total energy : a possible new mechanism to explain the unexpectedly high gravific masses of recently discovered BH mergers but also an attractive simple scenario to explain the six SN like enigmatic explosions of the single massive star iPTF14hls if they resulted from a succession of injections of antimatter from the dark side[40]. Such discontinuities in the vicinity of stars could also block matter accumulating in massive and opaque spherical shells around stars : a possible scenario to explain the reduced light signal from the recently discovered neutron stars merger.

Of course a Kerr type solution also remains to be established in our framework which is postponed for some future paper. But it is already clear that both conjugate metrics as well as the Minkowski metric in between them must be expressed in ellipsoidal coordinates (remind that our theory is generally covariant) hence in the form given by [45] Eq 21 for the Minkowski metric and Eq 22 or similar for the ensatz in input to our differential equations.

## 7. Primordial DG unification

### 7.1. *Problem statement*

At this point we have two separate theories, one that has the potential to describe all features of gravity except expansion, and a separate scalar-$\eta$ theory that challenges the LCDM evolution of the scale factor. However, at least in the linear domain, to reproduce all successes of the General Relativity theory of fluctuations not only inside the horizon but also on superhorizon scales we need equations of motion non trivially mixing background and perturbations just as in GR. Let's remind the first order cosmological perturbation GR equations for k=0 with the metric written in the Newtonian Gauge:

$$d\tau^2 = a^2(t)((1+2\Psi)dt^2 - (1-2\Psi)d\sigma^2) \tag{31}$$

The equations are : (4.4.169;4.4170;4.4.171 from [41]):

$$\nabla^2\Psi - 3H(\dot{\Psi} + H\Psi) = 4\pi G a^2 \delta\rho \tag{32}$$

$$\dot{\Psi} + H\Psi = -4\pi G a^2(\bar{\rho} + \bar{p})\mathbf{v} \tag{33}$$

$$\ddot{\Psi} + 3H\dot{\Psi} + (2\dot{H} + H^2)\Psi = 4\pi G a^2 \delta p \tag{34}$$

We must avoid to get similar equations because if we allow all the metric degrees of freedom to be dynamical, even if our dark side terms are small, our solutions will be asymptotically static as we realized earlier and we were obliged to introduce a dynamically restricted scalar-$\eta$ field just to be able to describe a non static background.

### 7.2. *The solution: emerging dynamics*

The concept of emerging dynamics will provide us with an elegant solution at least plainly valid and satisfactory as far as the physics of the very small fluctuations, tested through CMB studies, is concerned. The idea which is quite natural in a background dependent framework, is that some of the degrees of freedom which were frozen in the primordial metrics have only later gained their independence and have been released as dynamical dof either because the fluctuations became stronger than some threshold value or due to the scale factors differing from their initial value (at crossing point) by more than yet another fundamental threshold. We can actually already identify three metrics in our theoretical construction: the completely non dynamical background $\eta_{\mu\nu}$, the scalar-$\eta$ field which is a dynamically very limited metric having a single dof which is moreover constrained to be homogeneous, and the fully dynamical metric which degrees of freedom are all completely released in such a way that it's equations of motion constrain it to be asymptotically static. The idea of emerging dynamics is that there exists yet another dynamically intermediate metric between the last two defined as:

$$\Phi(t)\eta_{\mu\nu} + \Delta g_{\mu\nu}(\mathbf{r}, t) \tag{35}$$

where $\Delta g_{\mu\nu}(\mathbf{r}, t)$ stands for an in-homogeneous perturbation to the background but not yet a dynamical one in the sense that we shall still only require the action to be extremized by any variation of $\Phi(t)$ alone. We therefore again have a single scalar equation to be satisfied:

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \tag{36}$$

Now suppose we can write our metric in the Newtonian form (31) as in GR theory of cosmological fluctuations. From the zeroth order single equation (36) we then get the scale factor evolution equation (5) while the first order equation is all we need to describe the evolution of $\Psi(r, t)$. As we could check, neglecting dark side terms, this is unsurprisingly the same equation as the one obtained from combining the first and third equation of (34) to get $4\pi G a^2(t)(\delta\rho - 3\delta p)$ at the source. Because this equation is also valid within GR we obviously recover the same predictions for the evolution of $\Psi(r, t)$ as in the Standard Model in the linear regime of small fluctuations as far as the dark side terms can be neglected. However we need to keep in mind that this theory as well as GR in it's contracted form doesn't have enough equations to include modes other than the compressional ones described by

$\Psi(r,t)$. So the anisotropic dofs such as the radiative modes (gravitational waves) and rotational modes are not accounted for by such theory which fortunately only applies in the extremely weak field domain and therefore can only remain sustainable as long as B modes are not detected in the CMB.

We however need to justify the Newtonian metric form in DG. In GR it follows from neglecting anisotropic stress. In our case, even in the absence of anisotropic stress an equation is lacking which is eq 4.2.135 from [41]. For us a similar constraint originates from the way the dofs are frozen for our primordial metric to be in the most symmetrical form in our privileged coordinate system. Indeed, beyond the metric of the pure $scalar-\eta$ field, the next most symmetrical one we could consider is the metric in the isotropic form:[i]

$$d\tau^2 = a^2(t)(B(r,t)dt^2 + A(r,t)d\sigma^2) \tag{37}$$

All spatial coordinates are treated on the same footing in the expression of this metric and our additional extra constraint is the result of extending to space-time such kind of requirement on the form of the metric in our privileged coordinate system. This is achieved with the space-time exchange symmetry: a new kind of symmetry that was introduced and explained at length in section 6.2 of [3]. It implies that in our privileged coordinate system $B(r,t) = -A^{-1}(r,t)$[j]. Then our metric in the weak field approximation with $A(r,t) = 1 - 2\Psi(r,t)$ is just the same as the Newtonian Gauge metric.

### 7.3. *Space-time exchange symmetry breaking*

Eventually to understand the CMB fluctuations spectrum we need a unified field scalar equation (36) describing both the background and compressional fluctuations dynamics for an order two tensor field satisfying the space-time exchange symmetry in the privileged frame. This theory must be valid in the sufficiently weak gravity domain.

On the other hand, the observation of gravitational waves today means that the space-time exchange symmetry has been broken and that previously frozen dofs have emerged as truly dynamical field elements. The consequence is that the two separate theories that we have detailed earlier could actually only be valid at late times. We remind that one is the theory of the standalone scalar-$\eta$ homogeneous

[i]We are here interested in how the form assumed by the metric in our privileged coordinate system treats the various coordinates on the same footing rather than by isometries strictly speaking.

Moreover, if isotropy ensures the existence of a coordinate system in which the metric can be written in that simple isotropic form, there is also within DG the implicit understanding that this is as well the coordinate system in which the DG pivot metric satisfies $\eta_{\mu\nu} = (-1,1,1,1)$.

[j]Of course as it is written here, this is not a generally covariant constraint but we don't care as any non covariant equation can be considered to be the formed assumed by a generally covariant one in some particular coordinate system. Here we don't need to specify the generally covariant version of the equation as we shall remain in our privileged coordinate system.

field having it's own action and equation and the other only describes perturbations through a metric field which elements can all be varied independently hence have all the physical modes of GR (radiative, compressional, rotational) except it must remain asymptotically static in its domain of validity.

## 8. Late DG unification alternatives

### 8.1. *Unification through matter exchange*

It's worth writing the two Friedman type equations the conformal scale factor should satisfy for a gravitational field having all it's dofs dynamical:

$$a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{38}$$

$$\dot{a}^2 - \dot{\tilde{a}}^2 = 2K(a^4\rho - \tilde{a}^4\tilde{\rho}) \tag{39}$$

With $K = \frac{4\pi G}{3}$ the first one is of course also the equation 5 we got for the scale-factor within the scalar-$\eta$ theory. The time derivative of the second equation leads to:

$$a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{40}$$

$$a\ddot{a} + \tilde{a}\ddot{\tilde{a}} = K(a^4\frac{\dot{\rho}}{H} - \tilde{a}^4\frac{\dot{\tilde{\rho}}}{H} + 4\rho a^4 + 4\tilde{\rho}\tilde{a}^4) \tag{41}$$

with $H = \frac{\dot{a}}{a} = -\frac{\dot{\tilde{a}}}{\tilde{a}}$. The energy conservation equations on both sides being:

$$\frac{\dot{\rho}}{H} = -3(\rho + p) \tag{42}$$

$$\frac{\dot{\tilde{\rho}}}{\tilde{H}} = -\frac{\dot{\tilde{\rho}}}{H} = -3(\tilde{\rho} + \tilde{p}) \tag{43}$$

we can replace the corresponding terms in 41,

$$a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{44}$$

$$a\ddot{a} + \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) + \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{45}$$

then adding and subtracting the two equations we get the new equivalent couple of differential equations:

$$a\ddot{a} = Ka^4(\rho - 3p) \tag{46}$$

$$\tilde{a}\ddot{\tilde{a}} = K\tilde{a}^4(\tilde{\rho} - 3\tilde{p}) \tag{47}$$

which makes clear that the two equations are not compatible with $\tilde{a} = 1/a$ and any usual equation of state. However following an original ideal by Prigogin (see for instance [48] and multi-references therein) we can allow the gravitationally induced adiabatic creation or annihilation of particles on either side. Our above conservation equations then get modified[k]:

$$\frac{\dot{\rho}}{H} = (\frac{\Gamma}{H} - 3)(\rho + p) \tag{48}$$

$$\frac{\dot{\tilde{\rho}}}{\tilde{H}} = -\frac{\dot{\tilde{\rho}}}{H} = (\frac{\tilde{\Gamma}}{\tilde{H}} - 3)(\tilde{\rho} + \tilde{p}) \tag{49}$$

The next most natural assumption is to relate the creation rates through $\tilde{\Gamma} = -\Gamma$ (just as $\tilde{H} = -H$) in such a way that there is no actual creation or annihilation of particles but merely a transfer from one metric to the conjugate so that the baryonic number conservation is for instance globally insured. In [48] the creation/annihilation is done in such a way that the energy is covariantly conserved on the right side of the Einstein equation as required by the Bianchi identities: the energy is therefore somehow transferred from gravity to the created particles. This obviously requires that the energy momentum tensor at the source of Einstein equation be modified to include not only $\rho$ and p but also a creation pressure to be covariantly conserved. In our case the Bianchi identities are only approximately verified on the left hand side which implies that the right hand side can involve the energy-momentum conservation violating tensor (very weak violation when the ratio of the scale factors is very large) involving just $\rho$ and p alone. The adiabaticity is only a working assumption here allowing us to make use of the above relations from [48]: indeed the creation and annihilation rates are expected to be so small (at any time except near the origin of time) that an influx of particles with energies very different from the mean energy of particles in our universe should only disturb the thermodynamic properties of the cosmic fluid there in an almost negligible way.

Now replacing again in the differential equations and again adding and subtracting them we alternatively get:

$$a\ddot{a} = K(a^4(\rho - 3p) + \frac{1}{2}(C_r + \tilde{C_r})) \tag{50}$$

$$\tilde{a}\ddot{\tilde{a}} = K(\tilde{a}^4(\tilde{\rho} - 3\tilde{p}) + \frac{1}{2}(C_r + \tilde{C_r})) \tag{51}$$

including the creation/annihilation terms $C_r = a^4\frac{\Gamma}{H}(\rho + p)$, $\tilde{C_r} = \tilde{a}^4\frac{\Gamma}{H}(\tilde{\rho} + \tilde{p})$.

Now when our side density source terms dominate ($a^4 d >> \tilde{a}^4\tilde{d}$) where d (resp $\tilde{d}$) is any linear combination of densities $\rho$ and p (resp $\tilde{\rho}$ and $\tilde{p}$) alone, we just need $\frac{\Gamma}{H} << 1$ to recover from the first of these equations, the same evolution law of the scale factors we had before. The good new is that now the second equation can be compatible with this solution provided the $C_r$ term is dominant in the second equation : $\frac{\Gamma}{H} >> \frac{\tilde{a}^4\tilde{d}}{a^4 d}$. Then for instance in the matter dominated eras (before the

[k]The equations are as valid in conformal time as in standard time. The conformal time $\Gamma$ and H here are related to the standard time t' for our side metric $\Gamma'$ and H' according $\Gamma = a\Gamma'$ and $H = aH'$. The standard time being t" for the conjugate metric we also have $\tilde{\Gamma} = \tilde{a}\tilde{\Gamma}''$ and $\tilde{H} = \tilde{a}\tilde{H}''$

transition redshift ) on both sides, the equations simplify a bit:

$$a\ddot{a} \approx K a^4 \rho \tag{52}$$

$$\tilde{a}\ddot{\tilde{a}} \approx K \frac{a^4 \rho}{2} \frac{\Gamma}{H} \tag{53}$$

from which we get the required evolution of $\Gamma$:

$$\Gamma \approx 2H \frac{\tilde{a}\ddot{\tilde{a}}}{a\ddot{a}} = \frac{2H}{a^4} \left(\frac{1 - \frac{\dot{H}}{H^2}}{1 + \frac{\dot{H}}{H^2}}\right) \tag{54}$$

For a power law $a(t) \propto t^\alpha$ of the conformal scale factor,

$$\Gamma \approx \frac{2\alpha}{a^{4+1/\alpha}} \left(\frac{\alpha + 1}{\alpha - 1}\right) \tag{55}$$

is positive (transfer of particles from the conjugate to our side) for $\alpha > 1$ or $-1 < \alpha < 0$ and negative (transfer of particles from our to the conjugate side) otherwise. $\alpha$ positive (resp negative) translates to a decelerating (resp accelerating) universe in standard time t'. Hence in our cold matter dominated era before the transition reshift, $\alpha = 2$ implies that particles are transferred to the conjugate.

After the transition redshift, the conjugate scale factor dominates and roles are exchanged so:

$$\tilde{a}\ddot{\tilde{a}} \approx K \tilde{a}^4 \tilde{\rho} \tag{56}$$

$$a\ddot{a} \approx K \frac{\tilde{a}^4 \tilde{\rho}}{2} \frac{\Gamma}{H} \tag{57}$$

then,

$$\Gamma \approx 2H \frac{a\ddot{a}}{\tilde{a}\ddot{\tilde{a}}} = \frac{2H}{\tilde{a}^4} \left(\frac{1 + \frac{\dot{H}}{H^2}}{1 - \frac{\dot{H}}{H^2}}\right) \tag{58}$$

For a power law $a(t) \propto t^\alpha$ of the conformal scale factor, the sign of

$$\Gamma \approx \frac{2\alpha}{a^{-4+1/\alpha}} \left(\frac{\alpha - 1}{\alpha + 1}\right) \tag{59}$$

behaves as before but since now $\alpha = -2$ for the accelerating universe, this implies that particles are transferred from the conjugate to our side.

We see that DG equations can be solved for physically acceptable solutions, including a non static scale factor evolution even when all metric degrees of freedom are dynamical: for that we need to introduce the transfer of particles between the two conjugate metrics.

### 8.2. *Matter exchange or alternative mechanisms?*

The rate of matter exchange is, as we have seen, driven globally by the expansion rate but we would like to understand how this works locally. Adding the right specific new term in our local action coupling our to the dark sector as in ([32]) or

($^{47}$) should not do the job unless the new term is chosen explicitly non local and ad hoc.

Fortunately there is another more satisfactory way to address this issue in the sense that it would bring a better understanding of the physics behind matter-exchanges. If, for any reason, those transfer mechanisms were to be interrupted, the scale factor evolution would be frozen. This leads us to seriously consider the possibility that regions of our universe might indeed be completely frozen in a perfectly static background, all the more since, as we shall soon see, this is amazingly required by the most obvious interpretation of the Pioneer effect.

Following this idea, we may then have two kind of spatial domains. The evolving ones thanks to matter transfers and the frozen ones in which the metrics are asymptotically Minkowskian but rather in standard cosmological time coordinate (hence the expansion effects are switched off in such domains while their clock rates are not drifting with respect to clocks in the evolving domain). This is possible if high density regions, for instance about stars, cut-out of the rest of the expanding universe, implying a discontinuity at their frontier surface defining a new volume which is not anymore submitted to the expanding:

$$d\tau^2 = a^2(t)(dt^2 - d\sigma^2) = dt'^2 - a'^2(t')d\sigma^2 \tag{60}$$

cosmological metric ($d\sigma^2 = dx^2 + dy^2 + dz^2$), but to the new Minkowski metric.

$$d\tau^2 = a^2(t)dt^2 - C^2_{frozen}d\sigma^2 = dt'^2 - C^2_{frozen}d\sigma^2 \tag{61}$$

where $C_{frozen}$ stands for the reached value of the scale factor at the time it froze. This implies that in such domains our field equations apply but now with a new non dynamical Minkowski metric (1,-1,-1,-1) in standard time coordinate t' in between the Janus field metrics.

What is then new and crucial for us is that the domain of validity of the evolving background solutions according (60) has frontiers in such a way that all the local physics responsible for matter transfers may be taking place at those frontiers rather than in the bulk of the domain hence not requiring any additional coupling terms in our actions. We are of course strongly suspecting that the particle transfers could be taking place at our BH pseudo-horizons since this is where at least the $g_{00}$ elements of the conjugate metrics cross each other so this could be as well the frontier between an outside domain with evolving scale factor and the inside one with frozen scale factor.

However there is an even more fascinating alternative which would not require any actual transfer at all between our and the dark side. Indeed anything carrying energy-momentum crossing the frontiers of the evolving background domain on our side (resp on the conjugate side) could then contribute to the effective $\Gamma$ (resp $\tilde{\Gamma}$). And even more the frontiers could be dynamical, moving just in such a way as to contribute to these effective creation-annihilation operators as needed to insure the compatibility of our two cosmological differential equations. Then $\Gamma$ =- $\tilde{\Gamma}$ may be is no longer actually required.

The new question that arises then is what determines the density threshold for producing a frozen area and what determines the exact frontier of such domain. The answer might be related to quantum mechanics if the only contributors to the evolving domain are those particle wave functions that are dispersed rather than in their collapsed state. Indeed any object less than 1 micron (except may be a PBH) in the very rarefied intergalactic medium has a decoherence time more than 1 second (and more than 10 days for 0.1 micron particles) so that it's mass energy (we are following a realistic interpretation of QM) is most often diluted in a large volume insuring it should not represent a large fluctuation from the mean universe density which order of magnitude is atoms per cube meter. So most of the diffuse matter-energy in the form of gaz and dark matter should actually be in this un-collapsed state and would not produce frozen regions at the contrary to the collapsed forms of matter. At last any variation of the fundamental collapse triggering parameter will result in an increase or decrease of the fraction of energy matter in the evolving domain rather than in the static domains and then result in a contribution to the now effective $\Gamma$ and $\tilde{\Gamma}$. Eventually we are led to the fascinating idea that the physics of the QM wave function collapse is what could ultimately make possible the evolution of the scale factor in the Dark Gravity theory.

The existence of static domains is however not the solution to another problem that we did not already mention. At the transition from deceleration to acceleration regime of the universe the scale factors have exchanged their roles in such a way that the mean density of the dark side now leads the game because it is enhanced by a huge factor in equation 5. But, according what we explained earlier this also implies that any mass on our side should also have it's local gravitational field damped by a huge factor as it is now in the 1/C domain and corresponds to the $M_3$ kind of mass in equations (17) and (18). Certainly our earth, sun, and all stars of our galaxy do not belong to this type of mass as their gravity was never switched off and must still be of the $M_1$ kind of masses still in the C domain. So the question is : which ones are the actual energy-masses that must have flipped to the 1/C domain at the transition redshift resulting in switching off almost all the density of our side of the universe in the cosmological equation 5. The most natural answer to this question is that the part that flipped corresponds to the nearly homogeneous contributions of what we call dark matter whatever it is but also probably essentially most of, if not all of the diffuse intergalactic gas in the universe : the two contributions adding up to more than 99 % of the mass of the universe! As a result, from the transition redshift to now the gravific masses at work which effects we can probe in the universe are the fluctuations on the dark side (of type $M_4$) (we shall see in a next section that a void in that distribution can perfectly mimic a halo of dark matter on our side), but also the condensed forms of matter on our side (of type M1) : stars, planets...

Eventually the emergent picture of our universe is the following. At present our universe has regions still in the decelerated regime and others already in the accelerated expansion regime. In those regions that are still in the decelerating expansion regime, only the highly clustered forms of matter e.g. stars, planets, micro PBHs and may be up to even dust particles of a sufficient size are able to generate their own static domain of the scale factor evolution in their vicinity in which these can remain in the frozen regime described by (61). We shall later explore all the consequences and new related predictions among which the Pioneer effect as a natural outcome.

### 8.3. *Unification through discontinuities*

Because we want to understand the Pioneer anomaly, and for several other reasons discussed at the end of the previous section we are led to seriously consider that the static domains introduced in the previous section are real. These obviously require an alternative late DG unification mechanism which we shall detail now. In subsequent sections we shall focus on some of the very rich phenomenological related outcomes.

#### 8.3.1. *Actions and space-time domains*

We earlier explained why, after all previously frozen dofs have been completely released, anywhere we can't rely anymore on the matter exchange mechanism, the background can't evolve anymore. In such kind of space-time domain $D_{int}$ cut out from the expanding rest of the universe $D_{ext}$ we still have as usual the Einstein Hilbert (EH) action for the asymptotically Minkowskian Janus Field $g^{\eta}_{\mu\nu}$ added to SM actions for F and $\tilde{F}$ type fields respectively minimally coupled to $g^{\eta}_{\mu\nu}$ and $\tilde{g}^{\eta}_{\mu\nu}$ (the superscript here does not mean that the two sides of the Janus field are asymptotically identical but merely both asymptotically flat and static). However we may add to such action, an independent Einstein Hilbert action for a pure scalar-$\eta$ homogeneous and isotropic Janus field. The purpose of this action is just to extend to $D_{int}$ the background which dynamics was determined by extremizing the $D_{ext}$ action and solving the implied equations for the FRW ensatz to get the scale factor evolution $a(t)$. In other words in the $D_{int}$ action for the scalar-$\eta$ field the scalar field is not dynamical but enters as an external field which is just $a^2(t)$. Then the total action in $D_{int}$ is [1]:

$$\int_{D_{int}} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} + \tag{62}$$

$$\int_{D_{int}} d^4x(\sqrt{g}(R+L) + \sqrt{\tilde{g}}(\tilde{R}+\tilde{L}))_{g^{\eta}} \tag{63}$$

[1]There is may be one alternative possible way to obtain a background metric in $D_{int}$ in a fully dynamical way by adding source terms which densities would be averages over $D_{int}+D_{ext}$. Then the implied equations of motion for a dust universe, $\rho_{[D_{int}+D_{ext}]}/a^3_{[D_{int}+D_{ext}]}$=Const could still be compatible with $\rho_{D_{ext}}/a^3_{D_{ext}}$=Const, the scale factors $a_{[D_{int}+D_{ext}]}$ and $a_{[D_{ext}]}$ evolution being slightly different.

The advantage of adding a separate action for an independent background $\eta-scalar$ field in $D_{int}$ is not clear at this level because there is no shared field between the two kind of actions. The point is that $g^\eta$ is not only determined by its equations of motion. It could be asymptotically identical to any Minkowskian metric, for instance any of the form :

$$d\tau^2 = f^2(t)dt^2 - C^2 d\sigma^2 \tag{64}$$

in which the f(t) function is of course pure Gauge inside $D_{int}$ however it determines how clocks within $D_{int}$ may drift in time with respect to clocks in $D_{ext}$. Since f(t) is free as of now our purpose is to introduce a driving mechanism relating f(t) to a(t). We could just postulate these are equal to prevent the local clocks in $D_{int}$ to drift with respect to $D_{ext}$ clocks, however we are interested in a more involved mechanism actually allowing such drifts to occur at least momentarily as this would naturally produce Pioneer like effects. Our total action will be helpful just to later introduce such mechanism and establish a non trivial connection between f(t) and $a(t)$ at the frontier of $D_{int}$.

Instead of the always Minkowski metric of (64), in an earlier version of this work, we have been considering a metric of the kind

$$d\tau^2 = f^2(t)(dt^2 - d\sigma^2) \tag{65}$$

which is acceptable as long as f(t) would be a constant piecewise function of time. f(t) would be periodically discontinuously updated to $a(t)$ in such a way that it would closely follow the evolution of $a(t)$ through a series of fast discrete transitions on a regular basis. The idea is natural because f(t) is constrained to remain a mere integration constant C by the equations of motion in $D_{int}$ whereas it is also a boundary condition imposed at the boundary of $D_{int}$ requiring it to not remain constant but to actually evolve in time, for instance to follow the scale factor $a(t)$ from $D_{ext}$ so there are conflicting constraints on f(t). However the conflict can be solved if C can take different constant values in successive time slots, provided the actions and differential equations being only valid piece-wise i.e. only within those time slots. Only at the frontier between two such time slots or space-time domains do we need to apply new additional discrete rules to update the new C to the current value of the scale factor and accordingly to propagate the effect to all other physical quantities in $D_{int}$. The idea is fascinating because it just appears to be a genuine physically motivated quantization postulate that should shed light on the origin of quantum mechanics itself (remember that was one of our initial strongest motivations) [m] The quantization postulate however should be implemented carefully

[m] There is a striking analogy with what Quantum Field Theory actually describes : the succession of continuous local and discontinuous non local processes respectively described by the propagation of free fields according classical wave equations and the annihilation/creation of these fields wherever interactions take place, i.e. respectively propagators and vertices in the Feynman language. So our postulate is not at all a conceptually revolutionary one and we even feel tempted to name our discrete transition of C, a quantization rule even though it is quite an unusual one as it applies

to insure that the effect of the step by step evolving f(t) in a $D_{int}$ domain as for instance in our solar system will not be very different from those expected from GR. Indeed a naive implementation could lead to strongly excluded expansion effects of orbital planetary periods relative to atomic periods: the gravitational constant G would seem to vary at a rate similar to $H_0$ which is not the case[n].

In the following we shall stick to the always continuous evolution option of (64) rather than (65) but the results we shall obtain are also valid and straightforward to obtain in the other case. There is however an important difference, in one approach the metric is purely Minkowski in the solar system while in the other approach we would presumably (the full quantization program must be completed to get firm predictions) closely follow the predictions and expectations from GR with expansion effects only significant on scales beyond those of galaxy clusters and almost completely negligible but not strictly vanishing in the solar system.

### 8.3.2. *Field discontinuities*

If the mechanism which translates the $a(t)$ evolution into f(t) evolution is momentarily switched off, we expect a field discontinuity for the $g_{00}$ metric element at the frontier between a momentarily stationary scale factor domain $D_{int}$ and evolving outside $D_{ext}$ domain.

Let's stress that those new kind of discontinuities are not related at all to our permutation symmetry and the related discrete cosmological transition process that could trigger the acceleration of the universe. Now the usual conservation equations for matter or radiation apply when crossing such frontiers though in presence of genuine potential discontinuities. Indeed it's possible to describe the propagation of the wave function of any particle crossing this new kind of discontinuous gravitational potential frontier just as the Schrodinger equation can be solved exactly in presence of a squared potential well : we just need to require the continuity of the matter and radiation fields and continuity of their derivatives at such gravitational discontinuity. Since the differential equations are valid everywhere except at the discontinuity itself where they are just complemented by the former matching rules we obviously avoid the nuisance of any infinite potential gradients and eventually only potential differences between both sides of such discontinuity will physically matter. For instance we can now have have $(\rho a^3)_{before-crossing} = (\rho a^3)_{after-crossing}$ in contrast to what we had following the permutation transition ($\rho_{before-crossing} = \rho_{after-crossing}$).

to a zero frequency component in contrast to what we learned from the Planck-Einstein relations predicting vanishing quanta in the zero frequency limit.

[n]according to [28] "If G were to vary on a nuclear timescale (billions of years), then the rates of nuclear burning of hydrogen into helium on the main-sequence would also vary. This in turn would affect the current sun central abundances of hydrogen and helium. Because helio-seismology enables us to probe the structure of the solar interior, we can use the observed p-mode oscillation frequencies to constrain the rate of G variation." Again the relative variation of G at a rate similar to $H_0$ is completely excluded the precision being two orders of magnitude smaller.

### 8.3.3. *Space-time domains and the Pioneer effect*

The following question therefore arises: suppose we have two identical clocks exchanging electromagnetic signals between one domain submitted to the expanding $a(t)$ and another without such effect. The reader is invited to visit the detailed analysis in our previous publication [14] starting at page 71. We shall only remind here the main results. Electromagnetic periods and wavelengths are not impacted in any way during the propagation of electromagnetic waves even when crossing the inter-domain frontier. Through the exchange of electromagnetic signals, the period of the clock decreasing as $a(t)$ can then directly be tracked and compared to the static clock period and should be seen accelerated with respect to it at a rate equal to the Hubble rate $H_0$. Such clock acceleration effect indeed suddenly appeared in the radio-wave signal received from the Pioneer space-crafts but with the wrong magnitude by a factor two: $\frac{\dot{f}_P}{f_E} \approx 2H_0$ where $f_P$ and $f_E$ stand for Pioneer and earth clocks frequencies respectively. This is the so called Pioneer anomaly [11][12]. The interpretation of the sudden onset of the Pioneer anomaly just after Saturn encounter would be straightforward if this is where the spacecraft crossed the frontier between the two regions. The region not submitted to $a(t)$ (at least temporarily) would therefore be the inner part of the solar system where we find our earth clocks and where indeed various precision tests have shown that expansion or contraction effects on orbital periods are excluded during the last decades. Only the origin of the factor 2 discrepancy between theory and observation remains to be elucidated in the following sections as well as a PLL issue we need to clarify first.

### 8.3.4. *Back to PLL issues*

As we started to explain in our previous article [14] in principle a Pioneer spacecraft should behave as a mere mirror for radio waves even though it includes a frequency multiplier. This is because its re-emitted radio wave is phase locked to the received wave so one should not be sensitive to the own free speed of the Pioneer clock.

Our interpretation of the Pioneer effect thus requires that there was a failure of on board PLLs (Phase Lock Loop) to specifically "follow" a Pioneer like drift in time or even a failure that forced the analysis of the data in open loop mode. As for the first hypothesis, we already pointed out that nobody knows how the scale factor actually varies on short time scales: in [14] we already imagined that it might only vary on very rare and short time slots but with a much bigger instantaneous Hubble factor than the average Hubble rate. This behaviour would produce high frequency components in the spectrum which might have not passed a low pass filter in the on board PLL system, resulting in the on board clocks not being able to follow those sudden drifts. The on board clocks would only efficiently follow the slow frequency variations allowing Doppler tracking of the spacecrafts. Only when the integrated total drift of the phase due to the cumulative effect of many successive clock fast accelerations would reach a too high level for the system, this system would "notice" that something went wrong, perhaps resulting in instabilities and loss of lock at regular intervals [14]. This view would be even better supported if our clocks and rods are submitted to the scale factor evolution not continuously but rather through the succession of discontinuous steps we considered earlier. The failure of the PLL system is then even better understood for discontinuous variations of the Pioneer clock frequency with respect to the earth clock frequency. As a result, the frequency of the re-emitted wave is impacted by the Pioneer clock successive drifts and the earth system could detect this as a Pioneer anomaly.

#### 8.3.5. *Cyclic expanding and static regimes*

We are now ready to address the factor two discrepancy between our prediction and the observed Pioneer clock acceleration rate. We know from cosmology that, still in the same coordinate system, earth clocks must have been accelerating at a rate $H_0$ with respect to still standing electromagnetic periods of photons reaching us after travelling across cosmological distances (thus mainly in $D_{ext}$): this is nothing but the description of the so called cosmological redshift in conformal time rather than usual standard time coordinate.

On the other hand the Pioneer effect itself requires that not all regions have their clocks submitted to the same scale factor at the same time but some regions instead have their clocks drifting at rate $2H_0$ with respect to those from other regions.

This seems to imply that through cosmological times, not only earth clocks but also all other clocks in the universe, may have spent exactly half of the time in the $2H_0$ regime and half of the time in the static regime, in a cyclic way. It would follow that the instantaneous expansion rate $2H_0$ as deduced from the Pioneer effect is twice bigger than the average expansion rate (the average of $2H_0$ and zero respectively in the expanding and static halves of the cycle) as measured through a cumulative redshift over billions of years.

In our previous article we presented a very different more complicated and less natural explanation on how we could get the needed factor two which we do not support anymore. This article also discussed the expected field discontinuities at the frontier between regions with different expansion regimes, and likely related effects which we still support. Those discontinuities do not necessarily imply huge potential barriers even though the scale factors have varied by many orders of magnitude between the Big Bang and now. At the contrary they could be so small to have remained unnoticed as far as our cycle is short enough to prevent some regions to accumulate a too much drift relative to others. We are now at last ready, having introduced the main ideas, to detail the mechanism relating $f(t)$ to $a(t)$ in a $D_{int}$ domain.

## 9. Driving mechanism for f(t) and Frontier dynamics

- First postulate : A $D_{int}$ domain has it's own non dynamical Minkowski metric different from the non dynamical Minkowski metric in $D_{ext}$. This metric is just (61):

$$d\tau^2 = a^2(t)dt^2 - C_{frozen}^2 d\sigma^2 = dt'^2 - C_{frozen}^2 d\sigma^2 \tag{66}$$

instead of the $D_{ext}$ non dynamical Minkowski metric which is still :

$$d\tau^2 = dt^2 - d\sigma^2 \tag{67}$$

Obviously the dynamics of the background in $D_{ext}$ (the scale factor $a(t)$) is what determines the new non dynamical metric.

- Second postulate: The dynamical metric in $D_{int}$ is asymptotically successively:

$$d\tau^2 = D_{frozen}^2 dt^2 - C_{frozen}^2 d\sigma^2 \tag{68}$$

which is completely frozen and:

$$d\tau^2 = \frac{a^4(t)}{D_{frozen}^2} dt^2 - C_{frozen}^2 d\sigma^2 \tag{69}$$

in which clocks are found drifting at the double rate $2H_0$. $D_{frozen}$ in (69) stands for the last frozen value of $a(t)$ at the time the metric switched from (68) to (69). Of course $D_{frozen}$ has a new value at each cycle.
Therefore, in $D_{int}$ we have an alternate cyclic succession of what would seam to be the two sides of a new emergent Janus field about (66) except that at any time only one physically shows up and only as an asymptotic value of the $D_{int}$ dynamical field.
This field is of course always asymptotically Minkowskian at the contrary to the background of the Janus field in $D_{ext}$ just because this is required by the complete field equations in $D_{int}$ as we learned earlier. However as we also noticed earlier the asymptotic behaviour is not determined by those equations and as promised our postulates provide the needed constraints according to which $a(t)$ from $D_{ext}$ drives this asymptotic bahaviour.
The cyclic succession of (68) and (69) makes the $D_{int}$ dynamical field asymptotically evolve as (66) on cosmological times but this is a mean.
Of course the fact that metrics (68) and (69) look like the two sides of a new $D_{int}$ Janus field about (66) is not an accident. Presumably the existence of (69) is just the consequence of the existence of the other side (68) and (66) in between. In other words we have a kind of baby universe in $D_{int}$ which background is not (may be not yet) able to evolve by itself but which evolution is completely dictated by $D_{ext}$ according our postulates. Presumably the baby universe will eventually acquire it's full autonomy when the two sides really become the two sides of a genuine new dynamical Janus field starting it's own evolution according it's own action and derived field equations.

- Third postulate : In general the dynamical field is not necessarily asymptotically (68) or (69) in the whole domain $D_{int}$. Rather half of the time $D_{int}$

is in the static regime and the other half of the time the domain progressively passes in the double rate regime: when this occurs there is a domain frontier that scans the whole $D_{int}$: upstream (not yet reached area of) this propagating frontier we are still in the static regime while downstream all clocks have been synchronized and are in the double rate regime. At the end of the scan the whole $D_{int}$ is frozen again in the static regime for the next half cycle.
To describe this the action in $D_{int}$ is the one we have already written in (62) and (63) which we can rewrite now only retaining the double rate regime area in $D_{int}$ and the geometrical terms (the matter actions and static regime area play no role in the following so we drop them out hereafter just for the sake of conciseness)[o]:

$$\int_{D_{int:2H_0}} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} + \quad (70)$$

$$\int_{D_{int:2H_0}} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g^\eta} \quad (71)$$

Our third postulate is to require this action to be extremum i.e. stationary under any infinitesimal displacement of the hypersurface defined by the frontier of this action validity domain $D_{int:2H_0}$.

Our purpose is to understand the physics that governs the location of the frontier surface of $D_{int:2H_0}$ at any time. Of course determining it will at the same time determine the frontier of the complementary $D_{int:static}$ area. If such surface is moving it will of course scan a space-time volume as time is running out. Having extended the extremum action principle thanks to the third postulate allows to determine this hypersurface.

Indeed the arbitrarily displaced hypersurface might only differ from the original one near some arbitrary point, so that requiring the action variation to vanish actually implies that the total integrand should vanish at this point and therefore anywhere on the hypersurface. Eventually, anywhere and at any time at the domain boundary we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} + (\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g^\eta} = 0 \quad (72)$$

This equation is merely a constraint relating the Janus field gravity (terms 3 and 4) to the non dynamical metric (terms 1 and 2) at the hyper surface. Now provided one scale factor dominates the other side one we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} \approx \pm^{a>>\tilde{a}}_{a<<\tilde{a}}(\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} \quad (73)$$

[o]In the step by step Minkowskian alternative we would not need to introduce a new non dynamical Minkowski metric as is (61) since $a^2\eta$ that we already have is just what we need in that case.

and then we can make use of the contracted equation 4 to replace:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} \approx \pm_{a<<\tilde{a}}^{a>>\tilde{a}} 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T})_{g=a^2\eta} \tag{74}$$

in equation(72) and we can do the same for the $g^\eta$ part provided $C(t) = a^2(t)/D_{frozen}$ and $D_{frozen}$ dominate their inverse (the common order of magnitude of $C(t)$ and $D_{frozen}$ is simply named C hereafter). Then equation (72) becomes:

$$\pm_{a<<\tilde{a}}^{a>>\tilde{a}}(a^4 < \rho - 3p >_{ext} - \tilde{a}^4 < \tilde{\rho} - 3\tilde{p} >_{ext}) \tag{75}$$

$$\pm_{C<<\tilde{C}}^{C>>\tilde{C}}(C(t)\ D^3_{frozen}F(r)(\rho - 3p) - \tilde{C}(t)\tilde{D}^3_{frozen}\tilde{F}(r)(\tilde{\rho} - 3\tilde{p})) = 0 \tag{76}$$

The $F(r) = e^{2\Phi(r)}$ and $\tilde{F}(r) = e^{-2\Phi(r)}$ here account for the effect of a local assumed static isotropic gravitational potential $\Phi(r)$. The $<>_{ext}$ denote averages over $D_{ext}$. First and third terms involve a factor which currently has approximately the same magnitude as $a(t)$ in our cold side of the universe (even though third term is actually momentarily evolving at twice the rate of $a$ hence rather as $a^2$) while second and fourth terms involve a factor which currently has approximately the same magnitude as $\tilde{a}(t)$ (even though fourth term is actually momentarily evolving at twice the rate of $\tilde{a}(t)$ hence rather as $\tilde{a}^2(t)$) if the dark side is also in a cold matter dominated era.

The relative magnitudes of the local densities can be very different from the relative magnitudes of the averages $<>$ given the extremely non linear structures in the current universe. Is this enough to make the relative magnitudes of terms 1 and 2 in the opposite way to the relative magnitudes of terms 3 and 4 ? Unlikely at first sight given the huge expected current ratio $a(t)/\tilde{a(t)} \approx C(t)/\tilde{C(t)} \approx z^2_{crossing} >> 10^{18}$, if $z_{crossing}$ is the redshift of the conjugate scale factors equality probably much greater than the BBN redshift. Then as term 3 $>>$ term 4, just as term 1 $>>$ term 2 the equation today (with negligible pressures) simplifies to :

$$a^4 < \rho >_{ext} + C(t)D^3_{frozen}F(r)\rho = 0 \tag{77}$$

Such equation is satisfactory because the two terms don't evolve in the same way as a function of time: the first and second terms imply clocks drifting at rate $H_0$ and $2H_0$ respectively. So this can lead us to a trajectory r(t) for our hypersurface. Therefore, for instance in the external gravity of a massive spherical body, planet or star on our side, which radial a-dimensional potential is $\Phi(r) = -GM/rc^2$ and a quite uniform $\rho(r)$ so we may neglect it's radial dependency (for instance in the empty space surrounding a star), and using the fact that $C(t)$ momentarily evolves as $a^2(t)$ we are led to:

$$a(t) \propto e^{\frac{2MG}{rc^2}} \tag{78}$$

This equation gives us nothing but the "trajectory" r(t) of the hypersurface we were looking for. Here obtained in the conformal time t coordinate system, it is also valid in standard time t' coordinate since the standard scale factor and the "conformal scale factor" are related by $a(t) = a'(t')$. It is valid to PN order being understood that the exponential metric is here used for simplicity as a weak field PN approximation of a GR Schwarzschild solution rather than really the DG exponential Schwarzschild solution. This equation I=J implies $\dot{I}/I = \dot{J}/J$ so that:

$$H_0 = -2\frac{d\Phi}{dr}\frac{dr}{dt} \tag{79}$$

From this we learn that the frontier between the two domains is drifting at speed:

$$\frac{dr}{dt} = -\frac{1}{2}\frac{H_0}{[\frac{d\Phi(r)}{dr}]} \tag{80}$$

and therefore could involve a characteristic period, the time needed for the scale factor to scan $e^{\frac{2MG}{r}}$ from the asymptotic value to the deepest level of the potential at which point a new scan cycle is started. Thus we are able to understand both the Pioneer effect when we compare clocks in $D_{int:2H_0}$ and in $D_{int:static}$ but also the average $H_0$ expansion rate of the universe.

We may estimate an order of magnitude of the characteristic period of this cyclic drift assuming that the cycle is over when the frontier reaches the deepest potential levels. For collapsed stars such as white dwarfs or neutron stars this would give a far too long cycle exceeding billions of years because their surface potential is so deep and even much worse for black holes. But the majority of stars have very similar surface potentials even though there is a large variability in their masses and sizes. So we may take the value of our sun a-dimensional surface potential which is about $2.10^{-6}$ as indicative of a mean and common value. To that number we should add the potential in the gravitational field of the Milky Way and the potential to which the local cluster of galaxies is subjected. Knowing the velocities: 220 km/s of the sun about the center of the galaxy and 600km/s of the local cluster vs the CMB, the virial approximation formula $\frac{v^2}{c^2} \approx GM/rc^2$ may lead us to a crude estimation of each contribution and a total potential near $6.10^{-6}$. Then the order of magnitude of the cycle period would be in between $10^4$ and $10^5$ years[p].

## 10. The MOND phenomenology

As already pointed out DG crucially differs from GR in the way global expansion and local gravity work together. Any anomaly in the local physics of the solar system

[p]We also have a discontinuity for $g_{ii}$ metric elements (avoided however in the alternative step by step evolution scenario) because of presumably recently (i.e. not at high redshift) frozen $D_{frozen}$ but those can only produce hardly noticeable Shapiro delay or deflection of photons crossing it. It will prove interesting to check whether the implied distortions could explain the CMB low multipole anomalies (low quadrupole power and correlations with the ecliptic and galactic planes). Very much larger discontinuous barriers might however exist in the vicinity of compact star surfaces (white dwarfs, neutron stars or our pseudo Black Holes) because it takes much longer time for the scale factor to scan such star strong gravitational potentials so the expanding and stationary regions on either sides of the border can accumulate an extremely large relative drift relative to each other over such a long time.

or galaxy seemingly pointing to effects related to the Hubble rate is completely puzzling in the context of GR while it may be naturally explained within DG. Not only the Pioneer effect but also MOND phenomenology seem related to the $H_0$ value.

We derived in a former section the speed $\frac{dr}{dt} = -\frac{1}{2}\frac{H_0}{d\Phi(r)/dr}$ at which our local vs global frontier sitting at an isopotential between internal and external regions should radially propagate in the potential well of a given body. From this formula the speed of light $\frac{dr}{dt} = c$ is reached anywhere the acceleration of gravity equals $cH_0/2$. This appears to be the order of magnitude of the MOND acceleration and the corresponding radius even closer to the MOND radius beyond which gravity starts to be anomalous in galaxies [19][27]. Also remember that we assumed a radially uniform fluctuation to derive the speed formula for our hypersurface which amounts to consider that $d\Phi(r)/dr$ is its leading contribution so such estimation can only be very approximate. We are therefore tempted to suspect that something must be happening near the MOND radius due to frontier discontinuities propagating (and dragging matter) at a speed approaching the speed of light. Our best guess is that this is the radius beyond which our adiabatic particle exchanges allow a completely dynamical metric to take over. Another seemingly independent argument is that the mean universe density $\bar{\rho}$ should now be dominated by the conjugate one $\bar{\tilde{\rho}}$ by a $1.7^6 \approx 25$ factor given that equality was reached at the transition redshift $z \approx 0.7$. So at some distance from the center of galaxies we should also expect the local equality between $\tilde{\rho}(r)$ and $\rho(r)$ also defining a crucial radius as this is where the field asymptotic $C^2$ and $\frac{1}{C^2}$ must exchange their roles just because the cosmological permutation between $a(t)$ and $\tilde{a}(t)$ did not already take place below this radius. This, as we explained in a previous section would result in the gravitational field from the dark side in the region beyond such radius to be enhanced by a huge factor $C^8$ relative to the gravity due to our side matter in this region. Eventually this leads to a new picture in which only our side matter can be considered to be significantly gravific below the transition radius while only the dark side matter is significantly gravific beyond this radius. Then because a galaxy on our side implies a slightly depleted region on the dark side by it's anti-gravitational effects, even a slightly under-dense fluctuation[q] on the dark side would result in an anti-anti-gravitational effect on our side. This effect would exclusively originate from beyond the transition radius in such a way that it would be difficult to discriminate from the effect of a Dark Matter hallow! Also the most spectacular features of Dark Matter and MOND Phenomenology in galaxies such as galaxies that seem to be dominated at more than 99 percent by Dark Matter [20] or unexpectedly high acceleration effects in the flyby of galaxies [23] are more naturally interpreted in a framework where the gravitational

[q]though the dark side is in contraction which can boost growing of fluctuations especially on the largest scales, if it's density is similar to our side density at redshifts about 0.8, it's density was for instance much smaller at the time of the CMB emission so it's fluctuations must have started to grow significantly at much lower redshifts and much larger scales than on our side

effects from the hidden side are dominant beyond the MOND radius. At last, this would also mean that below the MOND frontiers the scale factor is still evolving according a decelerated expansion law.

## 11. Discrete symmetries, discontinuities and quantum mechanics

We earlier explained that in a theory with discrete symmetries having a genuine dynamical role to play, here global time reversal relating the two faces of a Janus field [6][13][14], discontinuities are expected at the frontier of space-time domains. All along this article we started to postulate various possible new discrete physical laws assumed to apply there: we can have discontinuous transitions in time when the conjugate scale factors exchange their roles, other kind of discontinuities in space at the frontier between static and expanding spatial regions, and in the expanding regions we might also postulate a succession of step by step discontinuous and fast periodic re-actualization of the local field piecewise constant asymptotic value allowing it to follow the evolution of the scale factor. We also already drew the reader attention to the harmlessness of discontinuous potentials as for the resolution of wave function equations in the presence of discontinuities. Of course the exploration of this new physics of discontinuities in relation to discrete symmetries is probably still at a very early and fragile stage and requires an open minded effort because it obviously questions habits and concepts we used to highly value as physicists.

Discontinuous and global fields as our scalar-$\eta$ field also put into question the validity of the Noether theorem implying the violation of local conservation laws wherever the new physics rules apply. However, we should remind ourselves that the most fundamental postulates of quantum physics remain today as enigmatic as they appeared to physicists one century ago: with the Planck-Einstein quantization rules, discontinuous processes came on to the scene of physics as well as the collapse of a wave function taken at face value obviously implies a violation of almost all local conservation laws. Based on these facts, a new theoretical framework involving a new set of discrete and non local rules which, being implied by symmetry principles are not anymore arbitrary at the contrary to the as well discontinuous and non-local quantum mechanics postulates, might actually be a chance. A real chance indeed as they open for the first time a concrete way to hopefully derive the so arbitrary looking quantum rules from symmetry principles and may be eventually relate the value of the Planck constant to the electrical charge, in other words compute the fine structure constant. We are certain that only our ability to compute the fine structure constant would demonstrate that at last we understand where quantum physics comes from rather than being only able to use it's rules like a toolbox.

In this perspective, it may be meaningful to notice that our Pseudo Black Hole postulated discontinuity at the pseudo horizon, which would lie at the frontier between approximate GR and DG domains, behaves as a wave annihilator for incoming GW waves and a wave creator for outgoing waves. In the DG domain the waves if any, carry almost no energy while in the GR domain they carry energy and momentum as usual. This is a fascinating remark because this would make it the only known concrete mechanism for creating or annihilating waves à la QFT or even a step toward a real understanding of the wave function collapse i.e. in line with a realistic view of quantum mechanics. Such collapse is indeed known to be completely irreducible to classical wave physics because it is non local, and in fact just as non local as would be a transition from GR $C >> 1$ to DG, C=1 in the inside domain. The latter transition is indeed non local because it is first of all driven by a transition of our global scalar-$\eta$ field which by definition ignores distances.

## 12. Stability issues about distinct backgrounds: C $\neq$ 1

### 12.1. *Stability issues in the purely gravitational sector*

Our action for gravity being built out of two Einstein Hilbert terms, each single one is obviously free of Ostrogradsky ghost. This also means that all degrees of freedom have the same sign of their kinetic term in each action.

There might still remain issues in the purely gravitational sector when we add the two actions and express everything in terms of a single dynamical field $g_{\mu\nu}$: everything is all right as we could demonstrate for C=1, but otherwise what we need to insure stability is that in the field equation resulting from the total action, all degrees of freedom will have their kinetic term tilting to the same sign. Again adopting $\bar{h}_{\mu\nu}$ from $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ and $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$ as the dynamical field puts forward that we have exactly the same quadratic (dominant) terms in $t_{\mu\nu}$ and $\tilde{t}_{\mu\nu}$ except that for $C > 1$ (resp $C < 1$) all terms in $t_{\mu\nu}$ are enhanced (resp attenuated) by a C-dependent factor while all terms in $\tilde{t}_{\mu\nu}$ are attenuated (resp enhanced) by a 1/C dependent factor, so that we will find in $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ all such quadratic terms tilting to the same sign, ensuring that the theory is still free of ghost in the purely gravitational sector.

Of course there remains an instability menace whenever $C \neq 1$ in the interactions between matters and gravity which we shall inspect now.

### 12.2. *Stability issues in the interactions between matter and gravity: the classical case*

Generic instability issues arise again when C is not anymore strictly equal to one. This is because the positive and negative energy gravitational terms $t^{\mu\nu}$ and $\tilde{t}^{\mu\nu}$ do not anymore cancel each other as in the DG C=1 solution. Gravitational waves are emitted either of positive or negative (depending on C being less or greater than 1) energy whereas on the source side of the equation we have both positive and negative energy source terms. Whenever two interacting fields (here the gravitational field and some of the matter and radiation fields) carry energies with opposite sign, instabilities would seem unavoidable (see [25] section IV and V for a basic description of the problem and [26] for a more technical approach) and the problem is even worsen by the massless property of the gravitational field.

Yet, the most obvious kind of instability, the runaway of a couple of matter particles with opposite sign of the energy, is trivially avoided in DG theories [4][7][8][5] [29][30][31][32][33][27] in which such particles propagate on the two different sides of the Janus field and just gravitationally repel each other.

It is also straightforward to extend the theory of small gravitational fluctuations to DG in the Newtonian approximation and neglecting expansion: the equations governing the decay or grow of compressional fluctuations are :

$$\ddot{\delta\rho} = v_s^2 \Delta\delta\rho + 4\pi G < \rho > (\delta\rho - \delta\tilde{\rho}) \tag{81}$$

$$\ddot{\delta\tilde{\rho}} = \tilde{v}_s^2 \Delta\delta\tilde{\rho} + 4\pi G < \tilde{\rho} > (\delta\tilde{\rho} - \delta\rho) \tag{82}$$

which in case the speeds of sound $v_s$ and $\tilde{v}_s$ would be the same on both sides allows to subtract and add the two equations with appropriate weights resulting in two new equations governing the evolution of modes $\delta^- = \delta\rho - \delta\tilde{\rho}$ and $\delta^+ = \delta\rho + \frac{<\rho>}{<\tilde{\rho}>}\delta\tilde{\rho}$.

$$\Box_s \delta^- = 4\pi G(< \rho > + < \tilde{\rho} >)\delta^- \tag{83}$$

$$\Box_s \delta^+ = 0 \tag{84}$$

Where $\Box_s$ is a fake Dalembertian in which the speed of sound replaces the speed of light. Because $\delta^+$ does not grow we know that $\delta\rho \approx -\frac{<\rho>}{<\tilde{\rho}>}\tilde{\delta\rho}$ and the two can grow according the growing mode of $\delta^-$. The complete study, involving different sound speeds, attenuation of gravity between the two sides and the effect of expansion (here represented by the evolution of C following the scale factor) will be the subject of the next section. It is already clear that in the linear domain anti-gravity by itself does not lead to a more pathological growth of fluctuations than in standard only attractive gravity: eventually we would expect the growth of a gravitational condensate on one side to proceed along with the corresponding growth of a void in the conjugate side and vice versa[r]. In other words our "instabilities" in the linear domain are nothing but the usual instabilities of gravity which fortunately arise since we need them to account for the growth of matter structures in the universe. These instabilities could be classified as tachyonic (the harmless and necessary ones for the formation of structures), non gradient (fortunately because those instabilities are catastrophic even at the classical level), and ghost (energy unbounded from below which is only catastrophic for a quantum theory) in the terminology of [36] reviewing various kind of NEC violations in scalar tensor theories which confirms that these are acceptable for a classical theory.

[r]The situation is less dramatic than Ref [25] section IV might have led us to think mainly because our leading order terms are linear in a gravitational field perturbation h whereas the leading order coupling term is quadratic in the lagrangian (22) of [25] leading to equations of motion of the form $\ddot{\Psi} \propto \Psi^3$.

From this it appears that DG is not less viable than GR in the linear domain as a classical theory and that the real concern with all DG models proposed to this date will actually arise for the quantized DG theories for which ghost instabilities are of course prohibitive, and may be in the strong field regime for the classical theories. Only then the real energy exchange between the gravitational field itself (it's kinetic energy quadratic terms) and other fields kinetic energies should start to become significant relative to the Newtonian like energy exchange between kinetic energy of the fields and their gravitational potential energy that drives the evolution of the compressional modes according Eq 81 and 82. In the strong field regime the problem is thus related to the radiation of gravitational waves when they are carrying non zero energy (for C≠1) while they can couple to matter sources with both positive and negative energies[s].

However, we expect that high density regions produced by compact objects on our side are always in the $C > 1$ domain (remind that the scale factors hence C permutation is triggered at the crossing of densities i.e. wherever the conjugate side density starts to dominate our side density) so that the interaction between this matter and the positive energy gravitational field (due to $C > 1$) is not a ghost interaction. For the same reason, high density regions produced by compact objects on the dark side are expected to remain in the $C < 1$ domain so that the interaction between the dark side negative energy (from our point of view) matter and the negative energy gravitational field (due to $C < 1$) is again not a ghost interaction. Eventually the only remaining ghost interactions with the gravitational field could be those from density fluctuations too small to locally flip the sign of C-1 in the safe direction, but these fluctuations do not produce strong gravity and therefore are not problematic, all the more since their gravity is expected to be suppressed by a huge $C^8$ factor.

### 12.3. *Stability issues in the interactions between matter and gravity: the quantum case*

#### 12.3.1. *Problem statement*

The next step is therefore to try to understand how we might solve stability issues in the quantum case. In the quantized theory the problematic couplings would produce divergent decay rates by opening an infinite space-phase for for instance the radiation of an arbitrary number of negative energy gravitons by normal matter

[s]This remains true even when great care is being taken to avoid the so-called BD ghost in the massive gravity approach particularly when the perturbations of the two metrics about a common background have different magnitudes i.e. when one parameter of the couple $\alpha$, $\beta$ dominates the other in [31]. By the way there is a much worse problem in models having two independent differential equations instead of one to describe the dynamics of two fields assumed independent, i.e. not related from the beginning by a relation such as Eq (1). Then the energy losses through the generation of gravitational waves predicted by each equation are different so that such models are inconsistent [4][7][8][5] [29] as shown in [15].

(positive energy) particles. To avoid such instabilities may be the most natural way would be to build the quantum Janus field operator also as a double-faced object, coupling it's positive energy face to usual positive energy particles and it's negative one (from our side point of view) to the negative energy particles (from our side point of view) of the dark side thereby avoiding any kind of instabilities. However the picture described by our classical Janus field equation which in principle really allows the direct exchange of energy between GW (with a definite sign of the energy depending on C $> 1$ or $C < 1$) and matter fields with different signs of the energy does not actually fit into such quantization idea. The most straightforward way to avoid such fatal quantum instabilities is to consider that the gravity of DG is not a quantum but remains a classical field. Semi-classical gravity indeed treats matter fields as being quantum and the gravitational field as being classical, which is not problematic as far as we just want to describe quantum fields propagating and interacting with each others in the gravity of a curved space-time (within GR) considered as a spectator background. To describe the other way of the bidirectional dialog between matter and gravity i.e how matter fields source gravity, semi-classical gravity promotes the expectation value of the energy momentum tensor of quantum fields as the source of the Einstein equation.

### 12.3.2. *The Janus field and the Quantum*

One often raised issue with semi-classical gravity is that this is incompatible with the Multi Worlds Interpretation (MWI) of QM since within the MWI the other terms of quantum superpositions which are still alive and represent as many parallel worlds would still be gravific as they contribute to the energy momentum tensor expectation value and should therefore produce large observational effects in our world. The MWI, considered as a natural outcome of decoherence is adopted by a large and growing fraction of physicists mainly because is considered the only alternative to avoid the physical wavefunction collapse. For this reason incompatibility with the MWI is often deemed prohibitive for a theory. Since we have nothing against a physically real wave function collapse (our theory even has opened new ways to hopefully understand it; discontinuity and non locality are closely related) we are not very sensitive to such argument. The wave function collapse might eventually be triggered at the gravitational level: a simple achievement of something similar to the Penrose idea (gravitationally triggered collapse) seems within reach in our framework, thanks to a transition to C=1 which is tantamount to a gravitational wave collapse. We are all the more supported in considering semi-classical gravity and the Schrodinger-Newton equation it implies [38] as the correct answers, as the usual arguments based on the measurement theory often believed to imply that gravity must be quantized have recently been re-investigated in [37] and the authors to conclude that "Despite the many physical arguments which speak in favor of a quantum theory of gravity, it appears that the justification for such a theory must be based on empirical tests and does not follow from logical arguments alone." This has even reactivated an ongoing research which has led to experiment proposals to test predictions of semiclassical gravity, for instance the possibility for different parts of the wave functions of a particle to interact with each other non linearly according classical gravity laws. However "together with the standard collapse postulate, fundamentally semi-classical gravity gives rise to superluminal signalling" [37] so the theoretical effort is toward suitable models of the wavefunction collapse that would avoid this superluminal signalling. From the point of view of the DG theory this effort is probably unnecessary because superluminal signalling would not lead to inconsistencies as long as there exists a unic privileged frame for any collapse and any instantaneous transmission exploiting it. We indeed have such a natural privileged frame since we have a global privileged time to reverse, so it is natural in our framework to postulate that this frame is the unic frame of instantaneity. Then the usual gedanken experiments producing CTCs (closed timelike curves) do not work any more : the total round trip duration is usually found to be possibly negative only because these gedanken experiments exploit two or more different frames of instantaneous signaling. Let's be more specific : Does instantaneous hence faster than light signalling unavoidably lead to causality issues ? : apparently not if there is a single unic privileged frame where all collapses are instantaneous. Then i (A) can send a message to my colleague (B) far away from me instantaneously and he can send it back to me also instantaneously still in this same privileged frame using QM collapses (whatever the relative motions and speeds of A and B and relative to the global privileged frame): the round trip duration is then zero in this frame so it is zero in any other frames according special relativity because the spatial coordinates of the two end events are the same: so there is no causality issue since there is actually no possible backward in time signalling with those instantaneous transmissions... in case there is some amount of time elapsed between B reception and re-emission, eventually A still receives it's message in it's future: no CTC here.

## 13. Evolution of fluctuations

### 13.1. *Evolution equations for negligible dark side gravity*

This is also the linear domain of the fluctuations and we already explained why it is straightforward to recover the same phenomenology as in LCDM even on superhubble scales as far as the compressional modes only are concerned.

We already pointed out that the evolution of the background before the transition to acceleration seems to require Dark Matter just as in the standard model to reach the cosmological critical density implied by k=0 and the measured value of the Hubble expansion rate. Presumably, this Dark Matter does the same good job as within LCDM to help the formation of potentials already in the radiative era and then thanks to these potentials the growth of baryonic fluctuations falling into these potentials. We then have potentially all the successes of CDM phenomenology on the largest scales with the bonus that we have a new natural candidate for Dark Matter and shall present it in an upcoming section.

### 13.2. *Evolution equations with dark side gravity*

Thus it remains to investigate whether the conjugate fluctuations from the dark side could now add new contributions on the smaller scales of galaxies to have the additional successes of MOND phenomenology there.

The dark side is also in a cold state with the same density as on our side at the transition redshift, but in contraction and therefore having started from a very low and presumably highly homogeneous mean density at z=1000. Therefore the radiative era is essentially the same as in LCDM (we have no effects related to the dark side at this epoch) and for instance we naturally have almost the same sound horizon even though a true singularity is avoided at t=0.

The dark side fluctuations could of course be boosted by the contracting scale factor especially on the largest scales but since the mean density was extremely small at high redshift with $\tilde{\rho} \approx z^{-6}\rho \ = 10^{-18}\rho$ at $z \approx 1000$, it is obvious that the growth of our side fluctuations starting from $\frac{\delta\rho}{\rho} \approx 10^{-5}$ of the CMB, could not be helped at high z. At low z, on the other hand, it is the weakness of the source term $\tilde{a^4}\tilde{\rho} \propto 1/a$ relative to $a^4\rho \propto a$ which makes the gravity from the dark side negligible with respect to our side matter gravity.

So we entirely need to rely on the extremely efficient new mechanism we introduced in the section devoted to the MOND phenomenology to see the gravitational effect of dark side fluctuations (voids) starting to play a significant role and produce the MOND empirical laws in galaxies.

What's indeed really new is that in accordance with what we also explained earlier each fluctuation has two regions : one central region where the gravity from our side $\delta\rho_{in}$ is hugely enhanced over the gravity from the dark side $\delta\tilde{\rho}_{in}$ and a peripheral one where at the contrary it is the gravity from the dark side $\delta\tilde{\rho}_{out}$ that hugely dominates that from $\delta\rho_{out}$. Moreover the permutation of the scale factors results in the same strength for $\delta\rho_{in}$ and $\delta\tilde{\rho}_{out}$ gravity in each equation.

As in LCDM, for the evolution of fluctuations the background evolution only becomes important in the matter dominated era arising as usual as an additional friction term $H\dot{\delta\rho}$ where H is the Hubble rate. So we can readily rewrite Eq (81) and (82) taking into account all non negligible effects depending on the scale factor.

$$\ddot{\delta\rho} + H\dot{\delta\rho} = 4\pi G < \rho > a^2(\delta\rho_{in} - \delta\tilde{\rho}_{out}) \tag{85}$$

$$\ddot{\delta\tilde{\rho}} - H\dot{\delta\tilde{\rho}} = 4\pi G < \tilde{\rho} > a^2(\delta\tilde{\rho}_{out} - \delta\rho_{in}) \tag{86}$$

We see that the interaction between the dark side and our side fluctuations can only be significant when $\tilde{\rho}$ is not too much smaller than $\rho$. So MOND like phenomenology would not be expected to arise well before the transition redshift. After the transition redshift on the other hand, even small fluctuations $\tilde{\delta\rho}$ in the dark side distribution relative to the dark side average density can lead to gravific effects much larger than what our side fluctuations $\delta\rho$ are able to do and all the effects of their dominant gravity is probably wrongly attributed to Dark Matter Halos within LCDM.

### 13.3. ***Cosmological Dark Matter reinterpretation***

We already pointed out that $H^2(t) \approx \frac{8\pi G\rho}{3}$ is also (this is an approximate version of the GR exact second Friedmann equation) valid according the DG cosmological equation (5) provided our side scale factor dominates the dark side one for $p \approx 0$. Therefore baryonic matter is, just as within GR, cosmologically not abundant enough to account for the measured Hubble rate, and we still need a "Dark Matter" cosmological density $\bar{\rho}_{DM}$.

#### 13.3.1. *Pseudo BH as DM candidates ?*

Primordial Black Holes (PBH) were recently considered a possible candidate for Dark Matter because these are collisionless, stable, and at least until recently, not yet completely ruled out by astrophysical and cosmological constraints. But much more likely is the possibility that the Dark Matter is made of our pseudo Black Holes and their remnants after death i.e. after the transition to C=1 we described earlier as a mechanism to stop the collapse to singularity. Indeed we could have many of these compact objects just as gravific as the, primordial or not, pseudo BH they originated from except that it's now their discontinuity that is gravific. Just as Black Holes, these could exist in any size but in principle the smallest ones would not escape the main observational constraint on the fraction of PBH: the Hawking radiation flashes (these would however evade detection by their microlensing signature, too small to be detected for small objects) expected to be the same for our pseudo black holes as for true black holes. This is however questionable if our PBHs can vanish i.e. completely disappear from our side point of view as their matter content is transferred to the conjugate side before the emission of the Hawking flash. Even in that case however those PBH could presumably not contribute a significant part of Dark Matter because if their masses spread over a range which upper bound is $10^{11}$kg (at this mass their lifetime is comparable to the age of our universe), as they are transferred to the dark side in a lifetime smaller than the age of the universe we would see the density on our side significantly deviating from the $\rho \approx 1/a^3$ conservation law. On the other hand if their mass range extends beyond $10^{11}$ kg they will not escape the exclusions from lensing experiments.

The exception is if for a yet unclear fundamental reason, it is the transition from deceleration to acceleration that triggered the transfer of matter from most primordial Massive Pseudo BH from our side to the dark side. Then not only the low abundance of these objects on our side today would be natural but as well the absence of detectable DM in nowadays universe. In that case we would need to explain all local effects usually attributed to DM, now rather by voids in the distribution of the dark side universe as we explained earlier. This scenario would also produce a much more accelerating effect resulting from the transition redshift. This is again an interesting alternative idea to help explain a recent acceleration higher than expected if this anomaly (recently measured high $H_0$ value) were to be confirmed.

#### 13.3.2. *Micro lightning balls as DM candidates ?*

In previous papers we also described objects called micro lightning balls (mlb) that would also be collisionless in their collapsed state (they would "decouple" from the baryon photon fluid due to their small "cross-section") and deserve much attention since these as well might be perfect Dark Matter candidates. Some of those objects, as well as pseudo BH, might have been created as the result of density fluctuations producing a gravitational potential rising above a fundamental threshold triggering the discontinuous potential trapping and stabilizing the object. Some are likely to behave as miniature stars, presumably as dense and cold as black dwarfs and extremely difficult to detect either through their black body radiation of an extremely cold object, their gravitational lensing given their surface gravity much smaller than that of a pseudo Black Hole of the same size and the absence of Hawking radiation even for the smallest of these objects. Of course a much more detailed characterization of long living micro lightning balls would be needed to make firm predictions as for both their spatial and mass distribution and the best way to detect them.

The intriguing possibility that our mlbs constitute dark matter is supported by figure 3 from [46] summarizing all existing constraints on the existence of Macros (massive Dark matter objects) possibly made of standard model particles assembled in a high density object (from beyond atomic to well beyond nuclear densities). Presumably this high density form of matter could have been injected in our universe in it's radiation dominated era (hence with a negligible influence on the scale factor evolution at this epoch) from pseudo black holes and compact stars of the dark side which was very cold at this time.

#### 13.3.3. *Heavy elements baryonic matter as DM candidate ?*

But figure 3 from [46] also leaves open the possibility that Dark Matter could be made of matter with usual atomic densities and heavy elements if this was again injected from the conjugate side Pseudo Black Holes in our radiative era (which also corresponds to the beginning of the contraction history of the dark side following a very long expansion era having resulted in a dark side universe in which most of the matter had been swallowed by Pseudo Black Holes). Then the distribution of this injected baryonic with high metallicity DM is expected to have been extremely inhomogeneous in our radiative era because highly concentrated on spots, much smaller than the Planck resolution, making their detection hardly possible [t]. This concentration of DM in spots with very high metallicity is needed to make the idea viable as otherwise we would hardly understand why the universe is almost everywhere we look nowadays at a very low level of metallicity (compatible with

[t] However, once dispersed more or less isotropically it might produce rings or spiral structures behaving as a new kind of CMB foreground. Penrose and colleagues have recently claimed again to be able to detect ring like signatures in the CMB, and it is natural in our framework to try to interpret them as features linked to injections of matter from the dark side.

the predictions of Big-Bang nucleosynthesis and stellar nucleosynthesis) both in the diffuse intergalactic gas as well as in stars. If this hypothesis is true the corresponding high metallicity and possibly dark regions remain to be discovered. The high metallicity is also required to insure that this matter has a low ratio charge over mass making it much less dragged by the primordial acoustic fluctuations and then contributing to DM rather than normal baryonic matter from the analysis of the CMB spectrum.

At last it is worth mentioning that discontinuities might have helped the fast formation of stars in general and large mass ones in particular leading to many large mass pseudo BH such as the ones recently discovered by Ligo or giant black holes at the centers of large galaxies. This is because the dragging effect of drifting discontinuities is presumably an effective mechanism to concentrate matter at all scales or to merge already formed pseudo BHs or their remnants.

Late Pseudo BHs as well as primordial ones might have resulted in a drop of the total detectable baryonic matter (the matter which is still free i.e. not captured by pseudo-BH, their remnants or micro lightning balls) hence a missing baryons effect.

### 13.4. *Frame dragging and gravitational waves anomalies*

We realize at this point that the reason why gravity loses it's GW and rotational degrees of freedom in the past (in the very homogeneous primordial universe) must also be related to the equivalent lost of those same degrees of freedom in the far future, near the end of the contracting phase. Indeed the contracted side of the universe is then as well expected to be very homogeneous on most scales except the smallest ones: those of remaining BH themselves after these have swallowed almost all of the available matter around...

Again the transition from a space-time regime in which the gravitational field possesses all its usual degrees of freedom to a regime in which the GW and rotational dofs are lost, and whatever the direction of this transition, must be spatially progressive: so we again must have drifting discontinuities in space at the frontiers between such regions.

We recently have been able to receive gravitational wave signals from far away sources, implying that most of the universe in which those GWs propagated is in the highly inhomogeneous regime allowing the propagation of these dofs.

However some areas might already have been able to switch to the future regime of a highly homogeneous space-time unable to propagate GWs or manifest frame dragging effects. This is expected for regions that are sufficiently empty of matter both on our and the conjugate side. Such regions can occur just because of the presence of a massive body in the vicinity of this body on our side just because the body efficiently repels matter from the dark side and at the same time also has already efficiently cleaned it's surrounding space through the accretion of all nearby matter on our side. Therefore the body is able to generate a high vacuum in it's vicinity on both sides.

Our bold hypothesis is that in the vicinity of stars or even planets such conditions could already have been reached very locally, triggering the vanishing of any kind of frame dragging or GW effects. This vanishing could even be transient and we are tempted to interpret the zero frame dragging effect which was initially observed by Gravity probe B on one of its four gyroscopes as evidence for this. See our section 12 devoted to gravitomagnetism and preferred frame effects in [3] for further details.

## 14. Last remarks and outlooks

We already pointed out that none of the faces of our gravitational Janus field could be seriously considered as a candidate for the spacetime metric. Yet, though the gravitational field loses this very special status (be the spacetime metric) it had within GR, it acquires another one which again makes it an exceptional field : it is the unic field that makes the connection between the positive and negative energy worlds (this definition is relative: for any observer the negative field is the one that lives on the other side), the only one able to couple to both the dark side SM fields and our side SM fields. This special status alone implied that the gravitational interaction might need a special understanding and treatment avoiding it to be quantized as the other interactions. Avoiding ghost instabilities related to the infinite phase space opened by any interaction between quantum fields that do not carry energies with the same sign, is a requirement which also confirms that the gravitational Janus field in Eq (1) and (2) could not interact with matter as a quantum field. So the old question whether it is possible to build a theory with a classical gravitational field interacting with all other fields being quantum, was back to the front of the stage just because the usual answer "gravity must be quantized because everything else is quantum" fails for the Janus theory of the gravitational field.

Another point that deserves much attention is that within DG, wherever the two faces of the Janus field are equal, vacuum energy terms trivially cancel out as we already noticed in [14] so we have good reasons to suspect that a mechanism is at work to insure that this cancellation is preserved even when the two faces depart from each other. First, cosmological constant terms are strictly constant within GR because of the Bianchi identities which is not necessarily the case in DG. Such terms might vary (because of varying cutoffs for instance) in order to preserve the cancellation between our side and the dark side vacuum energy terms. The context is anyway much more favourable than within GR where no such kind of cancellation could possibly occur.

At last, the issue of CTCs (closed timelike curves) is worth a few more words: in the context of GR it is known that a necessary condition to avoid CTCs is to ban negative energies at the source of Einstein equation (Hawking theorems). It is therefore interesting that in the limit of infinite C, in which DG tends to GR, negative energy terms also tend to decouple at the source. It is therefore left as an open mathematical problem whether for finite C values, the modification of the geometrical part of DG equations vs Einstein equations is just what we need to still avoid CTCs even in presence of negative energy source terms.

## 15. Conclusion

New developments of DG not only seem to be able to solve the tension between the theory and gravitational waves observations but also provide a renewed and reinforced understanding of the Pioneer effect as well as the recent cosmological acceleration. An amazing unification of MOND and Dark Matter phenomenology seems also at hand. The most important theoretical result is the avoidance of both the Big-Bang singularity and Black Hole horizon.

# Appendices

Field equations derivation

To get our field equation we demand that the action variation $\delta S$ should vanish under any infinitesimal variation $\delta g_{\mu\nu}$. But the variation of $g_{\mu\nu}$ implies a variation of $\tilde{g}_{\mu\nu}$ resulting in the following variation of the total action integrand which must vanish:

$$\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu})\delta g_{\mu\nu} + \sqrt{\tilde{g}}(\tilde{G}^{\mu\nu} + 8\pi G \tilde{T}^{\mu\nu})\delta \tilde{g}_{\mu\nu} = 0 \tag{87}$$

The variations are related by

$$\delta \tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}\delta g^{\rho\sigma} = -\eta_{\mu\rho}\eta_{\nu\sigma}g^{\rho\tau}g^{\sigma\kappa}\delta g_{\tau\kappa} \tag{88}$$

since the Minkowski metric not being dynamical, does not vary. Replacing in 87, we get :

$$\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu})\delta g_{\mu\nu} - \sqrt{\tilde{g}}(\tilde{G}^{\mu\nu} + 8\pi G \tilde{T}^{\mu\nu})\eta_{\mu\rho}\eta_{\nu\sigma}g^{\rho\tau}g^{\sigma\kappa}\delta g_{\tau\kappa} = 0 \tag{89}$$

Or, after a convenient renaming of the indices $(\mu, \nu) \leftrightarrow (\tau, \kappa)$ in the second term:

$$\left[\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu}) - \sqrt{\tilde{g}}(\tilde{G}^{\tau\kappa} + 8\pi G \tilde{T}^{\tau\kappa})\eta_{\tau\rho}\eta_{\kappa\sigma}g^{\rho\mu}g^{\sigma\nu}\right]\delta g_{\mu\nu} = 0 \tag{90}$$

The resulting single equation of motion can be reshaped in a more elegant form multiplying it by $\eta^{\delta\lambda}g_{\delta\mu}$, and using $\eta_{\kappa\sigma}g^{\sigma\nu} = \eta^{\sigma\nu}\tilde{g}_{\sigma\kappa}$ (inverse metrics).

$$\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu})\eta^{\delta\lambda}g_{\delta\mu} - \sqrt{\tilde{g}}(\tilde{G}^{\lambda\kappa} + 8\pi G \tilde{T}^{\lambda\kappa})\eta^{\sigma\nu}\tilde{g}_{\sigma\kappa} = 0 \tag{91}$$

Of course this field equation is invariant under the permutation of F and $\tilde{F}$ fields (both metrics and matter-radiation fields) just as the action we started from. We can also contract 90 with $g_{\mu\nu}$ to get:

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \tag{92}$$

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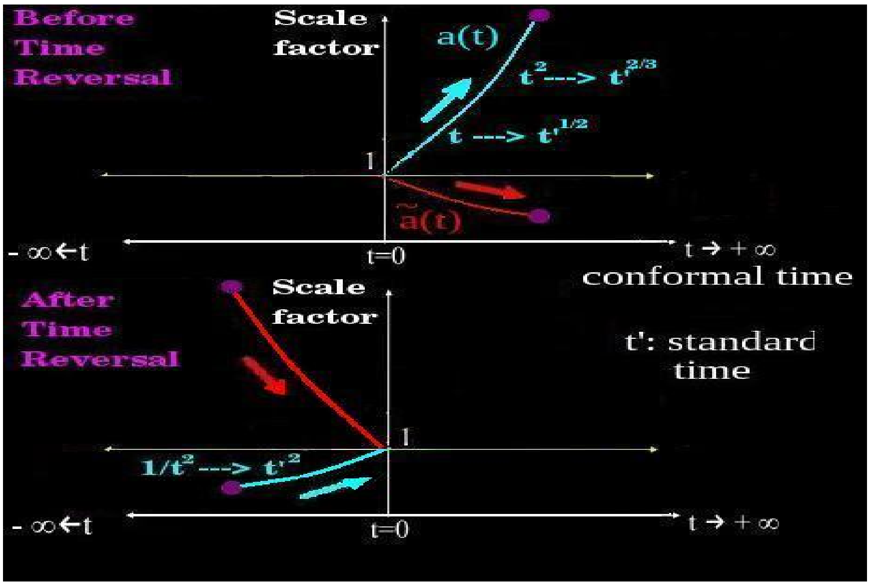


Fig. 1. Evolution laws and time reversal of the conjugate universes, our side in blue

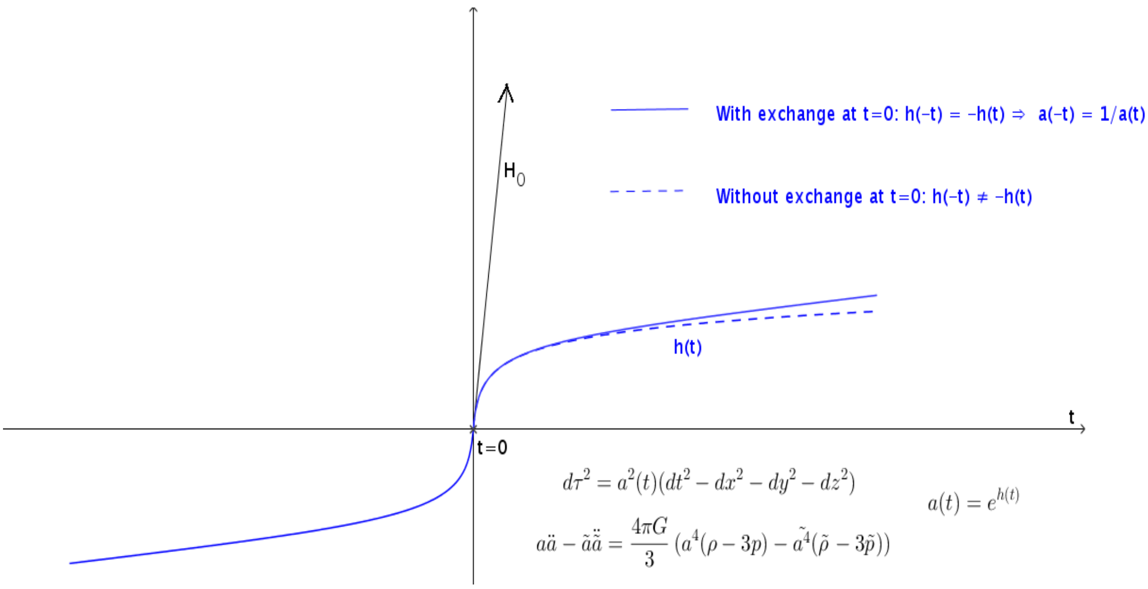


Fig. 2. h(t)

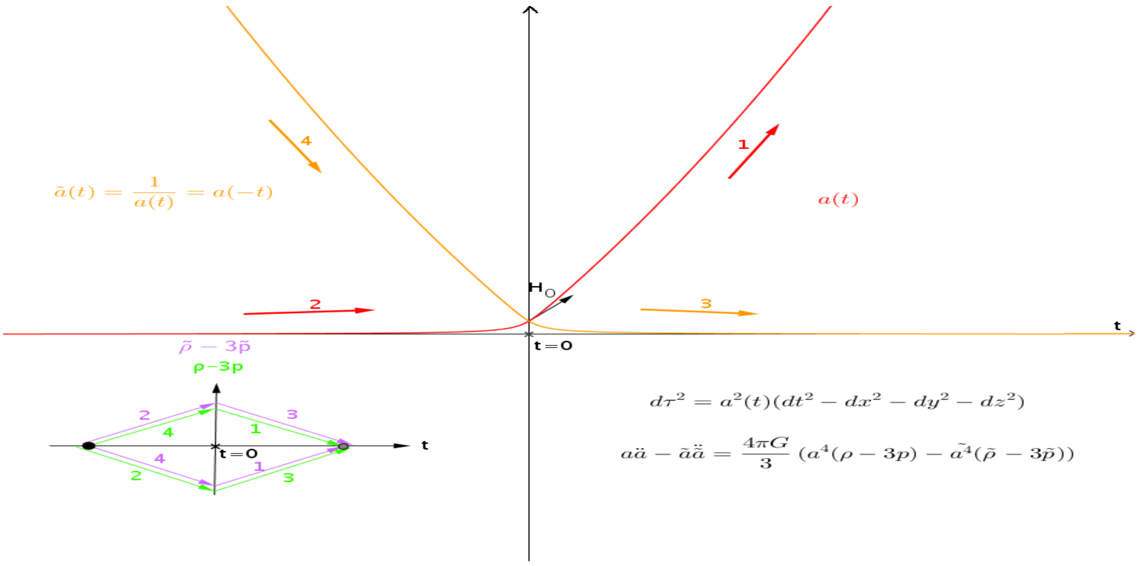


Fig. 3. Scale factors and densities evolution

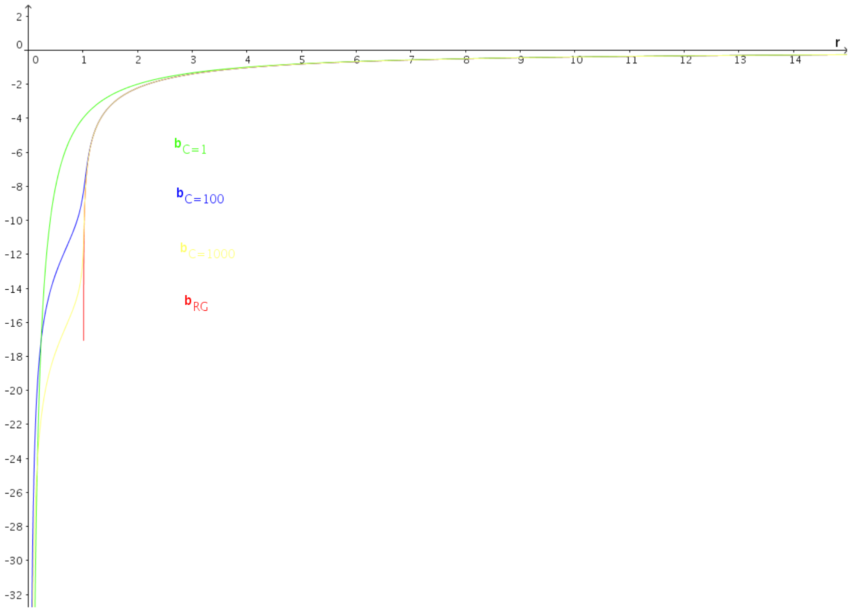


Fig. 4. b(r) near the Schwarzschild radius (r=1) for various C values